

# Institute of Actuaries of India

ACET June 2019 Solutions

## Mathematics

1. A.  $\frac{\log_{16} 625}{\log_8 125} = \frac{\log_2 5^4}{\log_2 5^3} = \frac{\log_2 5}{\log_2 5} = 1.$

2. D. In order that  $f(x) = \sqrt{\frac{x+2}{x-1}}$  is valid function,  $\frac{x+2}{x-1} \geq 0$  and  $x \neq 1.$

These imply:

Either  $x = -2$ , or  $(x + 2)$  and  $(x - 1)$  must have the same sign.

If  $(x + 2) > 0$  and  $(x - 1) > 0$ , then  $x > 1$

Similarly if  $(x + 2) < 0$  and  $(x - 1) < 0$ , then  $x < -2.$

Putting together, the domain for  $f(x)$  is  $(-\infty, -2] \cup (1, \infty)$

3. B.  $\cos^2 5 + \cos^2 85 = \cos^2 5 + \cos^2(90 - 5) = \cos^2 5 + \sin^2 5 = 1$   
Similarly,  $\cos^2 10 + \cos^2 80 = 1, etc.$

There are 8 such pairs leaving two terms  $\cos^2 45 = \frac{1}{2}$  and  $\cos^2 90 = 0.$

Hence, the sum is  $8\frac{1}{2}.$

4. B.  $f(x) = x^2 - 5; f'(x) = 2x; x_0 = 2.$

Choosing  $i = 0$  in  $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_{i+1})}$ , we have  $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-1}{4} = 2.25.$

5. A.  $f(x) = \frac{1}{1+x}; x$  real

$$f'(x) = -\frac{1}{(1+x)^2}; f'(0) = -1$$

$$f''(x) = \frac{2}{(1+x)^3}; f''(0) = 2$$

$$f'''(x) = 2\frac{-3}{(1+x)^4}; f'''(0) = 2(-3) \text{ and so on.}$$

The Maclaurin's series is:  $f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$

Hence,  $f(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$

6. C.  $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x+3)}$  implies

$$x^2 + 1 = A(x + 3) + B(x - 1)(x + 3) + C(x - 1)^2. \quad \dots (1)$$

Letting  $x = 1$ , in (1) we have,  $A = \frac{1}{2}$  and  $x = -3$ , we have  $C = \frac{5}{8}$

Equating the coefficients of  $x^2$ :  $B + C = 1$  and  $B = \frac{3}{8}$ . Hence  $A + 4B = 2$ .

Alternatively, differentiating (1) wrt  $x$ , we have  $2x = A + 2B(x + 1) + 2C(x - 1)$  and letting  $x = 1$  we have  $A + 4B = 2$ .

7. D.  $\binom{16}{2r} = \binom{16}{3r+1}$  implies  $2r = 16 - (3r + 1)$  [since  $\binom{n}{r} = \binom{n}{n-r}$ ]

Hence,  $r = 3$ . Therefore,  $\binom{4r}{3} = \binom{12}{3} = 220$ .

8. C. The typical term in  $\left(2x^2 + \frac{1}{x}\right)^9$  is  $\binom{9}{r}(2x^2)^r \left(\frac{1}{x}\right)^{9-r}$ .

In order that this term is a constant, we must have  $2r = 9 - r$ , i.e.  $3r = 9$  or  $r = 3$ .

Letting  $r = 3$ , in the typical term, we have  $\binom{9}{3}(2x^2)^3 \left(\frac{1}{x}\right)^{9-3} = \binom{9}{3}(2)^3 = 672$ .

9. B.  $\lim_{x \rightarrow 4} \frac{\log_e x - \log_e 4}{x - 4}$  is in  $\frac{0}{0}$  form.

By L'Hôpital's rule, we have,  $\lim_{x \rightarrow 4} \frac{\frac{1}{x}}{1} = \frac{1}{4}$ .

10. A.  $f(x) = \frac{2x}{\log_e x}$ ,  $x > 0$ . Hence,  $f'(x) = \frac{\log_e x \times 2 - 2x \times \frac{1}{x}}{(\log_e x)^2}$   
 $= \frac{2(\log_e x - 1)}{(\log_e x)^2} > 0$  if  $\log x > 1$  or if  $x > e$ .

11. B. If  $x = r \cos \theta$  then  $x^2 = r^2 \cos^2 \theta$  and  $y = r \sin \theta$ , then  $y^2 = r^2 \sin^2 \theta$ .

$$x^2 + y^2 = r^2(\cos^2 \theta + \sin^2 \theta) = r^2.$$

Hence,  $2r \frac{\partial r}{\partial x} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r} = \frac{r \cos \theta}{r} = \cos \theta$ .

12. A.  $\int x^3 \log_e x \, dx = \log_e x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx = \log_e x \cdot \frac{x^4}{4} - \frac{x^4}{16} = \frac{x^4}{4} \left( \log_e x - \frac{1}{4} \right) + c$ .

13. C.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, d(\sin x) = \frac{\sin^3 x}{3} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2}{3}.$$

14. C. The function  $|x| = -x$  if  $x < 0$  and  $x$  if  $x > 0$ . Hence,

$$\int_{-\infty}^{\infty} \frac{1}{2} e^{-|x|} \, dx = \int_{-\infty}^0 \frac{1}{2} e^x \, dx + \int_0^{\infty} \frac{1}{2} e^{-x} \, dx = \frac{1}{2} + \frac{1}{2} = 1.$$

15. D. Since  $\vec{a} \perp \vec{b}$ ,  $\vec{a} \cdot \vec{b} = 0$ .

$$\begin{aligned} \text{Hence, } (2\vec{i} + m\vec{j} + \vec{k}) \cdot (\vec{b} = \vec{i} - 2\vec{j} + \vec{k}) &= 0 \Rightarrow (2)(1) + (m)(-2) + (1)(1) = 0 \\ &\Rightarrow 2 - 2m + 1 = 0 \Rightarrow 2m = 3 \\ &\Rightarrow m = \frac{3}{2}. \end{aligned}$$

16. B. The projection of  $\vec{a}$  on  $\vec{b}$  is  $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{2+6-6}{\sqrt{1+4+9}} = \frac{2}{\sqrt{14}}$ .

17. D. The rank of the matrix  $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$  is 3, since  $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 4 \neq 0$

18. A. Given that:

$$\begin{aligned} A(x) &= \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}. \\ [A(x)]^{-1} &= \begin{bmatrix} (\cos x & \sin x)^{-1} & 0 \\ (-\sin x & \cos x) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = A(-x). \end{aligned}$$

## Statistics

19. D. Total number of ways =  $\binom{6}{4} + \binom{4}{1}\binom{6}{3} + \binom{4}{2}\binom{6}{2} = 15 + 80 + 90 = 185$ .

20. A.

$$P(A \cap B | A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)}.$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.3 - 0.1 = 0.7.$$

$$\text{So, } P(A \cap B | A \cup B) = \frac{0.1}{0.7} = \frac{1}{7}$$

21. C. Suppose Failure is denoted by  $F$ .

$$P(F|A) = 0.20, P(F|B) = 0.10, P(A) = 0.70, P(B) = 0.30. \text{ So}$$

$$P(A|F) = \frac{P(F|A)P(A)}{P(F|A)P(A) + P(F|B)P(B)} = \frac{0.20 \times 0.70}{0.20 \times 0.70 + 0.1 \times 0.30} = \frac{0.14}{0.17} = \frac{14}{17}.$$

22. C. Mean = 0. Standard deviation cannot be 0. The distribution is symmetric. Mode = 0 (since 0 has highest frequency).

23. B. Median of new observations =  $10 + 4 \times 12.8 = 61.2$ .

24. A.  $P(1 < X \leq 4) = P(X = 2) + P(X = 3) + P(X = 4) = 0.15 + 0.25 + 0.15 = 0.55$ .

25. B.  $E(40 - 2X - X^2) = 40 - 2 \times E(X) - E(X^2)$   
 $X \sim \text{Poisson}(3). E(X) = 3, \text{Var}(X) = 3$   
 $E(X^2) = \text{Var}(X) + (E(X))^2 = 3 + 9 = 12$

So,  $E(40 - 2X - X^2) = 40 - 2 \times 3 - 12 = 22.$

26. D.  $X \sim \text{Binomial}(10, 0.5). \text{Var}(X) = 10 \times 0.5 \times 0.5.$

$$\text{Var}(Y) = \text{Var}\left(\frac{X}{10}\right) = \frac{\text{Var}(X)}{10^2} = \frac{10 \times 0.5 \times 0.5}{10^2} = 0.025.$$

27. A. Find  $k$  from  $\int_0^\infty f(x)dx = 1.$

$$\int_0^\infty f(x)dx = \int_0^\infty kx^3 e^{-x/2} dx = 2^4 k \int_0^\infty u^3 e^{-u} du = 16k \times \Gamma(4) = 16k \times 3! = 96k.$$

So  $k = \frac{1}{96}.$

28. D.  $X \sim N(50, 12^2).$

$$P(X > 50) = P\left(\frac{X-50}{12} > \frac{75-50}{12}\right) = P(Z > 2.083) < P(Z > 1.96) = 0.025 \text{ [where } Z \sim N(0, 1)\text{]}.$$

29. B.  $\text{Var}(X_1 X_2) = E((X_1 X_2)^2) - (E(X_1 X_2))^2 = E(X_1^2 X_2^2) - (E(X_1 X_2))^2$   
 $= E(X_1^2)E(X_2^2) - (E(X_1))^2(E(X_2))^2$  ( $X_1$  and  $X_2$  are independently distributed).

$$E(X_1) = 0, \text{Var}(X_1) = 0.5, E(X_2) = 1 \text{ and } \text{Var}(X_2) = 0.8.$$

This implies  $E(X_1^2) = \text{Var}(X_1) + (E(X_1))^2 = 0.5.$

$$E(X_2^2) = \text{Var}(X_2) + (E(X_2))^2 = 0.8 + 1 = 1.8.$$

Hence  $\text{Var}(X_1 X_2) = 0.5 \times 1.8 - 0 \times 1 = 0.90.$

30. B.  $r_{XY} = 0.3. \text{Corr}(-1.5X, 2Y + 3) = \frac{\text{Cov}(-1.5X, 2Y+3)}{\text{sd}(-1.5X) \times \text{sd}(2Y+3)} = \frac{-1.5 \times 2 \times \text{Cov}(X, Y)}{1.5 \times \text{sd}(X) \times 2 \times \text{sd}(Y)} = \frac{-\text{cov}(X, Y)}{\text{sd}(X) \times \text{sd}(Y)} = -r_{XY} = -0.3.$

31. B. Two regression lines pass through  $(\bar{x}, \bar{y}).$

So  $\bar{x} + 3\bar{y} = 5$

$$4\bar{x} + 3\bar{y} = 8$$

This gives  $(\bar{x}, \bar{y}) = \left(1, \frac{4}{3}\right).$

## Data Interpretation and Data Visualization

32. C. Median = 50 percentile = 52000.

33. A. First quartile  $Q_1 = 28000$ . Third quartile  $Q_3 = 96000$ .

$$\text{Interquartile range} = Q_3 - Q_1 = 96000 - 28000 = 68000.$$

34. D. The percentage of families with income between 52000 and 140000 is 40.

35. B. Sales decrease from 3<sup>rd</sup> to 4<sup>th</sup>, 6<sup>th</sup> to 7<sup>th</sup>, 8<sup>th</sup> to 9<sup>th</sup> and 10<sup>th</sup> to 11<sup>th</sup>. So answer is 4.

36. C. Percentage increase:

$$2^{\text{nd}} \text{ to } 3^{\text{rd}} \text{ year: } (3/22) \times 100 < 20\%$$

$$4^{\text{th}} \text{ to } 5^{\text{th}} \text{ year: } (2/21) \times 100 < 20\%$$

$$5^{\text{th}} \text{ to } 6^{\text{th}} : (4/23) \times 100 < 20\%$$

$$7^{\text{th}} \text{ to } 8^{\text{th}} : (3/29) \times 100 < 20\%$$

$$9^{\text{th}} \text{ to } 10^{\text{th}} : (3/28) \times 100 < 20\%$$

$$11^{\text{th}} \text{ to } 12^{\text{th}} : (6/29) \times 100 > 20\%$$

37. B.  $n(P_1) = 35$ ,  $n(P_2) = 45$ ,  $n(P_1 \cup P_2) = 80 - 15 = 65$ .

$$n(P_1 \cup P_2) = n(P_1) + n(P_2) - n(P_1 \cap P_2).$$

So number of employees of who have opted both  $P_1$  and  $P_2$

$$n(P_1 \cap P_2) = 35 + 45 - 65 = 15.$$

38. C. The number of employees who have opted only  $P_1$

$$= n(P_1) - n(P_1 \cap P_2) = 35 - 15 = 20.$$

## English

39. B

40. C

41. A

42. C

43. A

44. C

45. C

46. A

47. A

48. A

49. C

- 50. C
- 51. D
- 52. C
- 53. B
- 54. D
- 55. A
- 56. A
- 57. C
- 58. C
- 59. B
- 60. C
- 61. D
- 62. C

## Logical Reasoning

- 63. D.
- 64. A.
- 65. C. The given words can be successive specifications of an address, starting from the Room number and specifying up to the district.
- 66. B.  $x$  weeks  $x$  days =  $(7x + x)$  days =  $8x$  days.
- 67. B. P's mother has taken legal steps to allow another person to act on her behalf. Therefore, this is the only choice that indicates that a power of attorney has been established.
- 68. D. One has to count only the cubes that lie completely inside. They form another cube with sides of length 2 cm. The volume is  $8 \text{ cm}^3$ , and so there are 8 smaller cubes in it.
- 69. D. At 4 o'clock the minute hand lags the hour hand by 20 minute spaces. In order that the two hands are in opposite directions, the minute hand has to have a net lead of 30 minute spaces. So there should be a gain of 50 minute spaces. The minute hand gains 55 minute spaces in 60 minutes. Therefore 50 minute spaces are gained in  $\frac{60}{55} \times 50 = 54 \frac{6}{11}$  minutes.
- 70. C.

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