Institute of Actuaries of India

CT1: Financial Mathematics

Indicative Solution

November 2008

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

Soln.1 Indicative Solution

(i) It denotes the present value of a continuously payable annuity of 1 per unit for *n* time units, deferred for *m* time units, where *m* is a non-negative number

(ii)

$$_{m|}\bar{a}_{\overline{n|}} = \int_m^{m+n} e^{-\delta t}dt$$

$$= e^{-\delta m}\int_0^n e^{-\delta s}ds$$

$$= \int_0^{m+n} e^{-\delta t}dt - \int_0^m e^{-\delta t}dt$$

Hence:

$$_{m|}\bar{a}_{\overline{n|}} = \bar{a}_{\overline{m+n|}} - \bar{a}_{\overline{m|}}$$

$$= v^m \bar{a}_{\overline{n|}}$$

[5]

Soln. 2

$$\int \frac{t^2}{100}dt = \frac{t^3}{300}$$

$$100e^{\left.t^3/300\right]_0^3} = 109.41743$$

$$(109.41743 + X)e^{\left.t^3/300\right]_3^6} - (109.41743 + X) = X$$

$$(109.41743 + X)(1.8776106) - 109.41743 - X = X$$

$$96.025894 = 0.1223894\,X$$

$$X = 784.59$$

[5]

Soln. 3

Price of the security

P=C an+ R vn where C is the coupon and R is the redemption amount

The DMT

DMT= [C (Ian) + R n vn]/P

The price of the 5 year stock would fall from 77.26 to 71.16 ie a fall of 7.9%

The price of the 25 year stock would fall from 45.54 to 37.25 ie a fall of 18.19%.

The change in interest rate has a greater effect on the longer 25 year stock, which has a
DMT of 11.81 years (based on 10% interest), than it has on the shorter 5 year stock, which has a DMT of 4.57 years.

[7]

Soln. 4

(i) Let I equal the interest rate used for discounting.
The expected net present value of investing in studio X
= 0.60 * (–300 + 120/I) + 0.40 * (–300 + 40/I)
= –300 + 88/I.
This expression is set equal to 800. Thus, I = 0.08.
The expected net present value of investing in shop Y
= 0.50 * (–200 + 100/0.08) + 0.50 * (–200 + 50/0.08) = 737.5 .

(ii) $100 + 200\, v^{n} + 300\, v^{2n} = 600\, v^{10}$
100 + 151.882 + 173.01
= 424.89 = 0.708 = v^{10} . i = 3.5%

[8]

Soln. 5

(a) Work in time $t = 0$ monetary values
$48{,}750 = 15000 *(341.4 /366.6 *v +341.4 / 382.0\, v^2+341.4 / 401.8 *v^3 + 341.4/421.5 * v^4)$
where v = 1/(1+i') with i' = real rate of return

Try 3% RHS = 48656.53

2.5% RHS = 49229.89

Implies i
= 0.025 + 0.005 * (49229.89- 48750)/(49229.89-48656)
= 0.0291 i.e. 2.9%

(b) $48{,}750 =15000\, a_{\overline{4}|}$ at i% p.a
$3.25= a_{\overline{4}|}$ at i% p.a
At 8%, $a_{\overline{4}|}$ = 3.3121
At 9% , $a_{\overline{4}|}$ = 3.2397
i = 0.08 + 0.01 * (3.3121-3.25)/(3.3121-3.2397)
= 0.089
i.e. 8.9%

(c)

We know that

$(1+ i)/ (1+e) \cong (1+i')$

where *e* = average annual rate of inflation over the period.

$1.089/(1+e) \cong 1.029$
which implies 5.8% p.a. inflation over the period

The Average inflation as per price index
= (421.5/341.4)^(1/4) = 5.4% , the difference is due to rounding

[10]

Soln. 6

(i) The equation of value is

$75(1+i) + 10\,[(1+i)^{11/12}+(1+i)^{10/12}+(1+i)^{9/12}+(1+i)^{8/12}+(1+i)^{7/12}+(1+i)^{6/12}+(1+i)^{5/12}+(1+i)^{4/12}+(1+i)^{3/12}+(1+i)^{2/12}+(1+i)^{1/12}+1] - [5(1+i)^{10/12}+25(1+i)^{6/12}+80(1+i)^{2.5/12}+35(1+i)^{2/12}] = 60$

As a first guess we could use the first order binomial expression replacing $(1+i)^n$ with (1+ni)

Therefore,
75+10*12+i [75+10(11/12+10/12 +9/12+8/12 +7/12+6/12 +5/12+4/12 +3/12+2/12+1/12] –i [5*10/12+25*6/12+80*2.5/12+35*2/12] = 60

i = 10/[75+10(11/12+10/12 +9/12+8/12 +7/12+6/12 +5/12+4/12 +3/12+2/12+1/12] - [5*10/12+25*6/12+80*2.5/12+35*2/12]=11.01%

Interpolating
i=11% LHS=60.02
i=12% LHS=75
Therefore i= 11%

(ii) Money-weighted rate of return is sensitive to the amounts and timing of the net cashflows. If, say, we are assessing the skill of the fund manager, this is not ideal, as the fund manager does not control the timing or amount of the cashflows – he or she is merely responsible for investing the positive net cashflows and realising cash to meet the negative net cashflows.

(iii)
- The disadvantages of both the time-weighted and money-weighted rates of return are that the calculation requires information about all the cashflows of the fund during the period of interest.
- In addition, the TWRR requires the fund values at all the cashflow dates.
- A disadvantage of the MWRR is that the equation may not have a unique solution – or indeed any solution.

[10]

Soln.7

Present Value of revenue at 1 Jan 2009

Working in units of quarter, the effective rate of interest per quarter is 5%

Present value of the grant

= 100 x v4

= 100 x 0.82270 = 82.270

Present value of income from toll charges at 1 Jan 2009

= 8 v8 [abarΓ1 + 1.0244v abarΓ1 + 1.0244^2 v2 abarΓ1 + 1.0244^3 v3 abarΓ1 + ---------------- + 1.0244^79 v79 abarΓ1] @ 5%

= 8 v8 abarΓ1 (1+1.0244v + 1.0244^2 v2 + 1.0244^3 v3 + ---------------- + 1.0244^79 v79)

=8 v8 abarΓ1 @ 5% (adueΓ80 at j%)

Where 1.0244/1.05 = 1/(1+j), implies j = 2.5% approx

abarΓ1 @ 5% = i/δ x aΓ1 = 1.024797 x 0.9524 = 0.97602

adueΓ80 @ 2.5% = 35.3222

= 8 x 0.67684 x 0.97602 x 35.3222 = 186.67

Present value of costs at 1 Jan 2009

Working in units of quarter, the effective rate of interest per month is 5%

Value of appraisal cost at 1 Jan 2009

= 0.50 (1.05)^2 = 0.55125

The Present value of costs of construction of the road at 1 Jan 2009 at the rate of 5% per quarter effective

= 150/4 * $a^{(3)}_{\Gamma 8}$ = 37.5 x i/i(3) $a_{\Gamma 8}$

i(3) = 3x[(1.05)(1/3) –1] = 0.049189

$i/i^{(3)}$ = 0.05/0.049189 = 1.016487

= 37.5 x 1.016487 x 6.4632 = 246. 36595

Present value of cost of road maintenance

= 0.30 v8 [$a^{(3)}_{\Gamma 1}$+ 1.0194v $a^{(3)}_{\Gamma 1}$ + 1.0194^2 v2 $a^{(3)}_{\Gamma 1}$+ 1.0194^3 v3 $a^{(3)}_{\Gamma 1}$ + ---------------- + 1.0194^79 v79 $a^{(3)}_{\Gamma 1}$] @ 5%

= 0.30 v8 $a^{(3)}_{\Gamma 1}$ [1 + 1.0194v + 1.0194^2 v2 + 1.0194^3 v3 + ---------------- + 1.0194^79 v79] @ 5%

=0.30 (v8 $a^{(3)}_{\Gamma 1}$) @ 5% x ($adue_{\Gamma 80}$ at j '%)

where 1.0194/1.05 = 1/(1+j'),implies j' = 3% apprx

v8 @ 5% = 0.67684

$a^{(3)}_{\Gamma 1}$) @ 5% = i/i(3) $a_{\Gamma 1}$ = 1.016487 x 0.9524 = 0.9681 adue$_{\Gamma 80}$ at 3%) = 31.0934

= 0.30 x 0.67684 x 0.9681 x 31.0934

= 6.1121

Let maximum x % of the toll charges is passed on to the government

The present value of the revenue given to the government

= x% of present value of income = x% of 186.67

For the project to be viable the

Present value of cost = Present value of revenue

0.55125 + 246. 36595 + 6.1121 + x/100 x 186.67

= 82.270 + 186.67

x = 8.52 %

[12]

Soln. 8

(i) *Advantages of issuing bonds at a fixed price*

- The government will know the price that investors will pay at outset and so it will know the cost of borrowing money.
- It will be administratively less difficult than by tender.

Disadvantages of issuing bonds at a fixed price

- Investors may be willing to pay more for the bond than the set price and so the money could be borrowed more cheaply if a tender is used.
- Insufficient investors may be prepared to pay the set price causing only part of the offer to be sold. The government may not meet its finance requirements.

(ii) Cashflows in ‘000 s

Time	Description	Cashflows
Jun-02	Purchase price	95
Dec-02	Coupon	2.04
Jun-03	Coupon	2.14
Dec-03	Coupon	2.22
Jun-04	Coupon	2.26
Jun-04	Redemption	113

(iii)(a) CGT due is : 0.35* (113 -95* 236/204) = 1.0843

(iii)(b) Yield is such that
95 = (2.04*v^0.5+ 2.14*v+2.22*v^1.5+ 2.26*v^2) *0.75 + (113-1.0843) *v^2
Roughly:
95 = 6.495*v +111.9157* v^2
Implies V =-6.495+(6.495^2+4*95*111.9157)^0.5

2*111.9157
implies i ~12.01
at 12% RHS is 94.8532
at 11% RHS is 96.5309
i = 11.91%

[12]

Soln.9

(i) We can find the loan outstanding prospectively or retrospectively. A prospective method involves finding the present value of future cashflows.
A retrospective method involves calculating the accumulated value of the initial loan less the accumulated value of the repayments to date.

(ii) The flat rate of interest is defined as the total interest paid over the whole transaction, per unit of initial loan, per year of the loan.

(iii)(a) Working in months
i= (1.12)^(1/12)-1=0.00948879

Let P be the monthly repayment amount

P abar120=500000

P= 500000/abar120=500000/71.45553=6997.36

The total repayment in year 5 is =12*6997.36=83,968.30

(b)

The capital o/s at start of year =P a72 =P*51.9949
The capital o/s at end of year =P a60 = P*45.58779

Capital repaid therefore is =P(51.9949-45.58779)=44832.84
Therefore interest repaid = 83,968.30-44832.84=39135.46

The capital o/s immediately after the 39th instalment is
Pa81=6997.36*56.3456=394,270.18

(c)

The interest paid in the 40th instalment is =
394,270.18 *0.00948879=3741.148107

The capital part therefore is = 6997.36- 3741.148107 = 3,256.21

We first need to find what is the o/s amount on his original loan, which is

(d) P a(120-84)=P a36=P*30.3748=212,543.05

So he now has 212543.05+100000=312543.05 to be paid off over 3 years
Therefore,
Pnew=312543.05/a36=312543.05/30.374754=10289.56641

[13]

Soln. 10

(i) The prevailing interest rates in investment markets usually vary depending on the time span of the investments to which they relate. The variation arises because the interest rates that lenders expect to receive and borrowers are prepared to pay are influenced by the following factors which are not normally constant over time:

Supply and demand
Interest rates are determined by market forces *ie* the interaction between borrowers and lenders. If cheap finance is easy to obtain or if there is little demand for finance, this will push interest rates down.

Base rates
In many countries there is a central bank that sets a base rate of interest, which provides a reference point for other interest rates. Investors will have a view on how this rate is likely to move in the future.

Interest rates in other countries
The interest rates in a particular country will also be influenced by the cost of borrowing in other countries because major investment institutions have the alternative of borrowing from abroad.

Expected future inflation
Lenders will expect the interest rates they obtain to outstrip inflation. So periods of high inflation tend to be associated with high interest rates.

Tax rates
If tax rates are high, interest rates may also be high, because investors will require a certain level of return after tax.

(ii)(a) Let i_t be the spot yield over t years:
One year: yield is 9% therefore $i1 = 0.09$
two years: $(1 + i2)^2 = 1.09*1.08$ therefore $i2 = 0.08499$
three years: $(1 + i3)^3 = 1.09*1.08*1.07$ therefore $i3 = 0.07997$
four years: $(1 + i4)^4 = 1.09*1.08*1.07*1.06$therefore $i4 = 0.07494$

(ii)(b) Price of the bond is 5[(1.09)^-1 + (1. 08499)^-2 + (1. 07997)^- 3]+ 105(1. 07494)^-4 = 91.4456

Find gross redemption yield (GRY) from

91.4456 = $5a_{\overline{4|}} + 100v^4$
try 7%;
try 7%; $a_{\overline{4|}} = 3.3872$; $v^4 = 0.76290$
gives RHS = 93.226

gives RHS = 93.226
GRY must be higher try 8%
RHS=90.0636
interpolate between 7% and 8%.
i = 0.0756

(ii)(c)

i. more demand → prices rise → yields fall
ii. prices rise → yields fall
iii. more issued →prices fall → yields rise
iv. demand falls → prices fall → yields rise

ii(d) The 8 year spot rate is
Y8= 0.10-0.05*e^(-0.1*8)= 0.077534

The 9 year spot rate is
Y9= 0.10-0.05*e^(-0.1*9)= 0.079672

Therefore the forward rate at time 8 is 9.69%
(1+f8)= (1+y9)^9/(1+y8)^8=1.0969

[18]

[100]

***********END*********