

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

10th November 2008

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q 1) i.) Describe what the term $m|\bar{a}_n$ denotes. (2)

ii.) Also prove that $m|\bar{a}_n = v^m \bar{a}_n$ (3)

[5]

Q 2) Manjit makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest of

$$\delta_t = \frac{t^2}{100}, t > 0.$$

The amount of interest earned from time 3 to time 6 is X . Calculate the value of X . [5]

Q 3) A government security offers coupon of 4% per annum payable annually and is redeemed at par.

a) How the price of the security will change if future rate of interest increases from 10% to 12% over all terms, given the term of the security as (a) 5 years and (b) 25 years? (4)

b) Calculate the discounted mean terms of the government security at the tenures mentioned above at 10%. Comment on your results. (3)

[7]

Q 4) i.) Ramona is evaluating to open one of the two equally risky studios, Studio X or Studio Y. She has collected the following information:

	Investment at time zero	Expected sales	Probability of expected sales	Expected annual cash flow beginning at time 1
Studio X	300	High	0.6	Perpetuity of 120
		Low	0.4	Perpetuity of 40
Studio Y	200	High	0.5	Perpetuity of 100
		Low	0.5	Perpetuity of 50

The net present value of expected cashflow in Studio X is 800. What is the net present value of expected cashflow in Studio Y? (4)

ii.) Ayesha can receive one of the following two payment streams:

a) 100 at time 0, 200 at time n , and 300 at time $2n$

b) 600 at time 10

At an annual effective interest rate of “ i ”, the present values of the two streams under (a) and (b) are equal. Given $v^n = 0.75941$, determine i . (4)

[8]

- Q 5)** During the period 1 Jan 02 and 1 Jan 06 the retail sale price index moved as shown in the table below:

Time t (years):	Retail price index:
1 Jan 02	341.4
1Jan 03	366.6
1Jan 04	382.0
1Jan 05	401.8
1Jan 06	421.5

Anjali purchased an annuity certain of Rs.15,000 payable annually in arrear for four years during 1 Jan 02 to 1 Jan 06 for Rs. 48,750.

- (a) Calculate the effective real rate of return she earned on her investment. (3.5)
- (b) Calculate the effective money rate of return she earned on her investment (3.5)
- (c) Using answer of part (a) and (b), calculate average inflation during the period of investment and compare this with average inflation from the price index above and comment. (3)

[10]

- Q 6)** i) Actuarial students study group had a fund balance of 75 on January 1 and 60 on December 31 in a particular calendar year. At the end of every month during the year, the group deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31. Calculate the money-weighted rate of return for the year. (5)
- ii) Explain why as a measure of investment performance the money-weighted rate of return is not entirely satisfactory? (2)
- iii) What are the disadvantages of both the time-weighted and money-weighted rates of return? (3)

[10]

- Q 7)** A construction company XYZ limited wants to bid for a project to build a 60-kilometer long eight lanes two-way road on build, operate and transfer basis. Under this project the company will construct the road and maintain it for 20 years from the date the road is opened for users and will earn revenue through toll charges from the users of the road over this period. The Company expects to complete the road and open it for users in exact 2 years' time from the date of start of construction. The construction would start from 1 January 2009, exactly 6 months after the date of appraisal.

The company conducted an appraisal of the project and estimated the following revenue and costs from the project:

Revenue:

A grant of 100 crore from the government after one year from the construction start date, income from toll charges received continuously at the rate of Rs. 8 crore per quarter in first quarter after the road is opened for users and increasing thereafter by 2.44% per quarter.

Cost:

One time appraisal cost of 50 lakh at the date of appraisal, Construction cost of 150 crore per annum payable monthly in arrears for 2 years, maintenance cost of 30 lakh per quarter payable monthly in arrears increasing by 1.94% per quarter

The company is also required to pay a percentage of its revenue through toll charges during the above 20 years period to the government.

The company considers the project viable if it provides a positive present value at rate of interest of 20% pa convertible quarterly. Calculate the maximum percentage of the revenue that can be paid to the Government. [12]

- Q 8)** (i) What are the main advantages and disadvantages to the government of issuing bonds at a set price as opposed to by tender? (4)
- (ii) Rahul purchased index linked bond of Rs.100,000 nominal at the issue date, at an issue price of Rs.95 per 100 nominal. He held it till redemption.

The details of the bonds are as under

Type	Indexed linked bonds
Issue date	1 June 2002
Redemption date	1 June 2004
Nominal Coupon rate	4 % per annum
Coupon payment frequency	Payable half yearly in arrears
Nominal Redemption price	Rs.100

The actual coupon and redemption payments were indexed according to the increase in the consumer price index from 6 months before the bond issue date till 6 months before the coupon or redemption payment dates.

The values of the consumer price index in the relevant months were:

Date	Consumer price index
Dec-01	200
Jun-02	204
Dec-02	214
Jun-03	222
Dec-03	226
Jun-04	236

Calculate his cash flows from this investment, before tax. (2.5)

(iii) Rahul is subject to income tax rate of 25% and capital gains tax rate of 35%. When calculating the capital gain amount which is subject to the capital gain tax, the price paid for the investment is indexed in line with the increase in the retail price index between the months in which the investment was purchased and the month in which it was redeemed.

a) Calculate the capital gains tax liability in respect of his investment. (1.5)

b) Calculate the net effective yield per annum obtained by him on these bonds. (4)
[12]

Q 9) (i) Describe the two methods of calculating the outstanding loan amount? (2)

(ii) Define the flat rate of interest. (2)

(iii) Sushil Kumar wants to build a gymnasium, for which he takes out a loan for Rs.500,000 for 10 years. The effective rate of interest on his loan is 12% per annum, and the repayments towards the loan are to be made monthly in arrears.

a) What is the monthly repayment towards his loan? (2)

b) What is the interest amount payable in the fifth year? (2)

c) What is the capital amount repaid in the 40th installment? (2)

d) He wants to make further additions to the equipments in his gymnasium and at the end of the 7th year wants to borrow a further amount of Rs.100,000. He wants to repay the total balance by monthly payments over a period of 3 years, assuming that there is no change in the rate of interest, calculate the revised monthly payment amount. (3)
[13]

- Q 10)** (i) What are the various factors that influence the variation in interest rates over time? (6)
- (ii)
- (a) Calculate the gross redemption yields (spot rates of interest) for 1-year, 2-year, 3-year and 4-year zero coupon bonds assuming the expectations theory explanation of the yield curve holds, where short-term, one-year annual effective interest rates are currently 9%; they are expected to be 8% in one years time, 7% in two years time and 6% in three years time. (4)
- (b) Calculate the gross redemption yield of a bond that is redeemed at par in exactly four years and pays a coupon of 5 per annum annually in arrears. The price of a coupon-paying bond is calculated by discounting individual payments from the bond at the zero-coupon bond yields in (a). (4)
- (c) Describe the relationship between price and yields of fixed income securities in the following situations. (2)
- i.) Institutional investors suddenly decide to invest less in equities and more in fixed-interest securities
- ii.) Bond prices rise
- iii.) Government issues many more stocks
- iv.) Demand for fixed-interest securities falls
- (d) If the “n” year spot rates can be approximated by the function $0.10 - 0.05 \exp(-0.1n)$, calculate the one-year forward rate at time 8. (2)

[18]
