# Institute of Actuaries of India

ACET January 2018 Solutions

## Mathematics

1. C. $[n(n+1)(2n+1)]/6$ is the sum of the squares of $n$ positive integers. Hence, $n(n+1)(2n+1)$ is a multiple of 6. The other choices are not always true.

2. A. $\log_k x \times \log_5 k = 3 \Rightarrow \log_5 x = 3 \Rightarrow x = 5^3 = 125$.

3. A. The following information given.

| $x$ | $F(x)$ |
|---|---|
| 2.5 | 0.3554 |
| 3.5 | 0.5221 |

Using linear interpolation, we have $\frac{3.5-3.0}{0.5221-F(3.0)} = \frac{3.5-2.5}{0.5221-0.3554}$ , giving $F(3.0) =$ 0.4387 approximately.
Alternative method: Since 3 is the mid-point of 2.5 and 3.5,

$$F(3.0) = \frac{0.3554 + 0.5221}{2} = 0.4387.$$

4. D. $\frac{x-2}{3x+1} > \frac{x-3}{3x-2}$ implies $\frac{x-2}{3x+1} - \frac{x-3}{3x-2} > 0$, giving $\frac{7}{(3x+1)(3x-2)} > 0$ .
This implies that either $(3x+1) > 0 \ and \ (3x-2) > 0$ or
$(3x+1) < 0 \ and \ (3x-2) < 0$
Thus for the inequality to hold $x\varepsilon \left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$.

5. A. If $\alpha$ and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then the sum of the roots is $-1$ and product of the roots is 1.Thus, if $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of an equation, then the sum of the roots is $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -1$ and the product of the roots is $\frac{1}{\alpha\beta}$ = 1. The only quadratic equation satisfying these conditions is choice A.

6. B. In the expansion of $\left(x + \frac{1}{x^3}\right)^{17}$, the general term is

$$S_{r+1} = C_r^{17} x^{17-r} \left(\frac{1}{x^3}\right)^r = C_r^{17} x^{17-4r}.$$

The coefficient of $x^5$ is the coefficient of $x^{17-4r}$ when $r = 3$, which is $C_3^{17} = 680$.

7. D. $\lim_{x\to a} \frac{x \sin a - a \sin x}{x-a} = \frac{0}{0}$ form. By L' Hospitals rule we have the given limit equal to $\lim_{x\to a} \sin a - a \cos x$= $\sin a - a \cos a$.

8. C. For the curve $y = x^3 - 3x + 1$, $y' = 3x^2 - 3$ and $y'' = 6x$. Equating $y'' = 0$ gives $x = 0$. Since $y''' > 6$, it is a minima of $y'$.

9. A. The maximum/minimum value of the function

$$f(x) = \frac{(1+x)^{0.3}}{1+x^{0.3}} \text{ in the interval } 0 \leq x \leq 1$$

is obtained by equating the first derivative of $f(x)$ to 0.

$$f'(x) = \frac{(1+x^{0.3})(0.3)(1+x)^{0.3-1} - (1+x)^{0.3}(0.3)x^{0.3-1}}{(1+x^{0.3})^2} = 0.$$

This implies:

$$(1 + x^{0.3})(0.3)(1 + x)^{0.3-1} - (1 + x)^{0.3}(0.3)x^{0.3-1} = 0$$

Or $(1 + x^{0.3})(1 + x)^{0.3-1} - (1 + x)^{0.3}x^{0.3-1} = 0$

Or $(1 + x)^{-0.7}[(1 + x^{0.3}) - (1 + x)^1x^{-0.7}] = 0$

Or $(1 + x^{0.3}) - (1 + x)^1x^{-0.7} = 0$, giving $x^{0.7} = 1$ *or* $x = 1$.

The second derivative is hard to compute, but it can be seen that $f'(x)$ has the same sign as $(1 + x^{0.3}) - (1 + x)^1x^{-0.7} = 1 - x^{-0.7}$, which is negative in the interval $0 \leq x < 1$. Therefore, $f(x)$ is decreasing over this interval and has a minimum at $x = 1$.

10. B. $u = \sqrt{x^2 + y^2}$, Then $\frac{\partial u}{\partial x} = \frac{1}{2}[x^2 + y^2]^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+y^2}}$

Similarly, $\frac{\partial u}{\partial y} = \frac{1}{2}[x^2 + y^2]^{-\frac{1}{2}}(2y) = \frac{y}{\sqrt{x^2+y^2}}$

Hence, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2 + y^2} = u.$

11. C If $y = x^3 + \tan x$, then $\frac{dy}{dx} = 3x^2 + \sec^2 x$;

$$\frac{d^2y}{dx^2} = 6x + 2\sec x \cdot \sec x \tan x = 6x + 2\sec^2 x \tan x.$$

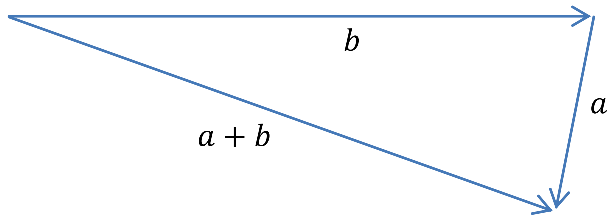
12. A. $\int_{-4}^{4}|x - 3|\,dx = \int_{-4}^{3}(3 - x)dx + \int_{3}^{4}(x - 3)dx = \int_0^7 ydy + \int_0^1 ydy = 25.$

13. B. $\frac{x+1}{(x+2)(x+3)} = \frac{P}{(x+2)} + \frac{Q}{(x+3)}$. This implies $x + 1 = P(x + 3) + Q(x + 2)$. Putting $x = -3$, we have $Q = 2$ and putting $x = -2$, we have $P = -1$.

$$\int \frac{x + 1}{(x + 2)(x + 3)}dx = \int \frac{-1}{(x + 2)}dx + \int \frac{2}{(x + 3)}dx = -\log(x + 2) + 2\log(x + 3).$$

14. B. $\int_0^\infty x^4e^{-2x^5}\,dx = \frac{1}{10}\int_0^\infty e^{-y}\,dy = \frac{1}{10}.$

15. D. The resultant $\vec{a}+\vec{b}$ has same magnitude as $\vec{b}$, and therefore the three vectors form an isosceles triangle. By considering half-base and one side of the triangle, we have angle between $\vec{a}$ and $\vec{b}$ as $\cos^{-1}\frac{1}{7}$.




16. C. The inverse of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$ is $\frac{1}{7}\begin{bmatrix} 6 & -5 \\ -1 & 2 \end{bmatrix}$

17. A. $B^3 = BABABA = BAAA = BAA = BA = B$.

18. B. $P = \begin{bmatrix} m & m & m \\ m & m & m \\ m & m & m \end{bmatrix} = m\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. The rank of this matrix is 1.

## Statistics

19. B. Arrangement: BBBBGGG or GGGBBBB. The number of ways the boys can sit together and so can all the girls is $2 \times 4! \times 3! = 288$. Required probability $= 288/7! = 288/5040$.

20. A. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)$ (Since $A$ is subset of $B$)
$= P(B)$
This implies $P(B) = 1/3$.
(Alternatively, since $A \subseteq B$, $A \cup B = B$. Therefore, $P(B) = P(A \cup B) = 1/3$.)

21. C. Let M: Male, F: Female, C: Colorblind.

$P(C|M) = 0.10$, $P(C|F) = 0.01$, $P(M) = 0.49$, $P(F) = 0.51$
Find $P(M|C) = \frac{P(C|M)P(M)}{P(C|M)P(M)+P(C|F)P(F)} = \frac{0.10 \times 0.49}{0.10\times0.49+0.01\times0.51} = 0.9057 \approx 0.91$

22. B. Median $= \frac{x+8}{2} = 5$. So $x = 2$. Hence mean is $(15 + 12 + 8 + 2 - 4)/6 = 33/6 = 5.5$.
23. D. Standard deviation of new observations is $s_y = |-4| \times 1.5 = 6$.
24. C. Accident cost over a year would be Rs. $6.5 \times 12 \times 20000 =$ Rs. $1560000$.
25. B. $P(X^2 = 9) = P(X = -3) + P(X = 3) = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$.

26. A. Suppose $M$ is the median of $Y$. Then $P(Y \leq M) = 0.5$.

This implies $P(2X \leq M) = 0.5, or\ P\left(X \leq \frac{M}{2}\right) = 0.5$.

So $1 - e^{-2\times\frac{M}{2}} = 0.5$. This implies $M = -\ln 0.5 = \ln 2$.

27. C. $E(Z^{17}) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} z^{17} e^{-z^2/2} dz = 0$ (because the integrand is an odd function).

28. A. $\text{Var}(Z) = V = p^2V_1 + (1-p)^2V_2$.

$\frac{dV}{dp} = 2pV_1 - 2V_2 + 2pV_2 = 0$. This implies $p = \frac{V_2}{V_1+V_2}$

$$\frac{d^2V}{dp^2} = 2(V_1 + V_2) > 0$$

29. D. $\sum_{i=0}^{1}\sum_{j=1}^{3} c(i+j) = 1$. This implies $c \times 15 = 1$ or $c = 1/15$.

30. B. Cov $(X, n-X) = -\text{Var}(X) = -np(1-p)$.

31. C. Least squares estimate of $a$ is obtained by minimizing $\sum_{i=1}^{n}(y_i - a\,x_i)^2$.

This gives $a = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{58}{39}$.

# Data Interpretation

32. A. $P = \{a, b, d, f\}, Q = \{b, d, c, e\}, R = \{d, e, f, g\}, Q \cap R = \{d, e\}, P \cup (Q \cap R) = \{a, b, d, e, f\}$.

33. D. It is clear from the frequency distribution.

34. A. The percentage of people below 30 years age = 7 +14 +7+7+7 = 42.

35. C. The percentage of people 55 and above = 10+6+6 =22. The percentage of people above 65 = 6+6 = 12. The percentage of people above 55 years will lie between 12 and 22.

36. A. The percentage of people 65 and above is 6+6 =12. The percentage of people in the age group [45, 65) = 14+10 =24. So the ratio is 12:24 = 1:2.

37. D. Inspecting the graph, it can be easily obtained.

38. B. By inspecting the graph, it can be easily obtained that maximum fall occurred in the year 1990.

# English

39. A
40. A
41. B
42. D
43. B
44. A
45. A
46. D
47. C
48. C
49. A
50. A
51. B
52. C
53. D
54. A
55. B
56. A
57. B
58. B
59. C
60. A
61. A
62. D

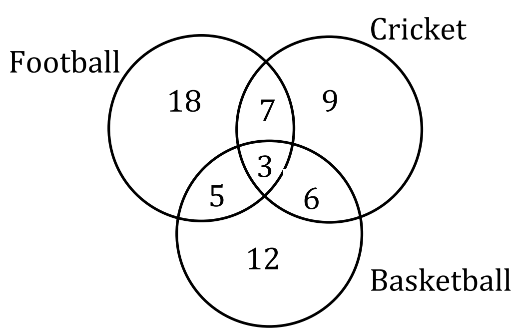
# Logical Reasoning

63. B
64. A
65. C 17 June 2017 (day of this ACET exam) is a Saturday. The number of years in between 17-6-2006 and 17-6-2017 is 11. Number of odd days = 11 + 3 =14 (3 leap years among the 11 years). Since 14 is a multiple of 7, 17 June 2006 must have been a Saturday too.

66. C. M, I, R, A, C, L and E are mapped to N, K, U, E, H, R and L, respectively. Therefore, in this code, RECLAIM would be coded as ULHREKN.
67. C. The correct order is :

| Plant | Cotton | Yarn | Cloth | Saree |
|---|---|---|---|---|
| 2 | 4 | 1 | 5 | 3 |

68. B. An ounce measures weight; the other choices measure length.
69. C.




Number of players who play only one game = 18 + 9 +12 = 39

70. C. The set of all terrorists is a subset of the intersection of the set of guilty and criminals. Hence, some guilty are criminals.