Institute of Actuaries of India

ACET June 2021 Solutions

Mathematics

1. A. $f \circ g(x) = f(g(x)) = f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right) + 1 = x$ and $g \circ f(x)$ $= g(f(x)) = \quad g(2x+1) = \frac{(2x+1)-1}{2} = x.$

2. B. The H.P is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} \ldots$; $\frac{1}{a} = \frac{1}{3}$ and $\frac{1}{a+d} = \frac{1}{5}$. These imply $a = 3$ and $d = 2$.

   Hence, the eighth term is $\frac{1}{a+7d} = \frac{1}{3+(7\times 2)} = \frac{1}{17}$.

3. B. $\log\sqrt{x} = \frac{1}{2}\log\ 16 \Rightarrow \log\sqrt{x} = \log\ 4 \ \Rightarrow \sqrt{x} = 4 \Rightarrow x = 16.$

   $\log y^2 = 2\log 3 \Rightarrow y^2 = 9 \Rightarrow y = -3$ since $y < 0$. Hence $x + y = 16 - 3 = 13$.

4. D. $f(2-) = 2, f(2+) = 2a + b$. In order that $f(x)$ is continuous at $x = \quad 2$, we must have $f(2-) = \ f(2+)$. This implies $2a + b = 2$.

   Similarly, for $f(x)$ to be continuous at $x = 8$, we have to have $8a + b = 18$. Solving these two equations, we get $a = \frac{8}{3}$ and $= -\frac{10}{3}$. At all other points $f(x)$ is continuous.

5. C. Let $\theta = \tan^{-1}\frac{2}{11}, \varphi = \tan^{-1}\frac{7}{24} \ \Rightarrow \ \tan\theta = \frac{2}{11}, \tan\varphi = \frac{7}{24}$.

   $\tan(\theta + \varphi) = \frac{\tan\theta + \tan\varphi}{1 - \tan\theta\tan\varphi} = \frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}} = \frac{1}{2} \Rightarrow \theta + \varphi = \tan^{-1}\frac{1}{2}$.

6. A. Let α and β be the roots of the equation $x^2 + ax + b = 0$, where $b \neq 0$. Then $\alpha + \beta = -a$ and $\alpha\,\beta = b$.
   If γ and δ are the roots of the equation $bx^2 + ax + 1 = 0$, then $\gamma + \delta = -\frac{a}{b} = \frac{\alpha+\beta}{\alpha\,\beta} = \frac{1}{\alpha} + \frac{1}{\beta}$ and $\gamma\delta = \frac{1}{b} = \frac{1}{\alpha} \times \frac{1}{\beta}$. No other choice holds among the roots.

   Alternatively, the second equation is equivalent to $b + a\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = 0$, which is the same as the the first equation with x replaced by $\frac{1}{x}$. Therefore, the second equation is satisfied by the values $\frac{1}{x} = \alpha$ and $\frac{1}{x} = \beta$.

7. B. Since $f(x) = x^3 - 4x - 9 = 0$, we have
$f(2) = 2^3 - 4 \times 2 - 9 = -9\ (-ve)$, and $f(3) = 3^3 - 4 \times 3 - 9 = 6\ \ (+ve)$.
Since $f(2) < 0$ and $f(3) > 0$, there lies a real root between 2 and 3.
The approximate root by first iteration of Bisection method: $\frac{2+3}{2} = 2.5$.

Now, $f(2.5) = 2.5^3 - 4 \times 2.5 - 9 = 15.625 - 10 - 9 = -3.375(-ve)$.
Since $f(2.5) < 0$ and $f(3) > 0$, there lies a real root in between 2.5 and 3.
The approximate root by second iteration: $\frac{2.5+3}{2} = 2.75$.

Further, $f(2.75) = 2.75^3 - 4 \times 2.75 - 9 = 20.797 - 11 - 9 = 0.797(+ve)$.
Since $f(2.5) < 0$ and $f(2.75) > 0$, there lies a real root in between 2.5 and 2.75.
The approximate root by third iteration: $\frac{2.5+2.75}{2} = \frac{5.25}{2} = 2.625$.

8. C. The general term is
$T_r = \binom{9}{r}\left(\frac{3}{2}x^2\right)^r\left(-\frac{1}{3x}\right)^{9-r} = (-1)^{9-r}\binom{9}{r}\frac{3^{2r-9}}{2^r}\ x^{3r-9}$ for $r = 0,1,\ldots,9$.
This term is independent of x when $3r - 9 = 0$, i.e., for $r = 3$.
The required term is $T_3 = \binom{9}{3}\left(\frac{3}{2}x^2\right)^3\left(-\frac{1}{3x}\right)^6 = \frac{9\times8\times7}{6} \times \frac{27}{8} \times \frac{1}{27\times27} = \frac{9\times7}{6\times27} = \frac{7}{18}$.

9. A. Let $x - \frac{\pi}{2} = h$. When $x \to \frac{\pi}{2}, h \to 0$. Hence,
$\lim_{h\to0}\frac{\tan 2\left(\frac{\pi}{2}+h\right)}{h} = \lim_{h\to0}\frac{\tan(\pi+2h)}{h} = \lim_{h\to0}\frac{\tan 2h}{h} = \lim_{h\to0}\frac{2\tan 2h}{2h} = 2 \times 1 = 2.$
Alternatively, by l'Hopital's rule, $\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}}\frac{2\sec^2 2x}{1} = 2$.

10. C.
$$\begin{aligned}\frac{d}{dx}\,y_n(x) &= \frac{d}{dx}\,e^{y_{n-1}(x)} = e^{y_{n-1}(x)}\frac{d}{dx}\,y_{n-1}(x) = y_n(x)\frac{d}{dx}\,y_{n-1}(x)\\ &= y_n(x)y_{n-1}(x)\frac{d}{dx}\,y_{n-2}(x) = \cdots = y_n(x)\cdots y_2(x)\frac{d}{dx}y_1(x)\\ &= y_n(x)\cdots y_2(x)y_1(x).\end{aligned}$$

11. D. $f(x) = (x+3)e^{-x}$. Since $e^{-x} > 0\ \forall\ x$, we have
$$\frac{d}{dx}\,f(x) = (x+3)e^{-x}(-1) + e^{-x} = e^{-x}(-x-3+1) = e^{-x}(-x-2)$$
$$\begin{cases} > 0\ if\ x < -2 \\ = 0\ if\ x = -2. \\ < 0\ if\ x > -2 \end{cases}$$
Thus, $f(x)$ is incresing in x in $(-\infty,-2)$ and decreasing in x in $((-2,\infty)$.

Alternatively, The function $f(x)$ is incresing in x wherever $\log f(x) = -x + \log(x+3)$ is incresing in x. This happens when the derivative is positive, i.e., $-1 + \frac{1}{x+3} > 0$, i.e., $x < -2$. By the same logic, $f(x)$ is decreasing when $x > -2$.

12. A. $f(x) = \log_{x^2} e^x = \frac{\log_e e^x}{\log_e x^2}$.

$\frac{d}{dx}\, f(x) = \frac{\log_e x^2 \times 1 - x \times \frac{2x}{x^2}}{(\log_e x^2)^2} = \frac{2\log_e x - 2}{(2\log_e x)^2} = \frac{2(\log_e x - 1)}{2^2(\log_e x)^2} = \frac{(\log_e x - 1)}{2(\log_e x)^2}$.

13. B. $\int \frac{1}{\cos^2 x\,(1-\tan x)^2}\, dx$. Put $1 - \tan x = t \Longrightarrow -\sec^2 x\, dx = dt$.

$$\int \frac{1}{\cos^2 x\ (1-\tan x)^2}\ dx = \int -\frac{1}{t^2} dt = t^{-1} + c = \frac{1}{1-\tan x} + c.$$

14. D. $\int_{-1}^{1} f(x)dx\ = \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx$.
$f(x) = \min(\,x^2 + k, x + k)$, x *real* and $k > 0$.
When $x \in (-1,0)$, $\min(x^2 + k, x + k) = x + k$ and
when $x \in (0,1)$, $\min(x^2 + k, x + k) = x^2 + k$.
Hence, $\int_{-1}^{0} f(x)dx = \int_{-1}^{0}(x+k)dx = \frac{x^2}{2} + kx\Big]_{-1}^{0} = -\left(\frac{1}{2} - k\right) = k - \frac{1}{2}$,
and $\int_{0}^{1} f(x)dx = \int_{0}^{1}(x^2+k)dx = \frac{x^3}{3} + kx\Big]_{0}^{1} = \frac{1}{3} + k$.
Thus, $\int_{-1}^{1} f(x)dx\ =\ k - \frac{1}{2} + \frac{1}{3} + k = \frac{12k-1}{6}$.

15. A. $I_1 = \int_{e^{-1}}^{e} \frac{1}{\log_e x} dx$. Put $\log_e x = t \Longrightarrow \frac{1}{x} dx = dt$.
When $x = e^{-1}$, $t = -1$ and when $x = e$, $t = 1$.
Therefore, $I_1 = \int_{e^{-1}}^{e} \frac{1}{\log_e x} dx = \int_{-1}^{1} \frac{1}{t} e^t dt =\ I_2$.

16. C. $\vec{a} =\ \vec{\imath} - 2\vec{\jmath} + 3\vec{k}$ and $\vec{b} =\ \vec{\imath} - \vec{\jmath} + \vec{k}$. Thus, $\vec{a} + \vec{b} = 2\vec{\imath} - 3\vec{\jmath} + 4\vec{k}$,

and $|\vec{a} + \vec{b}| = \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{29}$.
Hence the unit vector in the direction of $\vec{a} + \vec{b}$ is

$$\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{1}{\sqrt{29}}\ (2\vec{\imath} - 3\vec{\jmath} + 4\vec{k}).$$

17. D. $\frac{|\vec{a} \times \vec{b}|}{\vec{a} \circ \vec{b}} = \frac{|a||b||\sin\theta|}{|a||b|\cos\theta} = \tan\theta$ or $-\tan\theta$, depending on the sign of $\sin\theta$.

18. C. $A^2 = ABA = BA = A$. Also $B^2 = BAB = AB = B$. Hence

$$A^2 + B^2 = A + B.$$

19. B. $$\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0 \Rightarrow (c^2 - ab) - a(c - a) + b(b - c) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\Rightarrow \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0 \Rightarrow a = b = c.$$

Hence, the triangle *ABC* is equilateral with each of the angles at *A, B, C* equal to $60°$. Thus, $\cos^2 A + \cos^2 B + \cos^2 C = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$.

20. C. The characteristic equation of the matrix $A = \begin{bmatrix} -3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ is $|A - \lambda I| = 0$, which leads to the equation

$$\begin{vmatrix} -3 - \lambda & -9 & -12 \\ 1 & 3 - \lambda & 4 \\ 0 & 0 & 1 - \lambda \end{vmatrix} = 0.$$

Thus,

$$(1 - \lambda)[(-3 - \lambda)(3 - \lambda) + 9] = 0 \Rightarrow (1 - \lambda)\lambda^2 = 0.$$

Hence, the characteristic roots of the matrix are $(1,0,0)$.

Statistics

21. D. There are $6 \times 4 = 24$ ways to go by bus from P_1 to P_3. Since the man cannot use the same bus route from P_3 to P_2 and from P_2 to P_1. So, for every chosen route from P_1 to P_3 via P_2, there are $3 \times 5 = 15$ possible routes for the return journey. The require number of ways = $24 \times 15 = 360$.

22. C. The number of ways the programmers can be assigned to the jobs is

$$\frac{8!}{3!\,3!\,2!} = 560.$$

23. D. A. The number of observations with any particular value is even. The median lies strictly between the middle most values, which are 0 and 0.5 in the present case. As per common convention, the median is the simple average of these two values, which is 0.25.
B. For a symmetric distribution, median = (first quartile + third quartile) / 2. Therefore, the median is $(12 + 22)/2 = 17$.
C. The ordered values are 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7. The mode is the most frequently occurring value, which is 6.
D. The numbers are in AP. So the sum of the numbers is $\frac{n}{2}(57 + 111)$, where n is the number of observations. Therefore, the mean of the numbers is $\frac{57+111}{2} = 84$.

24. C. We have $y = 2 + 3x$.
Then $\bar{y} = 2 + 3\bar{x} = 2 + 3 \times 6.65 = 21.95$.
$\text{Range}(y) = 3 \times \text{Range}(x) = 3 \times 7 = 21$.

25. A. Let $\bar{x}_1$ and $\bar{x}_2$ denote the mean wages of male and female workers respectively.
Then $8 = \frac{\text{Rs.}20}{\bar{x}_1} \times 100 \Rightarrow \bar{x}_1 = \text{Rs.}\,250$.
Also $12 = \frac{\text{Rs.}15.6}{\bar{x}_2} \times 100 \Rightarrow \bar{x}_2 = \text{Rs.}\,130$.
Let n_1 and n_2 be the number of male and female casual workers respectively. Then the combined average of all the casual workers is

$$\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{n_1}{n_1 + n_2}\bar{x}_1 + \frac{n_2}{n_1 + n_2}\bar{x}_2 = 0.7 \times 250 + 0.3 \times 130 = \text{Rs.}\,214.$$

26. D. A and B are mutually exclusive: $P(A \cap B) = 0$.

$$P(A \cup B) = P(A) + P(B) = 0.25 + 0.50 = 0.75$$
$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - 0.75 = 0.25$$
$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B) = 1.$$

$$P(\bar{A}|\bar{B}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{0.25}{0.50} = 0.50.$$

27. C. $P\big((A_1^c \cap A_2^c) \cup A_3\big) = P(A_1^c \cap A_2^c) + P(A_3) - P(A_1^c \cap A_2^c \cap A_3)$.
Since A_1, A_2 and A_3 are mutually independent, $P(A_1^c \cap A_2^c) = P(A_1^c)P(A_2^c)$ and also $P(A_1^c \cap A_2^c \cap A_3) = P(A_1^c)P(A_2^c)P(A_3)$. Therefore, the required probability is $P(A_1^c)P(A_2^c) + P(A_3) - P(A_1^c)P(A_2^c)P(A_3) = \frac{3}{4}\times\frac{3}{4}+\frac{1}{4}-\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4} = \frac{9}{16}+\frac{1}{4}-\frac{9}{64} = \frac{43}{64}$.

28. B. Let X denote the age of the person.
$P(X \geq 65) = 0.6$ and $P(X \geq 75) = 0.2$.
$P(X \geq 75|X \geq 65) = \frac{P(X\geq 75 \cap X \geq 65)}{P(X\geq 65)} = \frac{P(X\geq 75)}{P(X\geq 65)} = \frac{0.2}{0.6} = \frac{1}{3}$.

29. C. Let D be the event that an item is defective.
Let S_i be the event that a randomly chosen item is produced in shift i, $i = 1, 2, 3$.
$P(D|S_1) = 0.02$, $P(D|S_2) = 0.03$ and $P(D|S_3) = 0.05$.
$P(D) = P(D|S_1)\,P(S_1) + P(D|S_2)P(S_2) + P(D|S_3)P(S_3) = 0.02 \times \frac{1}{3} + 0.03 \times \frac{1}{3} + 0.05 \times \frac{1}{3} = \frac{0.10}{3}$.
$P(S_1|D) = \frac{P(D|S_1)P(S_1)}{P(D)} = \frac{0.02\times\frac{1}{3}}{0.10\times\frac{1}{3}} = \frac{1}{5} = 0.2$.

30. B.
$$f(x) = \begin{cases} \frac{1}{2}, & \text{for } -1 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$
This is a symmetric distribution. So skewness = 0.
Alternatively, find $E(X) = 0$, $E(X^3) = 0$. So skewness = 0.

31. D.
A. If mean of a Poisson distribution is 1, then variance is also 1 and hence standard deviation is equal to 1. So mean and standard deviation can be equal.
B. If $X \sim Poisson\ (\lambda)$, $P(X = 1) = P(X = 2) \Rightarrow \lambda = 2$, i.e., $P(X = 0) = e^{-2}$.
C. $P(X > 0) = 1 - e^{-2} \Rightarrow P(X = 0) = e^{-2} \Rightarrow \lambda = 2$. So variance of the distribution is 2, and standard deviation is $\sqrt{2}$.
D. If $X \sim Poisson(3)$, then $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = e^{-3} + 3e^{-3} + \frac{3^2}{2}e^{-3} = 8.5\, e^{-3}$.

32. A. $P(X_1X_2 = 0) = P(X_1 = 0, X_2 = 0) + P(X_1 = 0, X_2 = 1) + P(X_1 = 1, X_2 = 0) = P(X_1 = 0)P(X_2 = 0) + P(X_1 = 0)P(X_2 = 1) + P(X_1 = 1)P(X_2 = 0) = (1 - p_1)(1 - p_2) + (1 - p_1)p_2 + p_1(1 - p_2) = 1 - p_1p_2$.

Alternatively, the possible values of X_1X_2 are 0 and 1. So $P(X_1X_2 = 0) = 1 - P(X_1X_2 = 1) = 1 - P(X_! = 1)P(X_2 = 1) = 1 - p_1p_2$.

33. A. $P(X \geq 1) = \int_1^\infty \lambda e^{-\lambda x} dx = e^{-\lambda} = 0.05 \Rightarrow \lambda = -\log_e 0.05$. Median of the distribution x_m satisfies the equation $P(X \leq x_m) = 0.5$, i.e., $1 - e^{-\lambda x_m} = 0.5$. Therefore, $x_m = -\frac{1}{\lambda}\log_e 0.5 = \frac{\log_e 0.5}{\log_e 0.05} = \frac{\log_e 2}{\log_e 20}$.

34. D. $P(X_1 = x) = \binom{2}{x}0.5^x(1-0.5)^{2-x} = \binom{2}{x}0.5^2, x = 0, 1, 2.$

$P(X_2 = x) = \binom{3}{x}0.5^x(1-0.5)^{3-x} = \binom{3}{x}0.5^3, \ x = 0, 1, 2, 3.$

$$\begin{aligned} P(X_1 + X_2 = 1) &= P(X_1 = 0, X_2 = 1) + P(X_1 = 1, X_2 = 0) \\ &= P(X_1 = 0)P(X_2 = 1) + P(X_1 = 1)P(X_2 = 0) \\ &= (0.5)^2 \times 3\,(0.5)^3 + 2(0.5)^2 \times (0.5)^3 = 5 \times (0.5)^5 \\ &= 0.15625. \end{aligned}$$

Alternatively, $X_1 + X_2$ has the binomial(5, 0.5) distribution, and therefore $P(X_1 + X_2 = 1) == 5 \times (0.5)^5 = 0.15625.$

35. A.

$$\begin{aligned} E(e^X) &= \int_{-\infty}^{\infty} e^x \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_{-\infty}^{\infty} e^{\mu + z\sigma} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}} dz \\ &= \int_{-\infty}^{\infty} e^{\mu + \frac{\sigma^2}{2}} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{(z-\sigma)^2}{2}} dz \\ &= e^{\mu + \frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{(z-\sigma)^2}{2}} dz = e^{\mu + \frac{\sigma^2}{2}} \times 1 = e^{\mu + \frac{\sigma^2}{2}}. \end{aligned}$$

36. B. $\text{corr}(2X, X^2) = \frac{\text{cov}(2X, X^2)}{\sqrt{\text{var}(2X)\text{var}(X^2)}}$.

$\text{cov}(2X, X^2) = E(2X \times X^2) - E(2X)E(X^2) = 2E(X^3) - 2E(X)E(X^2)$.

Since $X \sim N(0, 1)$, $E(X) = E(X^3) = 0$.

It follows that $\text{cov}(2X, X^2) = 2 \times 0 - 0 \times E(X^2) = 0$.

So $\text{corr}(2X, X^2) = 0$.

37. C.

		$X = x$				Total
		0	1	2	3	
$Y = y$	0	0.01	0.02	0.07	0.01	0.11
	1	0.03	0.06	0.10	0.06	0.25
	2	0.05	0.12	0.15	0.08	0.40
	3	0.02	0.09	0.08	0.05	0.24
	Total	0.11	0.29	0.40	0.20	1

$P(Y < 3|X = 2) = \frac{P(X=2, Y<3)}{P(X=2)}$.

$P(X = 2) = 0.40$.

$P(X=2, Y<3) = P(X=2, Y=0) + P(X+2, Y=1) + P(X=2, Y=2) =$
$0.07 + 0.10 + 0.15 = 0.32.$
So $P(Y<3|X=2) = \frac{0.32}{0.40} = 0.8.$

38. C. Since $y = 12 - x$, the correlation coefficient between x and y is -1.

Alternatively, find $\text{var}(x), \text{var}(y)$ and $\text{cov}(x,y) = \frac{1}{n}(\sum x_i y_i - n\bar{x}\bar{y})$.
$\bar{x} = \bar{y} = \frac{1+11}{2} = 6.$
$\text{var}(x) = \text{var}(y) = \frac{n^2-1}{12} = \frac{121-1}{12} = \frac{120}{12} = 10.$
$\text{cov}(x,y) = \frac{1}{11}(286 - 11 \times 36) = -10.$
$\text{corr}(x,y) = \frac{cov(x,y)}{\sqrt{var(x)var(y)}} = -\frac{10}{10} = -1.$

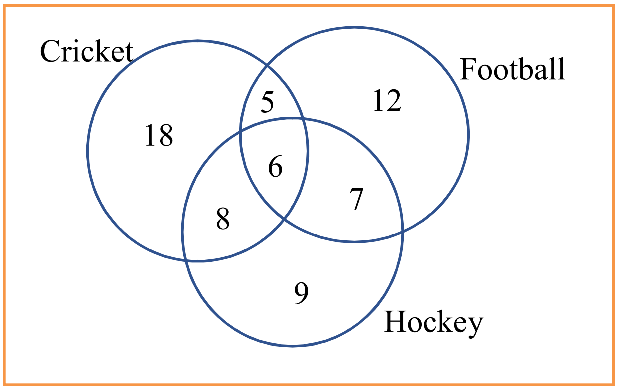
39. D. $r^2 = b_{yx} \times b_{xy} = -0.2 \times -0.18 = 0.36$, i.e., $r = \pm 0.6$.
The sign of r should be same as the regression coefficients. So $r = -0.6$.
Options A and B are not correct.
The regression equation of y on x is $y - \bar{y} = b_{yx}(x - \bar{x} \Rightarrow y = \bar{y} - 0.2(x - \bar{x})$.
The regression of x on y is $x - \bar{x} = b_{xy}(y - \bar{y}) \Rightarrow x = \bar{x} - 0.18\,(x - \bar{x})$.
Option C is not correct.
Option D is correct, since $b_{yx} = r.\frac{s_y}{s_x} \Rightarrow -0.2 = -0.6 \times \frac{s_y}{s_x} \Rightarrow s_x = 3s_y$.

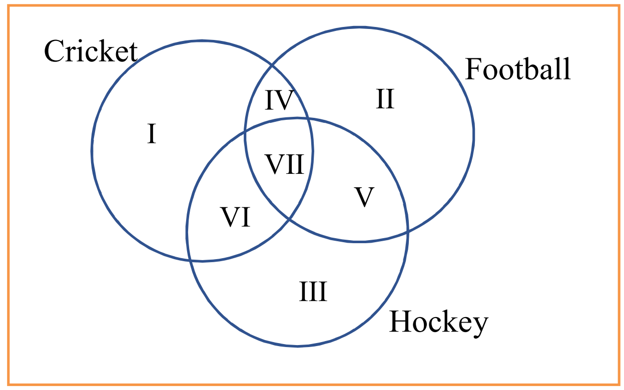
40. B. $4w = 2x + 7 \Rightarrow w = \frac{1}{2}x + \frac{7}{4}.$
$6z = 2y - 15 \Rightarrow z = \frac{1}{3}y - \frac{5}{2}.$
Regression coefficient of y on x is $b_{yx} = 3$.

Regression coefficient of z on w is $b_{zw} = \frac{\text{cov}(w,z)}{\text{var}(w)}.$

$$\text{cov}(w,z) = \text{cov}\left(\frac{1}{2}x + \frac{7}{4}, \frac{1}{3}y - \frac{5}{2}\right) = \frac{1}{2} \times \frac{1}{3} \times \text{cov}(x,y)$$
$$\text{var}(w) = \frac{1}{4}\text{var}(x)$$
$$b_{zw} = \frac{\frac{1}{2}\times\frac{1}{3}\times\text{cov}(x,y)}{\frac{1}{4}\text{var}(x)} = \frac{2}{3} \times \frac{\text{cov}(x,y)}{\text{var}(x)} = \frac{2}{3} \times b_{yx} = \frac{2}{3} \times 3 = 2.$$

Data Interpretation





41. A. The number of students who liked Cricket and Hockey but not Football lies in the region VI = 8.

42. D. The number of students who liked hockey or Football = II+III+IV+V+VI+VIII = 47.

43. B. The overall percentage change in price from January to December:

For Brand 1 $= \frac{180-125}{125} \times 100 = 44\%$

For Brand 2 $= \frac{195-1}{140} \times 100 = 39.29\%$

For Brand 3 $= \frac{216-155}{155} \times 100 = 39.35\%$

The percentage change in prince from Oct to Dec

Brand 1$= \frac{180-170}{170} \times 100 = 5.88\%$

Brand 2 $= \frac{195-185}{185} \times 100 = 5.40\%$

Brand 3$= \frac{216-205}{205} \times 100 = 5.37\%$

Table of price differences (for Questions 44 and 45)

Month	Brand 1	Brand 2	Brand 3	Brand 3-Brand1	Brand 3-Brand 2
Jan	125	140	155	30	15
Feb	125	142	160	35	18
Mar	130	145	163	33	18
Apr	135	152	172	37	20
May	145	155	178	33	23
Jun	148	162	185	37	23
Jul	153	165	188	35	23
Aug	160	180	192	32	**12**
Sep	165	182	200	35	18
Oct	170	185	205	35	20
Nov	170	187	210	**40**	23
Dec	180	195	216	36	21

44. D. The price difference between Brand 3 and Brand 2 is found to be the least in August.

45. C. The price difference between Brand 3 and Brand 1 is found to be the highest in November.

Summary computations (for Questions 46-48)

Region	Number of Families	Apr - Jun	Jul - Sep	Oct – Dec	Jan – Mar	Total	Average per family	Row total
R1	170	12	16	14.5	22.5	65	0.38	
R2	120	10	8	7	8.2	33.2	0.28	
R3	135	15	14	12	22	63	0.47	7.17
R4	160	18	20	45	34	117	0.73	13.32
R5	150	22	23	54	23	122	0.81	
R6	180	30	33.2	46.5	34	143.7	0.80	16.36
R7	110	26.5	28	22	45.2	121.7	**1.11**	
R8	200	45	62	40.5	65	212.5	1.06	24.20
Column total		178.5	204.2	241.5	**253.9**	878.1		

46. C. Average claimed amount per family over the year in a region

$$\frac{Total\ claim\ amount\ over\ the\ year\ in\ the\ region}{Number\ of\ families\ in\ the\ region} \times 100.$$

When calculated for the regions R4, R6, R7 and R8, it is seen to be highest in R7.

47. D. The quarter-wise total claim amounts are the column totals. It is found to be maximum (253.9) during Jan-Mar.

48. D. Total claim amount over the year for the regions R3, R4, R6, R8 is 878.1
Percentage claim amounts for the regions R3, R4, R6 and R8:
For R3, $\frac{63}{878.1} \times 100 = 7.17\%$.
For R4, $\frac{117}{818.1} \times 100 = 13.32\%$.
For R6, $\frac{143.7}{878.1} \times 100 = 16.36\%$.
For R8, $\frac{212.5}{878.1} \times 100 = 24.20\%$.
Only in R8, it is greater than 20%. Detailed calculations are not required. It can be obtained easily by looking at the figures.

49. A. In S1, number of oral complaints is about 900 and number of written complaints is about 3800. So the number of written complaints is more than four times the number of oral complaints.
S2: number of oral complaints = near 1000.
number of written complaints = 3500.
S3: numbers are very close - 1100 and 1300.
S4: number of oral complaints = about 750
number of written complaints = just above 1500
S5: number of oral complaints = near 700
number of written complaints = near 1500
S6: number of oral complaints = near 700
number of written complaints = near 1300
S7: number of oral complaints = about 700
number of written complaints = 2500
S8: number of oral complaints = about 1000
number of written complaints = about 3200
S9: number of oral complaints = about 950
number of written complaints = about 2450
S10: number of oral complaints = about 700
number of written complaints = about 1250.

50. A. Total number of complaints in S1 is about 900 + 3800 = 4700.
S2: 4600.
S8: 4300.
S9: 3400.

51. B. From the Figure it is clear that number of oral complaints is least in S5.

English

52. B.
53. C.
54. C.
55. D.
56. A.
57. B.
58. A.
59. B.
60. D.
61. A.
62. D.

Logical reasoning

63. D. The woman can be the wife or a sister-in-law.

64. B.

65. D. As the month begins on Saturday, the 1st, the 8th, the 15th, 22nd , and the 29th will be Saturdays, and the 2nd, the 9th, the 16th, the 23rd and the 30th will be Sundays. All five Sundays and two Saturdays (8th and 22nd) are holidays. Thus, there are 7 holidays in all.
Hence, number of working days = 30 – 7 =23.

66. A. The order is N R O P M.

67. B.

68. B. Every layer is triangular. The successive layers have 1 ball, 3 balls and 6 balls, respectively.

69. C. The faster clock runs 5 minutes faster in 1 hour.
The slower clock runs 5 minutes slower in 1 hour.
Therefore, in 1 hour, the faster clock will trace $5 + 5 = 10$ minutes more when compared to the slower clock.
In 6 hours, the faster clock will trace $10 \times 6 = 60$ minutes (an hour) more when compared to the slower clock.
In $6 \times 12 = 72$ hours, the faster clock will trace twelve hours more when compared to the slower clock. At this point, both the clocks would show the same time.

70. A. From the first sentence, it follows that there is either a partial overlap between bolts and nuts, or the nuts form a proper subset of the bolts. The second sentence says that the nuts form a subset of the screws. There is a possibility that all screws are bolts. Therefore, Statement I can be made logically.

The set of nuts need not be a proper subset of the set of screws. Therefore, Statement II cannot be made logically.