Credit Risk - Merton KMV and Altman Z Score Model

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Poll Question 1
Objective

- The purpose of this project is to conduct a **Quantitative Analysis** of the Credit Risk affecting companies in the different sectors in India, using various measurement models available.

- To study the **existing structural Credit Risk Models** for corporate defaults with a focus on the Merton/KMV Model and Altman Z Score Model.
What is Credit Risk?

Credit Risk is the possibility of a loss resulting from a borrower's failure to repay a loan or meet contractual obligations. Traditionally, it refers to the risk that a lender may not receive the owed principal and interest.

How it is Modelled?

For the purpose of our analysis, we will be concentrating on Structural Models.
The Merton Model

- Structural Model – Provides a relation between default risk & capital structure.
- The Model treats a company’s equity as a call option on its assets with strike price as Debt maturing in T years.
- Uses the Black Scholes Option pricing methodology.

Consider a Balance Sheet of a company:

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>D</td>
</tr>
</tbody>
</table>

Payoffs at time T:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Equity Holder</th>
<th>Bond Holder</th>
</tr>
</thead>
<tbody>
<tr>
<td>A &gt; D</td>
<td>A - D</td>
<td>D</td>
</tr>
<tr>
<td>A &lt; D</td>
<td>0</td>
<td>A</td>
</tr>
</tbody>
</table>
The Merton Model (Continued...)

Market value of firm’s asset value follows a geometric Brownian motion:

$$\frac{\partial V_A}{\partial t} = \mu V_A \partial t + \sigma_A V_A \partial z \quad \ldots (i)$$

- $V_A$ and $\partial V_A$ firm’s asset value and change in asset value respectively
- $\mu$ and $\sigma_A$ : firm’s asset value drift rate and volatility respectively
- $\partial z$ : Wiener Process

According to Black - Scholes Option Pricing Model :

$$V_E = V_A N(d_1) - e^{-rT} X N(d_2) \quad \ldots (ii)$$

- $V_E$ = Market Value of the firm’s Equity
- $X$ = Book Liability due at time $T$

$$d_1 = \frac{\ln \frac{V_A}{X} + (r + \frac{\sigma_A^2}{2}) T}{\sigma_A \sqrt{T}} \quad d_2 = d_1 - \sigma_A \sqrt{T}$$

- $r$ : Risk-free Rate of Return.
Estimating Firm’s Asset Value & Asset Volatility

- Black-Scholes option pricing model has two unobserved quantities: $V_A$ and $\sigma_A$
- Equity is interpreted as a call on the Asset Value: $E_t = \text{call}(t, V_t)$
- By differentiating: $\partial E_t = \partial \text{call}(t, V_t)$
  $\partial E_t = \mu_E E_t \partial t + \sigma_E E_t \partial W_t$ \hspace{1cm} (iii)
  - $\mu_E$: Expected Continuously compounded return on Equity.
  - $\sigma_E$: Volatility of the equity value
  - $\partial W_t$: Standard Brownian Motion
- From Ito’s Lemma:
  $\partial \text{call}(t, V_t) = (......) \partial t + \frac{\partial \text{call}}{\partial V} \sigma_A V_t \partial W_t$ \hspace{1cm} (iv)

From Equation (iii) & (iv):
$$\sigma_E E = \Delta_{\text{call}} \sigma_A V ...(v)$$
where $\Delta_{\text{call}} = N(d_1)$: the Delta Greek of the call

Finally we can solve equation (ii) & (v) using Solver function in excel, to calculate $V_A$ & $\sigma_A$ respectively.
Poll Question 2
Probability of Default (PD)

\[ P(V_A < X_t) = \Phi \left( \frac{\ln X_t - \ln V_A - \left( \mu - \frac{\sigma^2}{2} \right)(T-t)}{\sigma(T-t)} \right) \]

\[ = N\left[ - \frac{\ln \frac{V_A}{X_t} + \left( \frac{\mu - \sigma_A^2}{2} \right)}{\sigma_A \sqrt{t}} \right] \]

\[ = N\left( - d_2 \right) : \text{CDF of the Normal distribution} \]

To find the PD of a listed firm, below information will be required:

- Financial Statements.
- Market prices of the firms’ Debt & Equity.
- Subjective Appraisal of the firms’ prospect & risks.
Merton KMV Model

- It was developed in late 80’s and acquired by Moody Analytics in 2002.
- Derived from Merton Model
- Based on Structural approach to calculate Probability of Default (PD)
  - Uses firm’s stock price to determine the Value of Equity.
  - Market information contained in firm’s stock price & Balance Sheet are translated into an implied risk of default.

Assumptions

- Model works in accordance with the basic hypothesis of Merton Option Pricing Model.
- If A < D of a company – the company Defaults.
- Default Point is the sum of Short Term Debt and 50% of Long Term Debt
- Return on assets are fixed & known.
KMV defines the default point (DP) as the asset value at which the firm defaults and find that it lies somewhere between total debt (LTD) and current, or short-term debt (STD).

Default point, \( d = \text{short-term debt} + \frac{1}{2} \text{long-term debt} \)

Distance to Default (DD) - The number of standard deviations the asset value is away from default:

\[
DD = \frac{\text{Market Value of Assets} - \text{Default point}}{\text{Market Value of Assets} \times \text{Asset Vol.}}
\]

Bohn and Crosbie (2002)
Probability of Default Visualization

\[ V_T = V_0 \exp\left(\mu - \frac{\sigma_v^2}{2}\right)T + \sigma_v \sqrt{T} Z_T \]

\[ E(V_T) = V_0 e^{rT} \]

Default Region

Possible path of Asset value over the time period

\( \sigma_v \)
Poll Questions 3
Advantages and Limitations

**Advantages**

- High appealing feature of connecting Credit Risk to Structural Variables.
- Provides Economic Interpretation.
- Incorporates market based Option Pricing Methods such as Black Scholes Model.
- Adjusts Credit Cycle and quickly reflect any deterioration in Credit quality.

**Limitations**

- The assumption that the company can default only at time T and not before.
- The model does not distinguish among different types of debt according to their seniority, collaterals, covenants or convertibility.
- The model assumes that once management puts a debt structure in place, it remains unchanged even if the firm’s assets have increased.
- The assumption of a Constant and Flat Term structure of Interest Rate.
Implied Credit Spread

- Present Value of promised Debt: \( \bar{D}_0 = \bar{D}e^{-rT} \)

- Market Value of Debt at \( t = 0 \). (Where \( y \) is the yield to maturity on the debt): \( \bar{D} = \bar{D}e^{-yT} \)

- The yield to maturity on the debt is defined implicitly by: \( D_0 = \bar{D}e^{-yT} = \bar{D}_0 e^{(r-y)T} \)

- Recalling the definition of Leverage: \( L = \frac{\bar{D}_0}{A_0} \)

- Credit spread implied by the Merton Model:

\[
S = y - r = -\frac{\ln[N(d_2) + \frac{N(-d_1)}{L}]}{T}
\]

depends on the Leverage \( L \), the asset volatility \( \sigma_A \) and time to repayment \( T \)
Excel Calculation for Probability of Default for the company: Jindal Steel
ALTMAN Z Score

- **Formula of 5 basic financial ratios** which help us to determine the financial health of a company.

  \[ Z\text{-Score} = 1.2(A) + 1.4(B) + 3.3(C) + 0.6(D) + 0.99(E) \]
  
  where,

  - **A** = Working Capital / Total Assets (Measures liquidity of firm)
  - **B** = Retained Earnings / Total Assets (measures accumulated profits compared to assets)
  - **C** = Earnings Before Interest & Taxes / Total Assets (measures how much profit the firm’s assets are producing)
  - **D** = Market Value of Equity (Mkt. Cap. + Preferred Stock) / Total Liabilities (compares the company’s value versus it’s liabilities)
  - **E** = Sales / Total Assets (measures how much the company’s assets are producing in sales).

- **Interpretation of Z-Score Results:**
  - **Z-Score** < 1.81 represents a company in distress (Higher Chance of Default).
  - **Z-Score** between 1.81 and 3.0 represents the “caution” zone (Danger Zone).
  - **Z-Score** > 3.0 represents a company with a safe balance sheet (Away from Danger).
Output of Defaulted Companies Using KMV Model & Altman Z Score:

- **JINDAL STEEL**
Jindal Steel (Merton KMV Model)

- They defaulted on interest payments due on 30\textsuperscript{th} Sept, 31\textsuperscript{st} Oct & 30\textsuperscript{th} Nov in 2016. However, they shortly restructured the loan afterwards.

- This graph indicates the steep decline in cash from FY 2014-15 to FY 2015-16 as per the Financial Statement, however, the Probability of Default (PD) keeps on increasing.

- This graph indicates a negative correlation between Profit After Tax (PAT) and probability of Default (PD) over the financial years. Also, there is a positive correlation between Finance Cost and PD.
# Jindal Steel (Altman Z Score Model)

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Working Capital / Total Asset</td>
<td>1.2</td>
<td>-0.1159</td>
<td>-0.055</td>
<td>-0.092</td>
<td>-0.0353</td>
<td>-0.1139</td>
</tr>
<tr>
<td>Reserves (Retained Earning) / Total Asset</td>
<td>1.4</td>
<td>0.3204</td>
<td>0.3078</td>
<td>0.2811</td>
<td>0.269</td>
<td>0.3791</td>
</tr>
<tr>
<td>EBIT / Total Asset</td>
<td>3.3</td>
<td>0.1266</td>
<td>0.1029</td>
<td>0.0846</td>
<td>0.0892</td>
<td>0.0407</td>
</tr>
<tr>
<td>Net Sales / Total Asset</td>
<td>0.99</td>
<td>0.4028</td>
<td>0.3797</td>
<td>0.3183</td>
<td>0.2964</td>
<td>0.2099</td>
</tr>
<tr>
<td>MVO of Equity / Total Debt</td>
<td>0.6</td>
<td>3.5455</td>
<td>1.6666</td>
<td>1.2001</td>
<td>0.5486</td>
<td>0.2306</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Financial Year</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Score</td>
<td>3.257</td>
<td>2.084</td>
<td>1.6</td>
<td>1.188</td>
<td>0.876</td>
<td></td>
</tr>
<tr>
<td>Probability of Default (PD) *</td>
<td>0.06%</td>
<td>1.85%</td>
<td>5.48%</td>
<td>11.74%</td>
<td>19.05%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Remarks</th>
<th>Away from Default</th>
<th>Danger Zone</th>
<th>Higher Chance of Default</th>
<th>Higher Chance of Default</th>
<th>Higher Chance of Default</th>
</tr>
</thead>
</table>

*PD = NORMSDIST(-Z Score)
Advantages and Limitations

Advantages

• It uses fives financial ratios that are calculated on the basis of seven financial data which is easily available from the financial statements of any company.

• Being a quantitative model, it is very easy to draw insights from the outcome.

• Investors usually use it to measure the solvency of a company in order to decide whether to invest or not in that company.

Limitations

• It can only forecast the likelihood of failure only if the company is comparable to its database.

• The scoring system does not work well for new or emerging companies as their earnings are too low and will end up indicating high risk.

• The model works on garbage in garbage out method and so a misleading company financial will result in misleading Z score
Conclusions

• Altman Z score gives a **good fundamental analysis** of whether a company is risky or not.

• It is **difficult to time the year of default** by observing the PDs.

• A fundamental analysis of the company along with the **structural model is necessary to avoid risky investments**.

• Model cannot appropriately capture **Wilful Default** i.e., Not meeting the repayment obligation in spite of having the capacity to do so. example – Bombay Rayon (Year 2013 )
CPD Question 1
CPD Question 2
THANK YOU

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