Institute of Actuaries of India

ACET October 2020 Solutions

Mathematics

1. B. Required value is $(1+0.02)^8 = 1 + 8\times 0.02 + \frac{8\times 7}{2!}(0.02)^2 + \frac{8\times 7\times 6}{3!}(0.02)^3 + \frac{8\times 7\times 6\times 5}{4!}(0.02)^4 + \cdots = 1 + 0.16 + 0.0112 + 0.000448 + 0.00000112 = 1.1716$, which is correct up to 4 decimal places.

2. D. Tangent is parallel to x-axis. Hence, $f'(x)=0$, i.e., $7.\frac{3}{2}.x^{\frac{1}{2}} - 6.\frac{1}{2}.x^{-\frac{1}{2}} = 0$. Thus, $7.\frac{3}{2}.\sqrt{x} = \frac{3}{\sqrt{x}}$, i.e., $x = \frac{2}{7}$.

3. D. Here $g(x) = \sqrt{x+g(x)}$, i.e., $g^2(x) = x + g(x)$, i.e., $2g(x)g'(x) = 1 + g'(x)$. Therefore, $g'(x) = \frac{1}{2g(x)-1}$.

4. B. The given condition implies that $\left(\frac{\pi}{2} - \sin^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}y\right) = \pi$. Thus, $\sin^{-1}x + \sin^{-1}y = 0$.

5. A. Clearly $XY = I_3$, which means
$$\frac{1}{10}\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & c\\ 1 & -2 & 3\end{bmatrix} = I_3, \text{ i.e., } \frac{1}{10}\begin{bmatrix}10 & 0 & 5-c\\ 0 & 10 & c-5\\ 0 & 0 & c+5\end{bmatrix} = I_3.$$
By equating the (1,3) elements of XY and I_3, we have $\frac{5-c}{10} = 0$, i.e., $c = 5$. The same result may be obtained by considering any of the other elements of the third column.

6. B. Given expression = $9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots} = 9^{\frac{1}{3}/\left(1-\frac{1}{3}\right)} = 9^{\frac{1}{2}} = 3$.

7. C. Here $[a] = -1, [b] = 0, [c] = 1$. Thus the given determinant becomes $\begin{vmatrix}0 & 0 & 1\\ -1 & 1 & 1\\ -1 & 0 & 2\end{vmatrix} = 1$.

8. D. The given expression is equivalent to $1 - 1 + 1 - 1 + \cdots + (-1)^n$. Its value is either 0 or 1, depending on whether n is odd or even.

9. C. Here $2^m - 2^n = 112$, i.e., $2^n(2^{m-n}-1) = 112 = 2^4(2^{7-4}-1)$. After equating the odd and even factors of the sides, we obtain the unique solution $m - n = 3, n = 4$.

10. C. Required value $= \int_0^1 e^{x+[x]}dx + \int_1^2 e^{x+[x]}dx = \int_0^1 e^x dx + \int_1^2 e^{x+1}dx = (e-1) + (e^3 - e^2) = (e^2+1)(e-1)$.

11. C. As the three vectors are coplanar, the third column of the matrix $\begin{pmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{pmatrix}$ can be written as a linear combination of the first two. Therefore, the rank is at most 2. For the

rank to be equal to 1, both a and b should be equal to 1, which is not the case. Hence the rank is 2.

12. B. $\tan\left(\frac{10\pi}{3}\right) = \tan\left(3\pi + \frac{\pi}{3}\right) = \sqrt{3}$; $\tan\left(\frac{9\pi}{4}\right) = \tan\left(2\pi + \frac{\pi}{4}\right) = 1$; $\tan\left(\frac{7\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$. Clearly, $\sqrt{3}, 1, \frac{1}{\sqrt{3}}$ are in GP.

13. A. $\frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = -i$. Therefore $(-i)^m = -1 = (-i)^2$. The least positive integer is $m = 2$.

14. A. Clearly, $\frac{x+2}{x} > 0$, i.e. $x \in (-\infty, -2) \cup (0, \infty)$.
Also, $1 \geq \log_{\frac{1}{2}}\left(\frac{x+2}{x}\right) = \frac{\ln\left(\frac{x+2}{x}\right)}{\ln\left(\frac{1}{2}\right)} = \frac{\ln\left(\frac{x}{x+2}\right)}{\ln 2}$, i.e., $\frac{x}{x+2} \leq 2$. In view of the preceding range restriction on x, we have $x \in (-\infty, -4] \cup (0, \infty)$.

15. D. The n-th term of the given series $t_n = \frac{1}{1+2+3+4+\cdots+n} = \frac{2}{n(n+1)} = 2\left(\frac{1}{n} - \frac{1}{n+1}\right)$. The required sum is

$$\lim_{n\to\infty} \sum_{i=1}^{n} t_i = \lim_{n\to\infty} 2\left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right]$$
$$= \lim_{n\to\infty} 2\left(1 - \frac{1}{n+1}\right) = 2.$$

16. A. The given integral $I = \int_0^a \frac{dx}{1+e^{f(x)}} = \int_0^a \frac{dx}{1+e^{f(a-x)}} = \int_0^a \frac{dx}{1+e^{-f(x)}}$ (from the given condition), which simplifies to $\int_0^a \frac{e^{f(x)}dx}{1+e^{f(x)}}$. Hence $I + I = \int_0^a \frac{dx}{1+e^{f(x)}} + \int_0^a \frac{e^{f(x)}dx}{1+e^{f(x)}} = \int_0^a dx = a$, i.e., $I = \frac{a}{2}$.

17. A. A vector perpendicular (orthogonal) to the given vectors is the outer product

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 4\hat{\imath} - \hat{\jmath} - 3\hat{k}.$$

The corresponding unit vector is $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{1}{\sqrt{26}}\left(4\hat{\imath} - \hat{\jmath} - 3\hat{k}\right)$.
Two possible unit vectors orthogonal to the given vectors are $\pm\frac{1}{\sqrt{26}}\left(4\hat{\imath} - \hat{\jmath} - 3\hat{k}\right)$.
Acute angle with $\hat{k}$ means that the required vector has positive coefficient of $\hat{k}$.
Thus the required unit vector is $\frac{1}{\sqrt{26}}(-4\hat{\imath} + \hat{\jmath} + 3\hat{k})$.

18. C. The digit in the unit place is equivalent to the remainder after dividing the number by 10. We note that each of $5!, 6!, 7!, \ldots, 99!$ is divisible by 10. So also is their sum. Thus we get the remainder zero, when $5! + 6! + 7! + \cdots + 99!$ is divided by 10. Now $1! + 2! + 3! + 4! = 33$, which when divided by 10, gives the remainder 3. In this way we conclude that $1! + 2! + 3! + 4! + 5! + \cdots + 99!$ gives the remainder 3, when divided by 10. Therefore, the required digit in the unit place is 3.

19. D. Case $0 \leq \alpha < \beta$: Here $|x| = x, |\alpha| = \alpha, |\beta| = \beta$. Thus $I = \beta - \alpha = |\beta| - |\alpha|$.
Case $\alpha < \beta \leq 0$: Here $|x| = -x, |\alpha| = -\alpha, |\beta| = -\beta$. So $I = -(\beta - \alpha) = |\beta| - |\alpha|$.

Case $\alpha < 0 < \beta$. Here $|\alpha| = -\alpha, |\beta| = \beta$. Thus $I = \int_\alpha^0 \frac{|x|}{x} dx + \int_0^\beta \frac{|x|}{x} dx = \int_\alpha^0 (-1) dx + \int_0^\beta 1 dx = \alpha + \beta = |\beta| - |\alpha|$.

20. D. Let $Y = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, Z = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. The given equation is $YXZ = I_2$.

Pre-multiplying by Y^{-1}, we have $(Y^{-1}Y)XZ = Y^{-1}I_2$, i.e., $XZ = Y^{-1}$.

Post-multiplying by Z^{-1}, we obtain $X(ZZ^{-1}) = Y^{-1}Z^{-1}$, i.e., $X = Y^{-1}Z^{-1}$. Now

$$Y^{-1} = \frac{1}{4-3}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}, \qquad Z^{-1} = \frac{1}{9-10}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}.$$

$$\text{Thus,} \quad X = Y^{-1}Z^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

Statistics

21. B. Since variance ≥ 0, we have $\frac{1}{n}\Sigma x_i^2 \geq \left(\frac{1}{n}\Sigma x_i\right)^2$, i.e., $\frac{400}{n} \geq \left(\frac{80}{n}\right)^2$ or $n \geq 16$.

22. D. All the summary statistics except the minimum are smaller in the case of y.

23. C. The restricted sample space when the chosen number is known to be prime is $\{2,3,5,7,11,13,17,19,23\}$. Since there are just three prime numbers greater than 13 in this set, the probability that the chosen number is one of these is $3/9 = 1/3$.

24. D. Total number of arrangements = $\frac{10!}{2!}$, of which (i) two I's come together in 9! choices and (ii) do not come together in $\frac{10!}{2!} - 9!$ choices. Hence the required probability is

$$\frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = 1 - \frac{9!}{9! \times \frac{10}{2}} = \frac{4}{5}.$$

25. A.

$$P(X^c) = \frac{3}{5}, P(Y^c) = \frac{7}{10}, P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{5} + \frac{3}{10} - \frac{1}{5}$$
$$= \frac{1}{2}, P(X^c \cap Y^c) = 1 - P(X \cup Y) = \frac{1}{2}.$$

$$\text{Thus } P(X^c|Y^c) = \frac{P(X^c \cap Y^c)}{P(Y^c)} = \frac{1/2}{7/10} = \frac{5}{7}.$$

26. D. The mean is 15; the mode and the median are 10.

27. B. The probability of getting a spade is $\frac{13}{52} = \frac{1}{4}$ and that of a club is also $\frac{1}{4}$. The probability of getting a red card is $\frac{26}{52} = \frac{1}{2}$. In 6 draws, 2 spades, 2 clubs and two red cards can be selected in $\binom{6}{2} \times \binom{4}{2} \times \binom{2}{2} = 15 \times 6 \times 1 = 90$ ways. Thus the required probability is

$$90 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{90}{2^{10}}.$$

28. A. Let E_1 be the event that the ball transferred was white, and E_2 be the event that the ball drawn from urn B is white.

$$P(E_1|E_2) = \frac{P(E_2|E_1)P(E_1)}{P(E_2|E_1)P(E_1) + P(E_2|E_1^c)P(E_1^c)} = \frac{(2/7) \times (2/3)}{(2/7) \times (2/3) + (1/7) \times (1/3)}$$
$$= \frac{4}{5}.$$

29. B. Here $n = 8, p = \frac{1}{2}$. X follows binomial distribution.

$$
\begin{aligned}
P(|X-4| \leq 2) = P(2 \leq X \leq 6)& \\
&= 1 - [P(X=0) + P(X=1) + P(X=7) + P(X=8)] \\
&= 1 - \left[1 \times \left(\frac{1}{2}\right)^8 + 8 \times \left(\frac{1}{2}\right)^8 + 8 \times \left(\frac{1}{2}\right)^8 + 1 \times \left(\frac{1}{2}\right)^8\right] \\
&= 1 - \left(\frac{1}{2}\right)^8 \times (1+8+8+1) = 1 - \frac{18}{256} = \frac{119}{128}.
\end{aligned}
$$

30. C. Since the colour of the first draw is not known, it does not change anything. The probability of a red ball being chosen is $5/(5+4) = 5/9$.

Alternative method: Consider the events

E_1: a red ball is drawn in the 1st draw;

E_2: a blue ball is drawn in the 1st draw;

X : a red ball is drawn in the 2nd draw.

$P(E_1) = \frac{5}{9}, P(E_2) = \frac{4}{9}, P(X|E_1) = \frac{4}{8}, P(X|E_2) = \frac{5}{8}$. (Since 1st ball is not replaced)

$$P(X) = P(E_1).P(X|E_1) + P(E_2).P(X|E_2) = \frac{5}{9}.\frac{4}{8} + \frac{4}{9}.\frac{5}{8} = \frac{5}{9}.$$

31. A. The sum of the first sample is $15 \times 100 = 1500$. The combined sum is 3900. Therefore the sum of the second sample is $3900 - 1500 = 2400$, and the mean is $2400/150 = 16$.

The sum of squares of the first sample is $(3^2 + 15^2) \times 100 = 23400$. The combined sum of squares is 64200. Therefore the sum of squares of the second sample is $64200 - 23400 = 40800$. The second moment is $40800/150 = 272$ and the variance is $272 - 16^2 = 16$. Hence the standard deviation is 4.

32. C. Let the replacement time be T and the warranty period be w. We need

$P(T \leq w) = 0.02$, i.e., $P\big((T-91)/18 \leq (w-91)/18\big) = 0.02$,

i.e., $P(Z \leq (w-91)/18) = 0.02$,

i.e., $P(Z \leq -(w-91)/18) = 0.98$,

i.e., $-\frac{(w-91)}{18} = 2.05375$,

i.e., $w = 91 - 18 \times 2.05375 = 54$ months, or 4 years 6 months (approximately).

33. C. If the number of waiting customers is X, then $P(X = n) = e^{-3}3^n/n!$. We have the following table of probabilities.

n	0	1	2	3	4	5
$P(X = n)$	0.050	0.149	0.224	0.224	0.168	0.101
$P(X \leq n)$	0.050	0.199	0.423	0.647	0.815	0.916

Thus, 5 counters would suffice.

34. B. The number of necessary draws, X, has the geometric distribution with success probability $p = 0.85$. Therefore $E(X) = \frac{1}{p} = \frac{1}{0.85} = 1.176$.

35. B. The slope of the regression of Y on X (where Y is written in terms of X) is $r\frac{\sigma_Y}{\sigma_X}$, while the slope of the regression of X on Y (where X is written in terms of Y) is $r\frac{\sigma_X}{\sigma_Y}$. When the latter equation is written with Y expressed in terms of X, the slope is $\frac{1}{r} \times \frac{\sigma_Y}{\sigma_X}$, which is greater than $r\frac{\sigma_Y}{\sigma_X}$. Thus, the correct regression of Y on X is the one having smaller magnitude of coefficient of X when Y is written in terms of X.
Here, the first equation has smaller slope, and hence must be the regression of Y on X. From the given equations $b_{YX} = \frac{4}{5} = 0.8$; $b_{XY} = \frac{9}{20} = 0.45$. This implies $r^2 = b_{YX}b_{XY} = 0.8 \times 0.45 = 0.36$, that is $r = 0.6$ (positive since $b_{YX} > 0$).
Further, $b_{YX} = r\frac{\sigma_Y}{\sigma_x}$, i.e., $\sigma_Y = b_{YX}\frac{\sigma_x}{r} = 0.8 \times \frac{3}{0.6} = 4$.

36. C. The expected value of the numbers on the face that turns up from the throw of a single dice is $E(X) = \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6] = \frac{7}{2}$. Therefore, the sum of the expected values of two such random variables is 7.

37. A.

$$P(X > 6|X > 0) = \frac{P(X > 6, X > 0)}{P(X > 0)} = \frac{P(X > 6)}{P(X > 0)} = \frac{\int_6^\infty ce^{-x/3}dx}{\int_0^\infty ce^{-x/3}dx} = \frac{3ce^{-6/3}}{3ce^{-0/3}} = e^{-2}.$$

38. D.

39. A. The probability of any candidate having score above 30 is $\int_{30}^{100} \frac{1}{100} dx = 0.7$. Therefore, the number of candidates having score above 30 is a binomial random variable with $n = 1000$ and $p = 0.7$. Its variance is $np(1 - p) = 210$.

40. B. $Cov(X - Y, X) = Cov(X, X) - Cov(Y, X) = 3 - 0 = 3$.

Data Interpretation and Data Visualization

41. B. The maximum possible score by Rahul in Match 1 is 24, which is 10% of 240. In Match 3, 10% of 210 is 21, which is higher than 15. Hence 14 is the maximum possible score by Rahul in Match 3. Thus, the maximum contribution by Rahul is

$$\frac{(24+90+14+45)}{(240+270+210+170)} = \frac{173}{890} = 19.4\%.$$

42. D. The largest and smallest missing scores in each match are summarized below.

Missing score	Match 1	Match 2	Match 3	Match 4
Total	$240-78-67-50=45$	$270-68-65-90=47$	$210-100-15-70=25$	$170-45-50-48=27$
Largest	$0.1\times 240=24$	$0.1\times 270=27$	$15-1=14$	$0.1\times 170=17$
Smallest	$45-24=21$	$47-27=20$	$25-14=11$	$27-17=10$

The minimum missing scores of Rahul are $45-24=21$ and $25-14=11$ for matches 1 and 3, respectively (see table). Thus, his minimum total score is $21+90+11+45=167$. Likewise, minimum total score of Virat is $78+65+11+50=204$, that of Rohit is $21+20+100+10=151$, that of Laxman is $67+68+15+48=198$ and that of Vijay is $50+20+70+10=150$.

43. C. Refer to the maximum values of the missing scores given in the solution of the previous problem. Thus, the maximum possible score of Rahul, Virat, Rohit, Laxman and Vijay are $24+90+14+45=173$, $78+65+14+50=207$, $24+27+100+17=168$, $67+68+15+48=198$, $50+27+70+17=164$, respectively. Comparing these numbers with the minimum scores given in the solution of the previous problem, we conclude that Virat and Laxman undoubtedly occupy the top two ranks, but the ranks of the other players cannot be guessed because of the missing information.

44. A. Royalty to be paid is $(25000/15\%)\times 10\% = \text{Rs.}\,16667$.

45. D. Cost of paper per book is $(200/1.15)\times 30\% = \text{Rs.}\,52.2$.

46. A. Royalty is less than printing cost by $(15-10)/15 = 33.3\%$.

47. C. Total salary paid during the period is Rs. 1210 lakh. The total bonus paid during this period is Rs. 17 Lakh, which is 1.4% of Rs. 1210 lakh.

48. D. Total taxes paid during the period 2015-2019 is Rs. 401 lakhs. The total fuel and transport expenditure during this period is Rs. 536 lakh. The ratio is approximately $3:4$.

49. D. The largest percentage increase in salary happened in the year 2019, when it was $84/236 = 35.6\%$. The largest percentage increase in fuel and transport happened in the year 2018, when it was $32/91 = 35.2\%$. The largest percentage increase in bonus happened in the year 2017, when it was $1.32/2.52 = 52.4\%$. The largest percentage increase in interest on loans happened in the year 2016, when it was $9.1/13.4 = 67.9\%$.

50. B. Increase in turnover of Company B was $170 - 150 = \text{Rs.}\,20$ crores. Company C would have seen the same increase if its turnover in the second year had been $140 + 20 = \text{Rs.}\,160$ crores.

51. C. Difference of averages, which is the same as average of differences, is $(20 + 20 + 10 + 10 + 40)/5 = \text{Rs.}\,20$ crores.

English

52. D

53. A

54. C

55. B

56. B

57. C

58. B

59. A

60. B

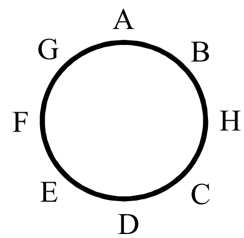
61. A

62. B

Logical Reasoning

63. B. Number of students in football team is $n(F) = 10$.
Number of students in science club is $n(S) = 14$.
Number of students in both football team and science club is $n(F \cap S) = 5$.
Therefore, number of students only in football team is $n(F) - n(F \cap S) = 10 - 5 = 5$.
Number of students only in science club is $n(S) - n(F \cap S) = 14 - 5 = 9$.
Number of students in only one of the two groups is $5 + 9 = 14$.

64. C. The following sitting arrangement emerges from the given description.



65. A.

66. C. The time gap between 3 pm and 7 am is 8 hours, which is two-thirds of 12 hours. Since 12 hours account for 360º, eight hours cover 240º.

67. D. Since 2004 was a leap year, the two given dates are separated by $366 = 7 \times 52 + 2$ days. Therefore, 8^{th} February 2004 was two days before Tuesday.

68. D. If Gopal belonged to union *A*, he could not have said that he always lies. If he belonged to union *B*, then the utterance becomes a truth that he could not possibly speak.

69. B. By negating the given implication, we can conclude that if any one of Sunil, Mohinder and Chandrasekhar is found not to attend the event, then Kapil would not have attended it.

70. D. There is no given information about the overlap between Lillies and flowers.