

Institute of Actuaries of India

ACET June 2021

Mathematics

1. Let $f(x) = 2x + 1$; x real and $g(x) = \frac{x-1}{2}$; x real. Then,

- A. $f \circ g = g \circ f$, for all x .
- B. $f \circ g \neq g \circ f$, for some x but not all x .
- C. $f \circ g > g \circ f$, for all x .
- D. $f \circ g < g \circ f$, for all x .

1 mark

2. The first and second term of a HP are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. Then the eighth term is

- A. $\frac{1}{15}$.
- B. $\frac{1}{17}$.
- C. $\frac{1}{19}$.
- D. $\frac{1}{21}$.

1 mark

3. If $\log \sqrt{x} = \frac{1}{2} \log 16$; $\log y^2 = 2 \log 3$, $y < 0$, then $x + y$ is equal to

- A. 11.
- B. 13.
- C. 15.
- D. 17.

1 mark

4. Let $f(x) = \begin{cases} 2 & \text{if } x < 2 \\ ax + b & \text{if } 2 \leq x < 8 \\ 18 & \text{if } x \geq 8. \end{cases}$

The values of a, b , for which $f(x)$ is continuous, are

- A. $-\frac{8}{3}, \frac{10}{3}$.

B. $-\frac{8}{3}, -\frac{10}{3}$.

C. $\frac{8}{3}, \frac{10}{3}$.

D. $\frac{8}{3}, -\frac{10}{3}$.

1 mark

5. The value of $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24}$ is

A. $\tan^{-1} \frac{1}{4}$.

B. $\tan^{-1} \frac{3}{4}$.

C. $\tan^{-1} \frac{1}{2}$.

D. $\tan^{-1} 1$.

1 mark

6. If α and β are the roots of the equation $x^2 + ax + b = 0$, where $b \neq 0$, then the roots of the equation $bx^2 + ax + 1 = 0$ are

A. $\frac{1}{\alpha}, \frac{1}{\beta}$.

B. α^2, β^2 .

C. β, α .

D. $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$.

1 mark

7. The approximate root that lies between 2 and 3 of $x^3 - 4x - 9 = 0$, by bisection method, based on three iteration is

A. 2.375.

B. 2.625.

C. 2.875.

D. 2.750.

2 marks

8. The term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is

A. $\frac{567}{16}$.

B. $-\frac{7}{18}$.

C. $\frac{7}{18}$.

D. $-\frac{567}{16}$.

1 mark

9. The value of $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

- A. is 2.
- B. is 1.
- C. is 0.
- D. does not exist.

1 mark

10. Let $y_1(x) = e^x$, $y_2(x) = e^{y_1(x)}$, $y_3(x) = e^{y_2(x)}$, ..., etc. In general, for any $n \geq 1$, $y_{n+1}(x) = e^{y_n(x)}$. Then $\frac{d}{dx} y_n(x)$ equals

- A. $y_n(x)y_{n-1}(x)$.
- B. $y_{n+1}(x)$.
- C. $y_n(x)y_{n-1}(x) \dots y_1(x)$.
- D. $y_n(x)y_{n-1}(x) \dots y_1(x)e^x$.

2 marks

11. The function $f(x) = (x + 3)e^{-x}$ is

- A. increasing in x for all x .
- B. decreasing in x for all x .
- C. decreasing in $(-\infty, -2)$ and increasing in $(-2, \infty)$.
- D. increasing in $(-\infty, -2)$ and decreasing in $(-2, \infty)$.

2 marks

12. Let $f(x) = \log_{x^2} e^x$. Then $\frac{d}{dx} f(x)$ equals

- A. $(\log_e x - 1)/2(\log_e x)^2$.
- B. $(\log_e x - 1)/(\log_e x)^2$.
- C. $(\log_e x - 2)/2(\log_e x)^2$.
- D. $\log_e x / 2(\log_e x)^2$.

2 marks

13. The value of the integral $\int \frac{1}{\cos^2 x (1-\tan x)^2} dx$ is

- A. $-\frac{1}{1-\tan x} + c.$
- B. $\frac{1}{1-\tan x} + c.$
- C. $\frac{1}{(1-\tan x)^2} + c.$
- D. $\frac{1}{\cos^2 x} + c.$

1 mark

14. Let $f(x) = \min(x^2 + k, x + k)$, x real and $k > 0$. Then the value $\int_{-1}^1 f(x) dx$ is

- A. 0.
- B. $\frac{12k+1}{6}.$
- C. $\frac{12k+5}{6}.$
- D. $\frac{12k-1}{6}.$

3 marks

15. If $I_1 = \int_{e^{-1}}^e \frac{1}{\log_e x} dx$ and $I_2 = \int_{-1}^1 \frac{e^x}{x} dx$, then

- A. $I_1 = I_2.$
- B. $I_1 = 2I_2.$
- C. $I_1 = -I_2.$
- D. $I_1 = \frac{I_2}{2}.$

2 marks

16. Let $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$. Then, the unit vector in the direction of $\vec{a} + \vec{b}$ is

- A. $\frac{1}{\sqrt{5}} (-\vec{j} + 2\vec{k}).$
- B. $\frac{1}{29} (2\vec{i} - 3\vec{j} + 4\vec{k}).$
- C. $\frac{1}{\sqrt{29}} (2\vec{i} - 3\vec{j} + 4\vec{k}).$
- D. $\frac{1}{\sqrt{14}} (\vec{i} - 2\vec{j} + 3\vec{k}).$

1 mark

17. If θ is the angle between \vec{a} and \vec{b} then $\frac{|\vec{a} \times \vec{b}|}{\vec{a} \cdot \vec{b}}$ is

- A. $\sin \theta$.
- B. $\cos \theta$.
- C. $\tan \theta$.
- D. either $\tan \theta$ or $-\tan \theta$.

1 mark

18. If A and B are matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ equals

- A. $2AB$.
- B. $2BA$.
- C. $A + B$.
- D. I .

1 mark

19. If in a triangle ABC the sides $AB = a$, $BC = b$ and $CA = c$ satisfy the condition

$$\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0, \text{ then } \cos^2 A + \cos^2 B + \cos^2 C \text{ equals}$$

- A. $\frac{3}{2}$.
- B. $\frac{3}{4}$.
- C. 0.
- D. 1.

3 marks

20. The characteristic roots of the matrix $A = \begin{bmatrix} -3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ are

- A. $(-3, 3, 1)$.
- B. $(0, 1, 1)$.
- C. $(0, 0, 1)$.
- D. $(0, -1, 1)$.

2 marks

Statistics

21. There are six bus routes between the places P_1 and P_2 and four bus routes between the places P_2 and P_3 . There is no direct bus service between P_1 and P_3 . A man can travel round-trip in a number of ways by bus from P_1 to P_3 via P_2 . If he does not use the same bus route more than once, the number of ways he can complete the round-trip journey is
- A. 18.
 - B. 20.
 - C. 248.
 - D. 360.

1 mark

22. A software development company has three jobs to do. The first two jobs require three programmers each, and the third one requires two. If the company employs eight programmers, how many different ways are there to assign them to the jobs, assuming no programmer is assigned more than one job?
- A. 70.
 - B. 144.
 - C. 560.
 - D. 580.

1 mark

23. Which of the following statements is true?
- A. If a variable x takes values -1, 0 and 0.5, 2 with equal frequencies, then the median of x is 0.5.
 - B. If for a symmetric distribution, the third quartiles are 12 and 22 respectively, then the median of the distribution is 16.
 - C. The mode of the values 5, 6, 3, 4, 4, 2, 5, 5, 7, 7, 6, 6, 7, 6 is 7.
 - D. The mean of the numbers 57, 60, 63, 66, ..., 111 is 84.

3 marks

24. The mean and range of 20 observations of a variable are 6.65 and 7. If each observation is multiplied by 3 and then 2 is added, then the new mean and the new range are
- A. 19.95, 7.
 - B. 19.95, 21.
 - C. 21.95, 21.

D. 21.95, 23.

1 mark

25. In an organization, the coefficients of variations of daily wages of casual male and female workers are 8% and 12%, respectively. The standard deviations are Rs. 20.00 and Rs. 15.60 respectively. Suppose 70% of the casual workers are male. Then the combined average wages of all the casual workers are

- A. Rs. 214.00.
- B. Rs. 222.60.
- C. Rs. 232.80.
- D. Rs. 240.00.

2 marks

26. If A and B are mutually exclusive events with $P(A) = 0.25$ and $P(B) = 0.50$, then

- A. $P(A \cup B) = 0.525$.
- B. $P(\bar{A} \cap \bar{B}) = 0.125$.
- C. $P(\bar{A} \cup \bar{B}) = 0.75$.
- D. $P(\bar{A}|\bar{B}) = 0.50$.

1 mark

27. Let the three mutually independent events A_1, A_2 and A_3 be such that $P(A_1) = P(A_2) = P(A_3) = \frac{1}{4}$. Then $P((A_1^c \cap A_2^c) \cup A_3)$ is

- A. $\frac{1}{16}$.
- B. $\frac{21}{32}$.
- C. $\frac{43}{64}$.
- D. $\frac{3}{64}$.

1 mark

28. Suppose that the probability of living to be older than 65 is 0.6 and the probability of living to be older than 75 is 0.2. If a person reaches her 65th birthday, what is the probability that she will celebrate her 75th birthday?

- A. 0.12.

- B. $1/3$.
- C. 0.8.
- D. Cannot be obtained from the given information.

1 mark

29. A factory runs three shifts. In a given day, 2% of the items produced by the first shift are defective, 3% of the second shift's items are defective, and 5% of the third shift's item are defective. Suppose the shifts produce same number of items. If an item produced in the factory is defective what is the probability that it was produced in the first shift?

- A. 0.02.
- B. 0.10.
- C. 0.20.
- D. 0.30.

2 marks

30. Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & \text{for } x < -1, \\ \frac{x+1}{2}, & \text{for } -1 \leq x \leq 1, \\ 1, & \text{for } x \geq 1. \end{cases}$$

The skewness of the distribution is

- A. -0.25 .
- B. 0.
- C. 0.25.
- D. 1.

1 mark

31. Which of the following statements is true?

- A. For a Poisson distribution mean and standard deviation can never be equal.
- B. If X follows a Poisson distribution with $P(X = 1) = P(X = 2)$, then $P(X = 0) = e^{-1}$.
- C. Probability that a Poisson variable takes positive values is $1 - e^{-2}$. Then the standard deviation of the distribution is 2.
- D. The number of commercial phone calls received per day by a person follows Poisson distribution with mean 3. The probability that person receives less than 3 commercial phone calls is $8.5e^{-3}$.

2 marks

32. Let X_1 and X_2 be two independent Bernoulli random variables with $P(X_1 = 1) = p_1$ and $P(X_2 = 1) = p_2$. The $P(X_1X_2 = 0)$ equals
- A. $1 - p_1p_2$.
 - B. $(1 - p_1)(1 - p_2)$.
 - C. $p_1(1 - p_2)$.
 - D. $(1 - p_1)p_2$.

1 mark

33. Let X be a random variable having probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$. If $P(X \geq 1) = 0.05$, then the median of the distribution is

- A. $\frac{\log_e 2}{\log_e 20}$.
- B. $\log_e 20$.
- C. $\log_e 2$.
- D. $\frac{\log_e 10}{\log_e 2}$.

1 mark

34. Suppose X_1 follows binomial(2, 0.5) and X_2 follows binomial(3, 0.5), and they are independently distributed. Then $P(X_1 + X_2 = 1)$ is
- A. 0.03125.
 - B. 0.6250.
 - C. 0.09375.
 - D. 0.15625.

1 mark

35. Suppose $X \sim N(\mu, \sigma^2)$. Then $E(e^X)$ equals

- A. $e^{\mu + \frac{\sigma^2}{2}}$.
- B. $e^{\mu + \sigma^2}$.
- C. e^μ .
- D. $e^{\mu - \frac{\sigma^2}{2}}$.

2 marks

36. Suppose $X \sim N(0, 1)$. Then the correlation coefficient between $2X$ and X^2 is
- A. -0.5 .
 - B. 0 .
 - C. 0.5 .
 - D. 1 .

1 mark

37. A delivery truck travels from point A to point B and back over the same route each day. There are three traffic lights on this route. Let X be the number of red lights the truck encounters on this way to delivery point B and Y be the number of red lights the truck encounters on the way back to delivery point A . The joint probability distribution of X and Y is given below.

		$X = x$			
		0	1	2	3
$Y = y$	0	0.01	0.02	0.07	0.01
	1	0.03	0.06	0.10	0.06
	2	0.05	0.12	0.15	0.08
	3	0.02	0.09	0.08	0.05

The probability that the truck encounters less than 3 red lights on the way back to A from B , given that the truck encounters 2 red lights on the way to delivery point B is

- A. 0.08 .
- B. 0.32 .
- C. 0.80 .
- D. 0.76 .

2 marks

38. Consider the following observations on (x, y) .

x	1	2	3	4	5	6	7	8	9	10	11
y	11	10	9	8	7	6	5	4	3	2	1

The correlation coefficient between x and y is

- A. 0 .
- B. 0.5 .
- C. -1 .
- D. 1 .

1 mark

39. If the regression coefficient of y on x is $b_{yx} = -0.2$ and regression coefficient of x on y is $b_{xy} = -1.8$, then which of the following statement is true?
- A. The correlation coefficient between x and y is -0.9 .
 - B. The correlation coefficient between x and y is 0.6 .
 - C. If \bar{x} and \bar{y} are means of x and y , the regression lines are $y = \bar{y} + 0.2(x - \bar{x})$ and $x = \bar{x} + 1.8(y - \bar{y})$.
 - D. The standard deviations of x and y are related as $s_x = 3 s_y$.

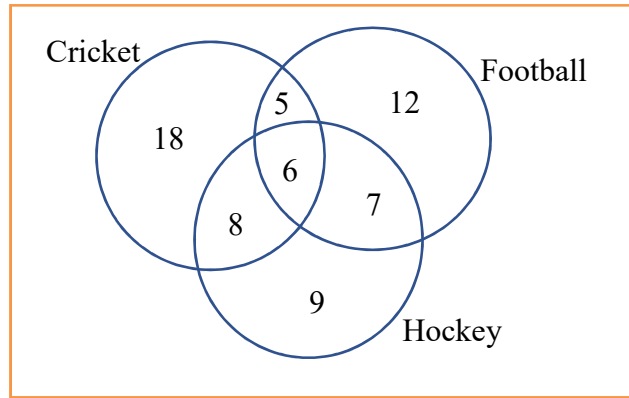
2 marks

40. If $4w = 2x + 7$ and $6z = 2y - 15$ and the regression coefficient of y on x is 3 , then regression coefficient of z on w is
- A. -2 .
 - B. 2 .
 - C. 3 .
 - D. 4.5 .

3 marks

Data Interpretation

A group of students were asked which of the sports – Cricket, Football and Hockey – they liked. The summary is provided in the Venn diagram given below. Please answer Questions 41 and 42 based on this diagram.



41. The number of students who liked Cricket and Hockey but not Football is

- A. 8.
- B. 17.
- C. 27.
- D. 35.

1 mark

42. The number of students who liked Hockey or Football is

- A. 9.
- B. 12.
- C. 28.
- D. 47.

1 mark

The following table gives the price (per liter) of three types of edible vegetable oils from January to December 2020. Please answer Question 43-45 based on this table.

Month	Brand 1	Brand 2	Brand 3
Jan	125	140	155
Feb	125	142	160
Mar	130	145	163
Apr	135	152	172
May	145	155	178
Jun	148	162	185
Jul	153	165	188
Aug	160	180	192
Sep	165	182	200
Oct	170	185	205
Nov	170	187	210
Dec	180	195	216

43. Which of the following statements is true?

- A. Overall percentage change in price from Jan to Dec is minimum for Brand 3.
- B. Overall percentage change in price from Jan to Dec is maximum for Brand 1.
- C. The percentage change in price from Oct to Dec is maximum for Brand 2.
- D. The percentage change in price from Oct to Dec is minimum for Brand 1.

2 marks

44. The price difference between Brand 3 and Brand 2 is the least in

- A. Jan.
- B. Feb.
- C. Mar.
- D. Aug.

1 mark

45. The price difference between Brand 3 and Brand 1 is the highest in

- A. Apr.
- B. Jul.
- C. Nov.
- D. Dec.

1 mark

The following table gives the amount (in lakhs) claimed under family health insurance scheme for four quarters of 2019 in eight regions. Please answer Questions 46-48 on based on this table.

Region	Number of Families	Apr - Jun	Jul - Sep	Oct - Dec	Jan - Mar
R1	170	12	16	14.5	22.5
R2	120	10	8	7	8.2
R3	135	15	14	12	22
R4	160	18	20	45	34
R5	150	22	23	54	23
R6	180	30	33.2	46.5	34
R7	110	26.5	28	22	45.2
R8	200	45	62	40.5	65

46. The average claim amount per family over the year is highest in region

- A. R4.
- B. R6.
- C. R7.
- D. R8.

2 marks

47. The total claim amount in all regions is maximum during the quarter

- A. Apr-Jun.
- B. Jul-Sep.
- C. Oct-Dec.
- D. Jan-Mar.

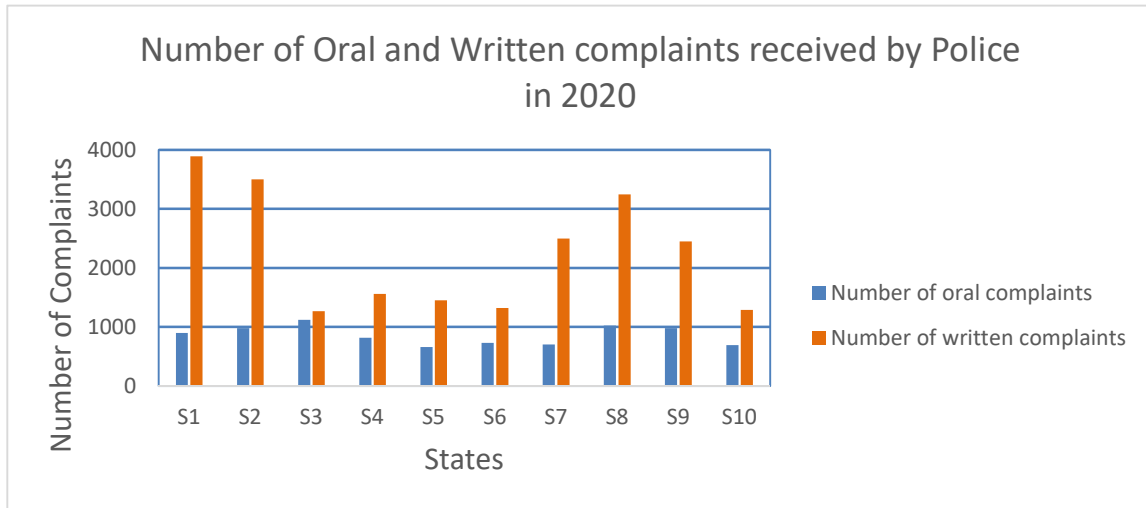
2 marks

48. More than 20% of the total amount claimed occurred over the year in region

- A. R3.
- B. R4.
- C. R6.
- D. R8.

1 mark

The following graph shows the number of oral and written complaints received by Police during 2020 in ten states. Please answer Questions 49-51 based on this graph.



49. The number of written complaints is more than four times the number of oral complaints in state

- A. S1.
- B. S2.
- C. S7.
- D. None of the above.

2 marks

50. The total number of complaints is highest in state

- A. S1.
- B. S2.
- C. S8.
- D. S9.

1 mark

51. The number of oral complaints is least in state

- A. S4.
- B. S5.
- C. S7.
- D. None of the above.

1 mark

English

52. Select the word that most appropriately completes the following sentence: If you work beyond your capacity, you will naturally feel _____.

- A. Drowsy.
- B. Tired.
- C. Confident.
- D. Giddy.

1 mark

53. Select the phrase that most appropriately completes the following sentence: “This book is quite similar _____.”

- A. with the "Treasure Island".
- B. of that film, we saw at school.
- C. to the one I read last week.
- D. than a story told by our teacher.

1 mark

54. Select the phrase that correctly completes the following sentence: “He did not like engineering and left the college _____.”

- A. for now.
- B. for then.
- C. for good.
- D. for always.

1 mark

55. Select the word that is opposite in meaning to the word ‘Humdrum’.

- A. Insipid.
- B. Dim.
- C. Routine.
- D. Exciting.

1 mark

56. Choose the most appropriate antonym of the word 'Quarantine'.

- A. Participation.
- B. Activity.
- C. Abandonment.
- D. Isolation.

1 mark

57. Meaning of the phrase “to knock off” is

- A. to knock at the door.
- B. to stop working at the end of the day.
- C. to close the door while leaving.
- D. to start a fight with someone.

1 mark

58. Meaning of the phrase “to hang out” is

- A. to spend time in relaxing or socializing.
- B. to spend time in important work.
- C. to spend time in office work.
- D. to roam around as a traveller

1 mark

59. Meaning of the phrase “to check off” is

- A. to delete name(s) from a list.
- B. to mark with a tick.
- C. to fire from a job.
- D. to vacate a house.

1 mark

60. Choose the correct sequence from the options to complete the given sentence: Stung by years of criticism that it has pampered _____

- i. Wall Street criminals, the Justice Department issued new policies
- ii. to turn over evidence against their executives
- iii. on Wednesday that prioritizes the prosecution of individual employees
- iv. not just their companies, and put pressure on corporations

- A. i, ii, iii, iv.
- B. i, ii, iv, iii.
- C. i, iii, ii, iv.

D. i, iii, iv, ii.

2 marks

61. Choose the correct sequence from the options below to make a complete sentence:

- i. a task, I fear we are letting ourselves
- ii. merely put a lot of effort into
- iii. by defining “our best” as the thing we did when we
- iv. off the hook

- A. iii, ii, i, iv.
- B. iii, ii, iv, i.
- C. iv, i, iii, ii.
- D. iv, ii, i, iii.

2 marks

Read the passage below and answer Question No. 62.

Iguacu Falls, which sits on the border between Argentina and Brazil, is said to make Niagara look like a leaky faucet. Great cataracts stretch for two and a half miles across lushly foliated rocky outcroppings before plunging a staggering two hundred and thirty feet into the river below. The falls region is densely forested and is home to a wide variety of plants and animals, including several endangered ones. It is a paradise where parrots dive and swoop through the spray, butterflies cavort among the tropical plants and coatis, and giant otters and anteaters amble through the trees. The foliage itself varies between tropical and deciduous with orchids blushing in the shade of pines and ferns nodding gracefully in the shadow of fruit trees.

Depending on rainfall and water flow, between 100 and 300 individual falls tumble over the cliffs creating a stunning panoply of churning water. Small wonder that this natural powerhouse attracted the attention of developers. At the top of the falls on the Parana River sits Itaipu Dam, the world’s largest operational hydroelectric power plant. The dam is often numbered among the wonders of the modern world.

The falls’ superlatives don’t stop with technology. Many consider the cataract system itself one of the natural wonders of the world. In 1986, UNESCO (United Nations Educational, Scientific, and Cultural Organization) concurred when it declared the falls a World Heritage site to ensure its preservation in the face of continuing technological development.

- I. The comparison in the first sentence is meant to emphasize the
 - i. Abundant beauty of Niagara Falls.
 - ii. The smallness of Niagara Falls relative to Iguacu Falls.
 - iii. The distance of Iguacu Falls from civilization.
- II. The author's attitude toward Iguacu Falls can best be described
 - i. Overweening pride.
 - ii. Positive appreciation.
 - iii. Mild acceptance.
- III. What can most reasonably be inferred from the information in the final two paragraphs?

- i. Iguacu Falls' success as a hydroelectric site could potentially cause a threat to its longevity.
- ii. UNESCO considers the preservation of Iguacu Falls crucial to the balance of the world ecosystem.
- iii. Developers consider any opportunity to invest in hydroelectric power a lucrative business venture.

62. The correct answers to I, II and III are

- A. i, ii, iii, respectively.
- B. ii, i, i, respectively.
- C. ii, i, iii, respectively.
- D. ii, ii, i, respectively.

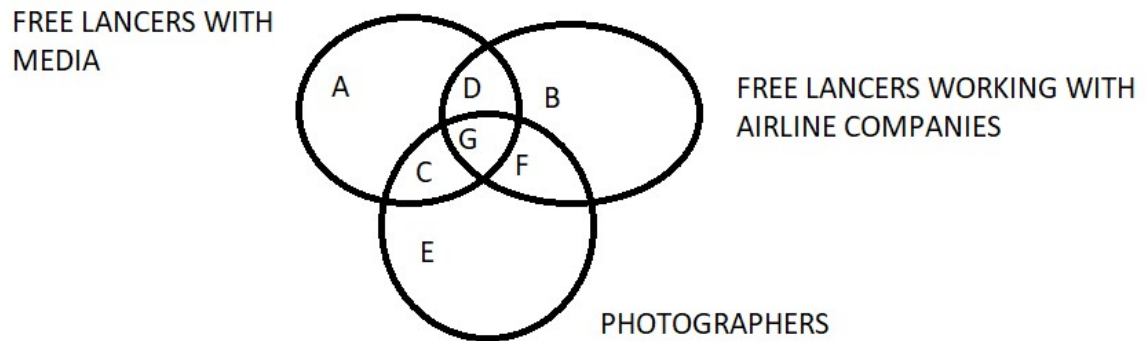
3 marks

Logical Reasoning

63. A man said, “the lady in the picture is a daughter of my wife’s grandfather’s only child”. How is the woman related to the man?
- A. Wife.
 - B. Sister-in-law.
 - C. Sister.
 - D. Data inadequate.

1 mark

64. Which of the following represent free lancers with media who also are free lancers with airline companies but are not photographers?



- A. D and G.
- B. Only D.
- C. G and F.
- D. Only A.

1 mark

65. The first day of a month is a Saturday and is a working day and the month has 30 days. If every alternate Saturday and all Sundays are holidays, then how many working days are there in that month?
- A. 25.
 - B. 22.
 - C. 24.

D. 23.

1 mark

66. Five ladies are sitting in a row. P is sitting to the left of M and to the right of O. R is sitting to the right of N but to the left of O. Who is sitting in the middle?

- A. O.
- B. R.
- C. P.
- D. M.

1 mark

67. If PALE is coded as 2134, EARTH is coded as 41590, how is PEARL coded?

- A. 29530.
- B. 24153.
- C. 25413.
- D. 25430.

1 mark

68. Balls of identical size are tightly arranged in the form of a pyramid. Each face of the pyramid is triangular, with each side of every triangle consisting of three balls. How many balls are there in the pyramid?

- A. 9.
- B. 10.
- C. 12.
- D. 15.

1 mark

69. The time displayed on both the watches is 12.A.M right now. The first clock gains five minutes every hour, whereas the second one is slower by five minutes every hour. After how many hours will both the watches show the same time again?

- A. 48.
- B. 60.
- C. 72.

D. 84.

2 marks

70. Which of the two statements may be made logically from the sentences given below?

All bolts are not nuts.

All nuts are screws.

Statement I : All screws being bolt is a possibility.

Statement II: Some screws are not nuts.

- A. Only I.
- B. Only II.
- C. Both I and II.
- D. Neither I nor II.

2 marks
