

Advantage of Quantile-based risk measure over VaR: An elementary review

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Abstract: Here we discuss quantile-based risk measures (QBRMs) that have recently been developed in the financial risk and actuarial/insurance literatures. The measures considered include the Value-at-Risk (VaR) and its limitation. After that we are trying to focus on coherent risk measures theoretically like CTE, and expected short fall/ Expected tail loss. We discuss and compare the properties of VaR and one of the other coherent risk measure named Expected tail loss, and point out the limitation of the VaR over coherent risk measure and shown that VaR is seriously flawed. We then discuss how QBRM can be estimated in a graphical/pictorial representation, and discuss the way the measure might be applied to insurance risk and other financial risk problems.

Key Words: VaR, QBRM, Limitation of VaR, Coherent Risk Measure. Tail VaR, Expected Shortfall, and Expected tail loss.

Introduction

Financial risk measurement has been one of the main prerequisites of actuaries and insurance practitioners for a very long time. Financial risk management is involved for many different types of insurance problems these are determination of reserves or capital, the setting of premiums and the estimation of magnitudes such as expected claims, expected losses and probable maximum losses; they also manifest themselves implicitly in problems involving shortfall and ruin probabilities. In each of these cases, we are interested, explicitly or implicitly, in quantile of some loss function or, more generally, in quantile-based risk measures (QBRMs). VaR emerges in the mainstream financial risk management (FRM) area. And after the development of a number of newer risk measures, of which the best known are coherent risk measures. Increased interest in risk measurement also arises from deeper background developments, such as: the impact of financial engineering in insurance, most particularly in the emerging area of alternative risk transfer (ART); the increasing securitization of insurance-related risks; the increasing use of risk measures in regulatory capital and solvency requirements; the trend toward convergence between insurance, banking and securities markets, and the related efforts to harmonize their regulatory treatment; and the growth of enterprise-wide risk management.

We draw on both the mainstream FRM literature and the actuarial/insurance literature. Both literatures have witnessed important developments in this area, but the amount of cross-fertilization between them has also been curiously imbalanced, as the actuarial/insurance community has tended to pick up on developments in financial risk management much more quickly than financial risk managers have picked up on developments in actuarial science. Indeed, important developments in the actuarial field – such as the theory of distortion risk measures – are still relatively little known outside actuarial circles. In comparing the various risk measures and discussing how they might be estimated and applied, we wish to make three main arguments, which will become clearer as we proceed. There are many QBRMs that have respectable properties and are demonstrably superior to the VaR, but the choice of 'best' risk measure(s) is a subjective one that can also depend on the context. The estimation of any QBRM is a relatively simple matter, once we have a good VaR estimation system.

This is because the VaR is itself a quantile, and any calculation engine that can estimate a single quantile can also easily estimate a set of them, and thence estimate any function of them. This implies, in turn, that it should be relatively straightforward for institutions to upgrade from VaR to more sophisticated risk measures.

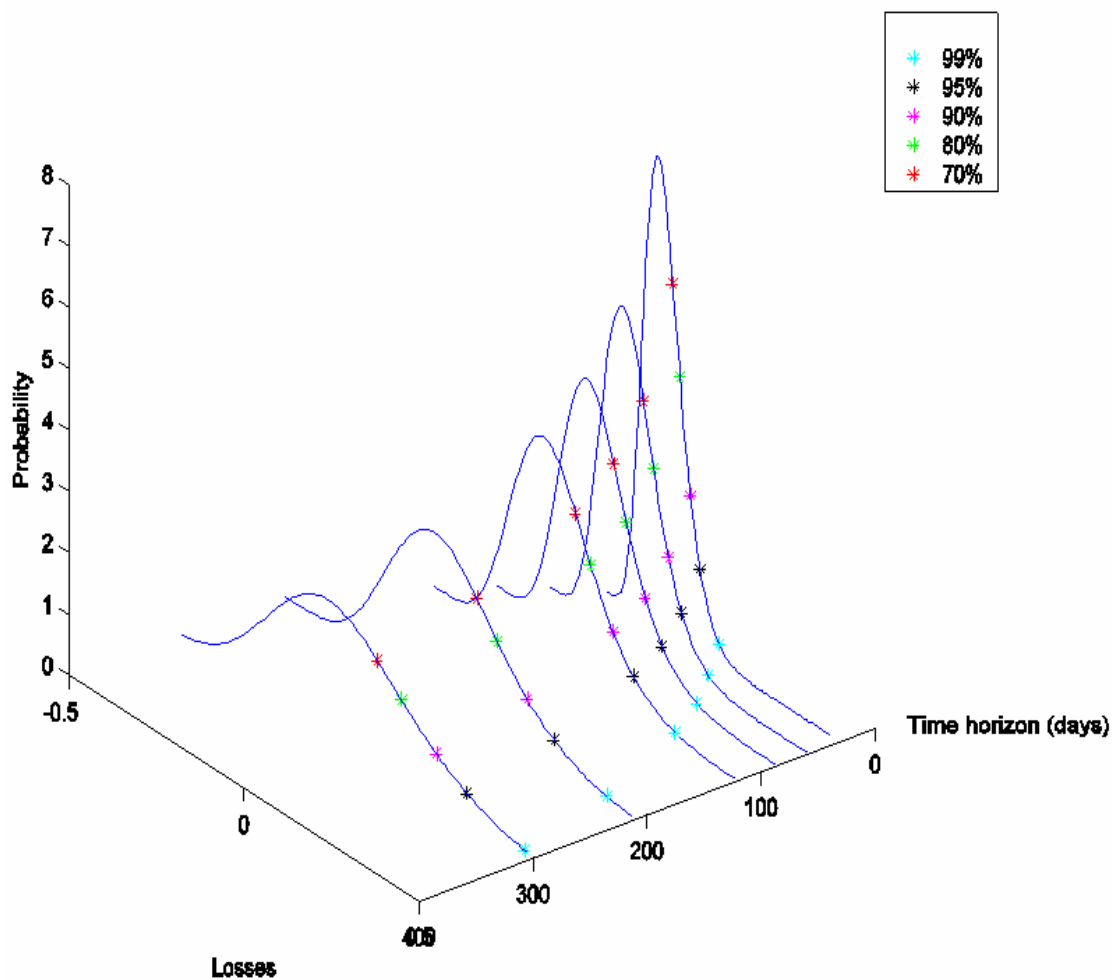
Insurance risk measurement problems are often extremely complex. This complexity is due to many. For reasons of space we restrict ourselves to QBRMs and ignore other types of risk measure (e.g., the variance, semi-variance, mean absolute deviation, entropy, etc.). We also have relatively little to say on closely related risk measures that are well covered in the actuarial literature, such as premium principles, stop-loss measures and stochastic ordering. A good case in point here is the slowness with which axiomatic theories of financial risk measurement – of which the theory of coherent risk measures is the most notable example – have been accepted across the FRM community, despite highly persuasive arguments that coherent measures are superior to the VaR. This slowness to adopt superior risk measures seems to be due to the fact that many FRM practitioners still do not understand the axiomatic theories of financial risk measures, and has led to the patently unsustainable situation that the VaR continues to be the most widely used risk measure despite the fact that it is now effectively discredited as a risk measure different factors (which we will address in due course), and implies that the overwhelming majority of insurance risk measurement problems need to be handled using stochastic simulation methods.

Visit to Value at Risk (VaR)

In common business term Value at risk expressed as “What is the loss level that we are X% confident it will not be exceeded in N business days?” On the other hand Value at risk defined as a statistical measure of possible portfolio losses– A percentile of the distribution of outcomes. Value at Risk (VaR) is the amount of loss that a portfolio will experience over a set period of time with a specified probability under normal market conditions. Thus, VaR depends on some time horizon and a desired level of confidence

Other relevant points about VaR

- VaR depends on the choice of time period (t) and a probability (x). Without knowing t and x , a value at risk number is meaningless
- Longer t and lower x lead to higher VaR
- Implicit assumption is that the portfolio is constant over the time period t . t should be chosen to reflect portfolio turnover



Identifying the Market Factors

There are several methods to calculate VaR, but the first step is to identify the "market factors". Market factors are the variables that impact the value of the portfolio like stock prices, interest rates, exchange rates, pricing of product, product value of insurance policy etc

VaR is usually defined as: "VaR is the maximum potential loss that a portfolio can suffer within a fixed confidence level (cl) during a holding period."

$$VaR_{cl}(X) = \sup\{x \mid P[X \geq x] > cl\}$$

This definition can be misleading because VaR does not represent the "maximum" loss - a portfolio can lose much more than suggested by VaR depending on the shape of the tail of the distribution.

A better definition of VaR:

"VaR is the minimum potential loss that a portfolio can suffer in the 100(1-cl) % worst cases during a holding period.

But VaR is not the most appropriate measure to describe the risks associated with holding a certain position.

Other Methods available for Calculating VaR

1. Parametric simulation
2. Extreme value theory
3. Back testing
4. Monte Carlo simulation - Generate scenarios to determine changes in Portfolio value
5. Historical simulation- Apply recent experience to current portfolio
6. Variance and co-variance method- Assume a normal distribution and use the Statistical properties to find VaR

This type of calculations can be misleading because VaR does not represent the "maximum" loss - a portfolio can lose much more than suggested by VaR depending on the shape of the tail of the distribution. Now we will show some limitations of VaR on the basis of risk measure for different type of markets including insurance market.

Limitation of Value at Risk

1. VaR was developed to represent single number, which could provide the information, which this property is the mathematical equivalent of the diversification effect. For risk measures that are not sub-additive may happen that diversified portfolio require more regulatory capital than less diversified portfolios. Especially in the area of credit risk the sub additivity property plays a fundamental role. Could provide the encapsulate information about the risk portfolio.
2. VaR is not the answer all the risk management challenges like risk limit, regulatory capital, internal capital allocation and performance
3. VaR is not the optimal tool for decision making statement
4. VaR does not measure "event" (e.g., market crash) risk. That is why portfolio stress tests are recommended to supplement VaR.
5. VaR does not capture liquidity difference among instruments
6. VaR does not capture the model risk, which is why model reserves are also necessary.
7. VaR does not capture all relevant information about market risk.
8. VaR is not coherent
9. VaR is not sub additive

Coherent Risk Measure:

A coherent risk measure ρ assigns to each loss X a risk measure $\rho(X)$ such that the following conditions are satisfied:

$\rho(tX) = t\rho(X)$	(homogeneity)
$\rho(X) \geq \rho(Y)$, if $X \leq Y$	(monotonicity)
$\rho(X + n) = \rho(X) - n$	(risk-free condition)
$\rho(X) + \rho(Y) \leq \rho(X + Y)$	(sub-additivity)

These conditions guarantee that the risk function is convex, which in turn corresponds to risk aversion:

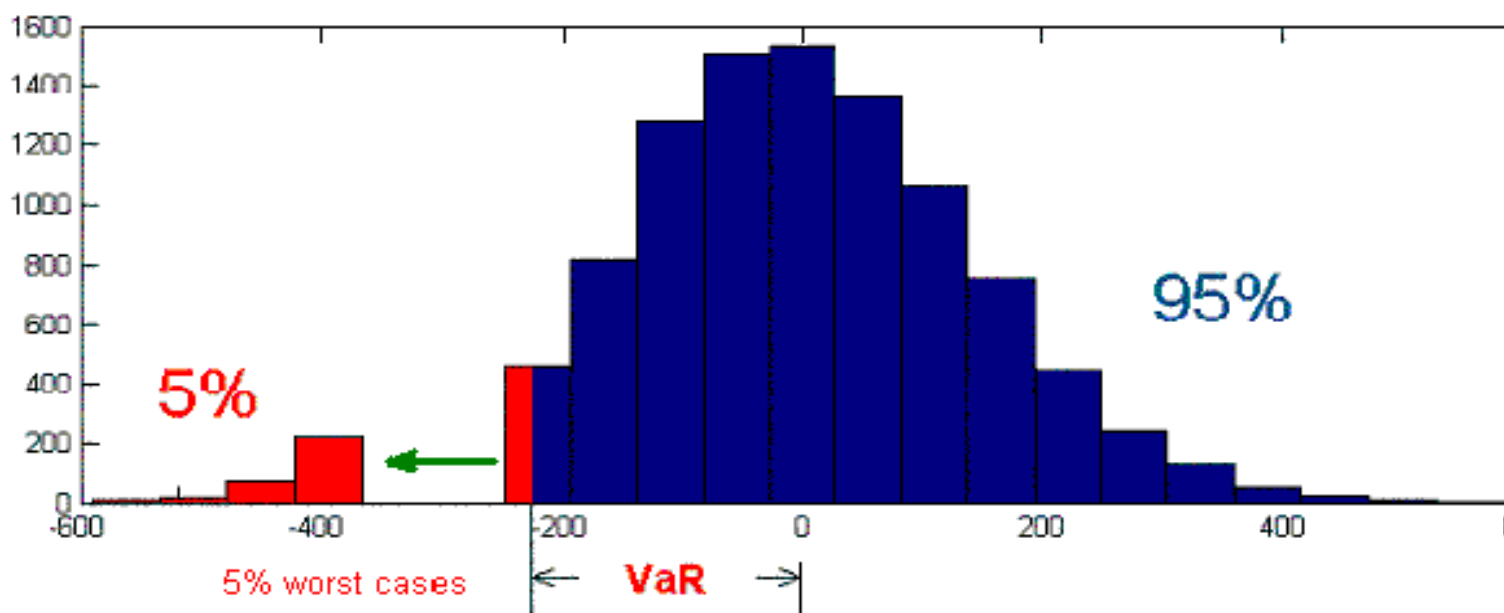
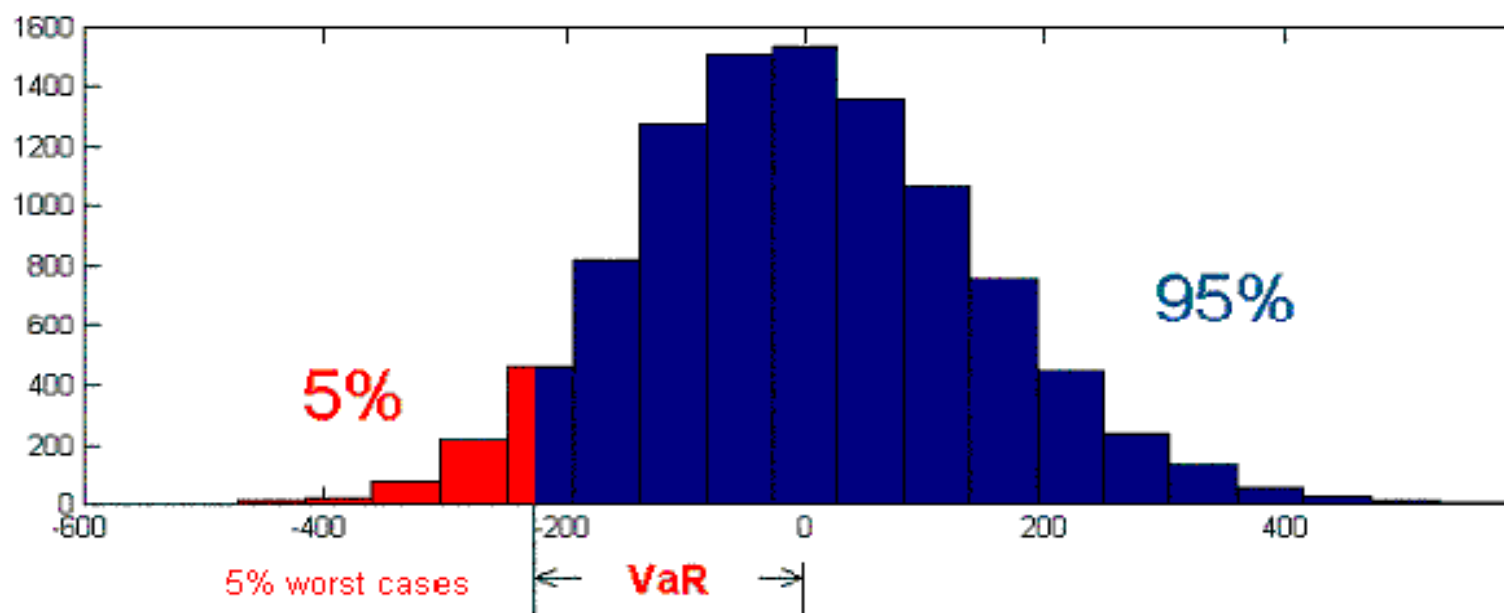
$$\rho(tX + (1-t)Y) \leq t\rho(X) + (1-t)\rho(Y)$$

VaR is not a coherent risk measure because it does not necessarily satisfy the sub-additivity condition. VaR can only be made sub-additive if a usually implausible assumption is imposed on returns being normally distributed. For a sub-additive measure, which ETL is, portfolio diversification always leads to risk reduction, while for VaR; diversification may produce an increase in its value even when partial risks are triggered by mutually exclusive events. Sub-additivity matters because: Adding risks together would give an overestimate of combined risk - a sum of risks can be used as a conservative estimate of combined risk. If regulators use non-sub-additive risk measures to set capital requirements, an insurance company might be tempted to break itself up to reduce its regulatory capital requirements. Non-sub-additive risk measures can inspire traders to break up their accounts, with separate accounts for separate risks, in order to reduce their margin requirements. VaR provides no handle on the extent of the losses that might be suffered beyond the threshold amount. VaR is incapable of distinguishing between situations where losses in the tail are only a bit worse, and those where they are overwhelming. Expected Tail Loss quantifies the losses that might be encountered in the tail.

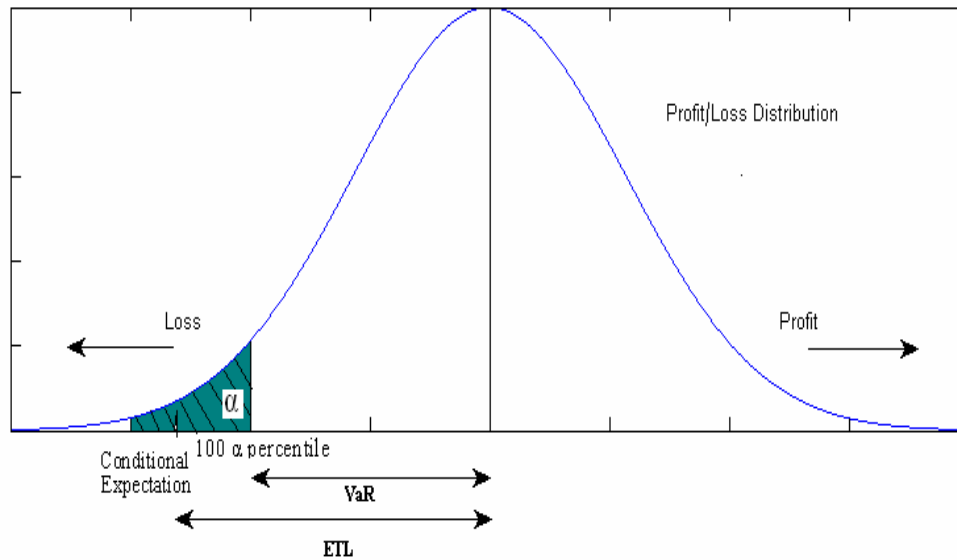
Expected Tail Loss is the expected value of the loss of the portfolio in the $100(1-cl)$ % worst cases during a holding period.

$$ETL_{cl}(X) = E[X \mid X \geq VaR_{cl}(X)]$$

Profit & Loss distribution function

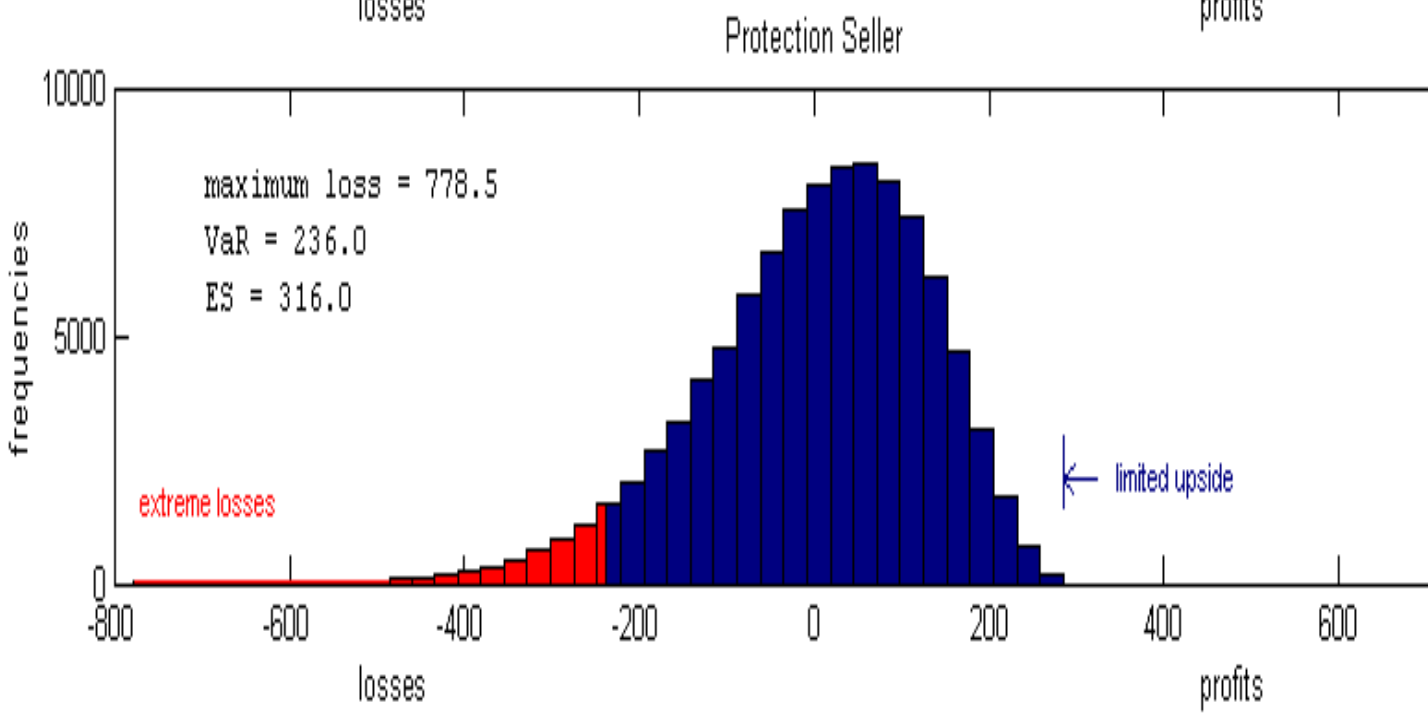
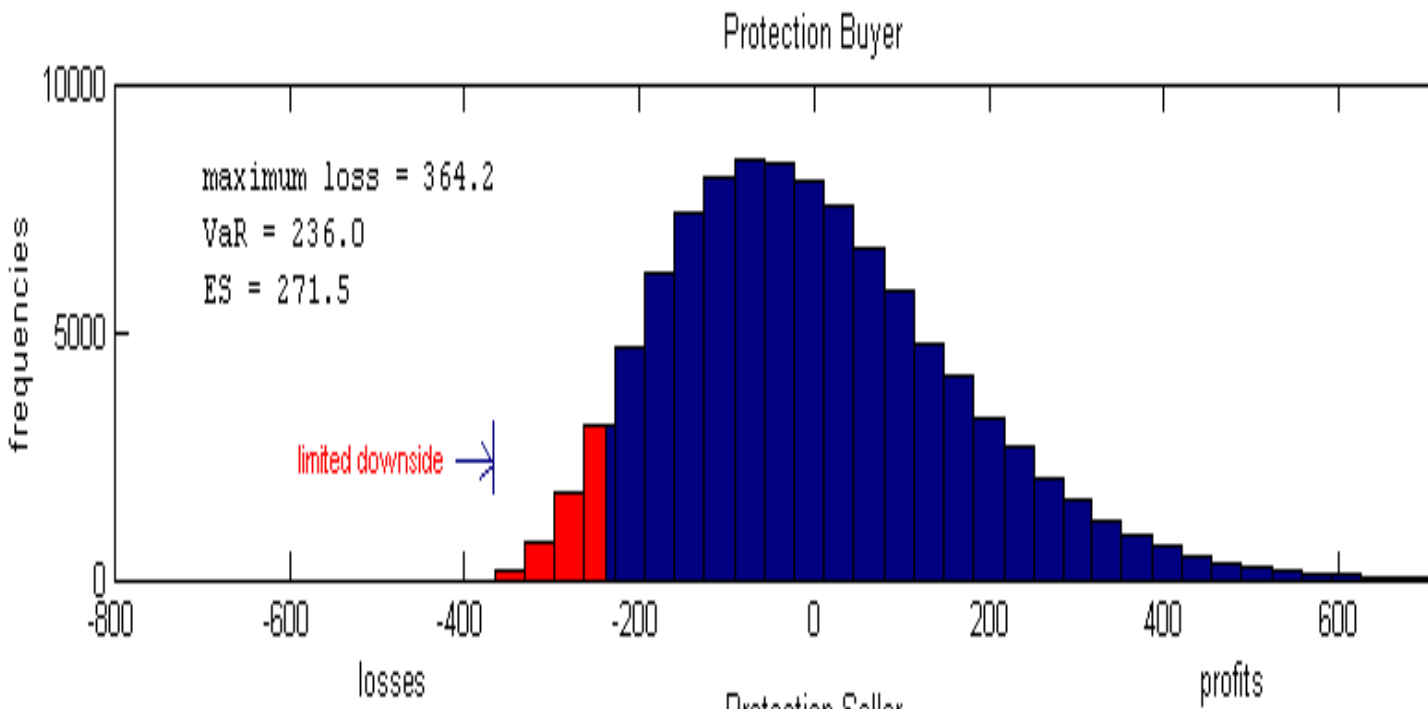


Profit and Loss Distribution:



Advantage Coherent risk measure (Expected Tail Loss) over Value at Risk:

We are faced with two different portfolios with one being clearly riskier than the other. Despite this, VaR tells us that we face exactly the same risk when investing in these two portfolios. This is because VaR ignores extreme losses if they are rare enough. Other way ETL weights extreme losses it can differ between such portfolios and correctly identify the riskier one. ETL is far more sensitive to extreme events no matter how rare they are.



Conclusion:

Though VaR is getting advanced day to day it should not be lost with the (probable) adoption of coherent risk measures into regulatory framework. Superior quality of VaR techniques should yield superior coherent risk measure like Expected Tail Loss forecasts showing that VaR and Expected Tail Loss should be regarded as basic important tools and each is supported by others.

Disadvantages of risk measurement models cannot be ignored and they will continually arise even if we switch from one risk measure to another.

In the developing and developed stock markets and insurance markets simpler VaR models consistently fail their task - provide risk managers with falsely optimistic data about the levels of risk that the financial institutions/insurance institute is exposed to.

Advantages and disadvantages of every model remain with them and that is why knowledge obtained in developing VaR models must not be wasted. VaR techniques can easily be adopted to serve a new "superior" risk measure that is Expected Tail Loss which is broadly called as coherent risk measure. Complete information from the market like insurance market (pricing data, product data) can serve as a solid basis for decision making in financial institutions and reveal actual risk exposure both to investors and regulators.

Financial Risk measurement process has moved on, and we now have many 'respectable' risk measures available in the market to choose from : these include coherent risk measures spectral risk measures, distortion risk measures, and many others. Indeed, in some ways, we now have *too many* risk measures available to us, calculating any of these risk measures is straightforward and we can easily estimate any of the Quantile based risk measures. This way we can upgrade from a Value at Risk to a more 'respectable' risk measure using few more concepts and few lines of code to our program if it is applicable.

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