INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

05th November 2007

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Do not write your name anywhere on the answer sheets. You have only to write your Candidate Number on each answer sheet/s.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4. *Fasten your answer sheet/s together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

5. *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer sheet/s and this question paper to the supervisor separately.

Q. 1]
  I) **a)** What is the difference between real rates and money rates of interest? (2)
       b) What is the relationship between the two rates in the period of positive inflation, negative inflation and zero inflation? (1.5)
       c) List the two factors in deciding the usefulness of any one of two rates—real rate and money rate? (1.5)

  II) Saira has invested a sum of Rs.40 lakh. Exactly one year later, the investment is worth Rs.44.4 lakh. An index of prices has a value of 224 at the beginning of the investment and 240 at the end of the investment. She pays tax at 30% on all money returns from investment. Calculate:

       a) The money rate of return per annum before tax.
       b) The rate of inflation.
       c) The real rate of return per annum after tax. (4)

       [9]

Q. 2]
  I)   **a)** What is an annuity? (1)

              b) Define $\overline{a}_{\overline{n}|}$ in words? (2)

              c) Derive the formula for $\overline{a}_{\overline{n}|}$. (3)

  II) A perpetuity costs 77.1 and makes annual payments at the end of the year. The perpetuity pays Re.1 at the end of year 2, Rs.2 at the end of year 3,…… , Rs.n at the end of year (n+1). After year (n+1), the payments remain constant at Rs.n. The annual effective interest rate is 10.5%. Calculate n. (4)
       [10]

Q. 3]
  I)   **a)** Describe the two methods of finding the loan outstanding. (4)
          b) Define flat rate of interest. (2)
          [6]

  II) Kabir Kumar takes a fixed-interest housing loan of Rs.300000 for a 25 year term from HDC bank The loan is repayable by monthly instalments paid in advance. The instalments are calculated using a rate of interest of 6% per annum effective. After ten years, he has the option to repay any outstanding loan and take out a new loan, equal to the amount of the outstanding balance on the original loan, if interest rates on 15-year loans are less than 6% per annum effective at that time. The new loan will also be repaid by monthly instalments in advance.
          a) Calculate the amount of the monthly instalments for the original loan. (3)
          b) Calculate the interest and capital components of the 25th instalment. (4)

c) He takes advantage of the option to repay the loan after ten years. The rate of interest on housing loans of length 15 years has fallen to 2% per annum effective at that time. The first loan is repaid and a new loan is taken out, repayable over a 15-year period, for the same sum as the capital outstanding after ten years on the original loan.

    i. Calculate the revised monthly instalment for the new loan. (3)

    ii. Calculate the present value of the reduction in instalments at a rate of interest of 2% per annum effective, if Kabir exercises the option. (3)

[13]

Q. 4]

I) What are the main advantages and disadvantages to the government and to investors of issuing bonds at a set price as opposed to by tender (4)

II) Give the names of three securities, which have uncertain income and why is it uncertain? (6)

III) For each of the following calculate the equivalent effective annual rate of interest:

    i. an effective rate of interest of 12.7% paid every 2 years (1)

    ii. a nominal rate of discount of 6% pa convertible quarterly (1)

    iii. a rate of interest of 14% pa convertible every 2 years. (1)

[13]

Q. 5] Surya Insurance Company has just written contracts that require it to make payments to policyholders of Rs.1,000,000 in five years time. The total premiums paid by policyholders amounted to Rs.850,000.

The insurance company is to invest half the premium income in fixed interest securities that provide a return of 3% per annum effective. The other half of the premium income is to be invested in assets that have an uncertain return. The return from these assets in year t,i_t, has a mean value of 3.5% per annum effective and a standard deviation of 3% per annum effective. $(1 + i_t)$ is independently and log normally distributed.

    a) Deriving all necessary formulae, calculate the mean and standard deviation of the accumulation of the premiums over the five-year period. (9)

    b) A director of the company suggests that investing all the premiums in the assets with an uncertain return would be preferable because the expected accumulation of the premiums would be greater than the payments due to the policyholders. Explain why this still may be a more risky investment policy (2)

[11]

Q. 6]

I) An investor is considering making an investment in one or both of two projects. The cash flows associated with the projects are as follows. The unit of time is years.

Project A: Initial payments of Rs.2 million at time zero and Rs.4 million at time 2 are made. In return a sum of Rs.900, 000 per annum is paid continuously from time 5 to time 25.

Project B: Regular payments of Rs.100, 000 are made at the start of each year for 10 years. In return, amounts of *X, 2X , 3X* and so on are made annually for 10 years, the first payment being made at time 11.

    a) Find the net present value of Project A at an effective annual interest rate of 10%. (2)

    b) Show that the internal rate of return for Project A is 9.38% *pa.* (2)

    c) Find the value of *X* if the internal rate of return for Project B is the same as that for Project A. (3)

    d) Find the value of *X* if both projects are to have the same net present value at 10% *pa.* (3)

    e) The investor proposes to borrow all the money needed for the project. Funds are available at an interest rate of 7% per annum effective. Repayments can be made at any time, and positive cash balances can be invested to yield 3% per annum.If *X* =Rs.45,000, find the accumulated value of each project at the end of the 25 year period. (12)

                                                                                [22]

    II)   **a).** In respect of an investment project, define:
              i   the discounted payback period
              ii   the payback period (3)

      b). Discuss why both the discounted payback period and the payback period are inferior measures compared with the net present value for determining whether to proceed with an investment project. (3)

                                                                                           [6]

Q. 7] A man makes payments into an investment account of Rs.200 at time 5, Rs.190 at time 6, Rs.180 at time 7, and so on until a payment of Rs.100 at time 15. [Unit of time is year]**.** Assuming an annual effective rate of interest of 3.5%, calculate:
          i.   the present value of the payments at time 4 (4)
          ii.   the present value of the payments at time 0 (1)
          iii.   the accumulated value of the payments at time 15 (1)

                                                                                        [6]

Q. 8] On 1 January 2005 an investor agrees to pay Rs.6000 in four years' time for a security. The security pays no interest and the price of the security at the time of the agreement was Rs.5360.On 1 July 2006 the price of the security is Rs.5600. Calculate the value of the forward contract on 1 July 2006. (4)

                                                                                                     [4]

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