General Insurance
Ratemaking Principles

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Pricing Considerations - recap

• For each segment, total premium should cover total costs
• For largest risks, use individual ratemaking
• When not feasible, set rates for homogeneous classes
• Other considerations: marketing, operational, regulatory
• Once population is subdivided, calculate rate differential to base
  – If applied multiplicatively, then called relativity
  – If applied additively, then called additive
• “Class” refers to group of insureds for a single or multiple rating variables
  – e.g., age, gender, marital class
• Companies that differentiates risks finer can achieve favorable selection and gain a competitive advantage
Choosing Rating Variables

• Statistical
  – Statistical significance
  – Homogeneity
  – Credibility

• Operational
  – Inexpensive to administer
  – Verifiable and Affordable

• Social and Legal
  • Causality
  • Privacy concerns
One-way classification Ratemaking

- *One-way classification ratemaking* involves grouping risks with similar loss potential and charging different rates for each group to reflect the differences in loss potential.

- For instance, in a motor insurance context, data might be grouped by different driver age bands (18-25, 25-30, etc.) to reflect different levels of risk. Pure premium or loss ratios are then computed for each of these different levels of driver age.

- This process is typically repeated across all rating factors, and the pure premium or loss ratios are compounded (additively or multiplicatively or combination of both) to arrive at the actual premium charged for a particular risk.
Failings of traditional one-way classification ratemaking

• Simple, yet the traditional one-way ratemaking approach is often flawed.

• It does not allow for the effect of other rating variables -- different correlations between factors

• Example:
  – One-way analysis may suggest male drivers are twice as risky as female drivers, and town drivers are twice as risky as countryside drivers.
  – This does not necessarily imply that country male drivers are twice as risky as country female drivers and so on.
  – We need to evaluate the interactions.
One-Way vs Multi-way

- Need to separate “signal” from “noise”
- Avoid double counting correlated exposures
- Address variable interactions
- The signal is usually made up of inter-related effects
- Hence, a multi-variate approach is preferred
Benefits of Multi-Variate Analysis

1. Main benefit - They allow for the effect of other rating variables and the correlations between them.

2. They allow for the “nature of the random process”. Many multivariate methods allow various distributional assumptions to be included within the models. For example:
   - Claim frequency can be modeled assuming a Poisson distribution
   - Claim severity can be modeled assuming a Gamma distribution
   - Probability of claim can be modeled assuming a Binomial distribution

3. They allow consideration of the interaction or interdependency between two or more rating variables. This is when the effect of one variable varies according to the levels of another variable. For example, motor claim frequencies for male and female drivers are known to vary differently with driver age.

4. More sophisticated methods produce detailed information on:
   - Model diagnostics
   - Credibility of results
   - Statistical appropriateness of the fitted model.
Linear Models

• $Y = X\beta + \varepsilon$ is a typical formulation of a linear model
• $Y$ is the observation (the loss cost for an exposure) being predicted
• It is considered to be random
• $X$’s are the Predictor variable -- age, sex, etc
• $\beta$’s are the parameters to be estimated
• $\varepsilon$ is the error in the prediction
Underlying Assumptions

- A standard linear model implies a set of assumptions (that may not be valid)
- Random Component: observations are independent and from a normal distribution with common variance
- Systematic Component: the predictor variables are related as a linear sum, i.e. \( \eta = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = E(Y) \)
- The Link Function is the identity function
Generalized Linear Models

• What if Y cannot be predicted by the *additive* combination of the X variables -- for example if the predictors combine *multiplicatively*.  
  \[ E[Y] = (\beta_1 x_1) \times (\beta_2 x_2) \times (\beta_3 x_3) \times \ldots \]

• Need to use a Generalized Linear Model
  
  \[ E[Y] = \mu = g^{-1}(X.\beta + \xi) \]
  
  \[ \text{Var}[Y] = \phi.V(\mu) / \omega \]
GLM Assumptions

• GLM formulation allows more lenient assumptions
• The predictor variables are still related as a linear sum
• Link Function: The expected value of $Y$ is equal to a transformation of the linear predictor, i.e. $E[Y] = g^{-1}(\eta) = g^{-1}(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)$

• For example: using a log-link where $g(x) = \ln(x)$, $E[Y] = \exp(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)$
GLM Formulation

• **X** defines the explanatory variables to be included in the model
  – could be continuous variables - "variates"
  – could be categorical variables - "factors"

• **\( \beta \)** contains the parameter estimates which relate to the factors / variates defined by the structure of **X**

• Invert the “link function” to obtain the predicted value of the random variable
Choosing Distributions

• Standard Linear Model ("normal" assumptions) is probably the easiest version of GLM with "unity" link function

• GLM requires that Y be from a distribution from the exponential family (e.g., Normal, Poisson, Gamma, Exponential, Binomial, Negative Binomial, Beta, etc)

• Strategy - continuous distributions for severity and discrete distributions for frequency
Meaning of Linear

• Linear model \( Y_i = \mu_i + \text{error} \)
• \( \mu_i \) based on linear combination of measured factors
• Which factors, and how they are best combined is to be derived
• Which one of the following is Linear?
  • \( \mu_i = \alpha + \beta \cdot \text{age}_i + \gamma \cdot \text{age}_i^2 + \delta \cdot \text{height}_i \cdot \text{age}_i \)
  • \( \mu_i = (\alpha + \beta \cdot \text{age}_i) \cdot \exp(\delta \cdot \text{height}_i \cdot \text{age}_i) \)
Model Building

• How are GLMs used in practice?
  – Typically, GLMs are fitted to each claim type (for example, accidental damage, third party, etc.)
  – This is commonly done in two ways:
    • Model fitted on overall claims cost directly for each claim type
    • Models fitted on claims frequency and severity separately, before combining both models to create the “risk premium” or expected claims cost model for that claim type
  – The latter approach requires more work but is preferable, because it allows separate analyses of frequency and severity trends. For example, frequency and severity may exhibit opposing trends, which would not be picked up in the “direct” approach.
Tests for Appropriateness

• How do we determine appropriateness of model fit?
  – Use only those factors which are predictive
    • standard errors of parameter estimates
    • F tests / $\chi^2$ tests on deviances (p-values)
    • consistency over time
    • human intuition
  – Make sure the model is reasonable
    • residual plots (histograms / Q-Q / residual vs fitted value etc)
    • leverage
Some Test Questions

• Are ordered categorical variables well behaved?
• Can you believe it, given correlations with other factors?
• Can the underwriters believe it?
• How different is it to the one-way?
• What does this factor do in other frequency/amounts models and for other claim types?