

**Project 1**

From: [Investment.colleague@work](mailto:Investment.colleague@work)  
To: [Student@CA2.exam](mailto:Student@CA2.exam)  
Subject: Historical Market Data Analysis

To increase our market share of endowment products we are considering offering equity-linked policies. We have identified two indices that we are prepared to use and need a thorough comparison of their historical data as well as a simple method to simulate these indices.

The first index is a broad based stock index. The second is a commodities index. Historical observations for each index are compiled in the attached spreadsheet.

Our current computer systems are able to simulate normal random variables. For this product we would need to simulate possible index return paths. Parameterise the Normal model for each of the indices, assuming that returns are normal (or lognormal), and simulate a set of possible index return paths.

I look forward to seeing your comparison at Monday's planning meeting.

.

**Core reading:**

The following core reading may be useful for this project:

CT3: for histograms, mean/variance/moments/skewness, distributions, sample statistics, estimators

CT5: for index-linked annuities

CT8: for the lognormal model, stochastic simulation

**Instructions to CA2 candidate**

- (i) Read the email above, which describes the background to this project.
- (ii) Copy the indices data to a new worksheet (“Data preparation”) and then carry out the following investigation using an Excel spreadsheet.
  - (a) Re-base both series of data, with 100 representing the value of the data series on the first date for which there is data. Plot the data appropriately.
  - (b) Calculate the daily geometric returns, and give figures for the mean, variance, skewness and kurtosis of the geometric returns. Comment on your results.
  - (c) Calculate the correlation co-efficient for the relationship between the daily returns for the two indices and plot an appropriate scatter diagram. As well, plot a histogram of each index return against a normal distribution. Draw any conclusions you think relevant about the market.
  - (d) What values of  $\mu$  and  $\sigma$  would you use to simulate an index return path, if you assume index returns follow a normal (or lognormal) distribution? Produce a set of simulated paths, and provide some statistical analysis of your simulations including a range of percentiles.

*You are given the following formulae:*

The skewness of a continuous probability distribution is defined as:

$$S = \int_{-\infty}^{\infty} (x - \mu)^3 f(x) dx$$

It is a measure of the extent to which a distribution is asymmetric about its mean. For example, the normal distribution is symmetric about its mean and therefore has zero skewness, whereas the lognormal distribution is positively skewed.

The kurtosis of a continuous probability distribution is defined as:

$$K = \int_{-\infty}^{\infty} (x - \mu)^4 f(x) dx$$

It is a measure of the “peakedness” or “pointedness” of a distribution.

*Note that there are other formulae relevant to this project in the Formulae and Tables for Actuarial Examinations.*

- (iii) Prepare a set of five summary pages. You should cover the following subjects:

Data

Graph the re-based data and comment.

Analysis

Outline the statistics for the series and comment.

Results

Comparison of Indices: correlation/relation between the indices.  
Comparison of Indices with the Normal Distribution

Conclusions

State your conclusions about the two indices and comment on the simulated results and possible next steps.