INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

07th November 2007

Subject ST6 — Finance and Investment B

Time allowed: Three hours (14.15* pm – 17.30 pm)

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer sheet.*
2. ** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *The answers are not expected to be any country or jurisdiction specific However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
4. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor.*
5. *Mark allocations are shown in brackets.*
6. *Attempt all questions, beginning your answer to each question on a separate sheet.*
7. *Candidates should show calculations where this is appropriate.*
8. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

Professional Conduct:

It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI.

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answersheets and this question paper to the supervisor seperatly.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

Q. 1) Infosys is interested in using a futures contract maturing at time T to hedge a positive foreign currency exposure at time t_1. Define r as the interest rate (flat yield curve) on rupee and r_f as the interest rate (flat yield curve) on the foreign currency. Assume r and r_f are constant and zero transaction costs.

a) Show that the optimal hedge ratio is $e^{(r_f - r)(T - t_1)}$ (3)

b) Show that, when the company wishes to hedge against exchange rate movements over the next day, the optimal hedge ratio is S_0 / F_0 where S_0 is the current spot price of the currency for the contract maturing at time T (1)

[4]

Q. 2)

a) State Bank of India offers Infosys a choice between borrowing cash at 13% per annum and borrowing gold at 3% per annum. (If gold is borrowed interest and principal must be repaid in gold. Thus 100 ounces borrowed today would require 103 ounces to be repaid in one year.) The risk free interest rate is 8% per annum and storage costs are 0.5% per annum.
Discuss whether the rate of interest on the gold loan is too high or too low in relation to the rate of interest on the cash loan.
(The interest rates on the two loan are expressed with annual compounding where as the risk free interest rate and storage cost are expressed with continuous compounding.) (6)

b) Consider the variable S that follows the process $dS = \mu\, dt + \sigma\, dz$
For the first 3 years $\mu = 2$ and $\sigma = 3$; for the next 3 years, $\mu = 3$ and $\sigma = 6$. If the initial value of the variable is 10 what is the probability distribution of the value of the variable at the end of the year 6? (4)

[10]

Q. 3) Consider a derivative that pays off S_T^n at time T where S_T is the stock price at that time. When the stock price follows geometric Brownian motion, it can be shown that its price at time t (t<=T) has the form $h(t,T)\, S_t^n$ where h is a function of t and T.

a) By substituting into the Black-Scholes partial differential equation derive an ordinary differential equation satisfied by $h(t,T)$. (3)

b) What is the boundary condition for the differential equation for $h(t,T)$? (1)

c) Derive the function $h(t,T)$. (4)

[8]

Q. 4)

a) Give a precise definition of a tradable security. (2)

b) If S_t is a tradable Black Scholes stock price under the martingale measure Q , where $S_t = e^{\sigma \tilde{W}_t + (r - \frac{1}{2}\sigma^2)t}$ with cash bond $B_t = e^{rt}$, show that $X_t = S_t^2$ is non tradable but $X_t = S_t^{-\alpha}$ where $\alpha = \dfrac{2r}{\sigma^2}$ is tradable.

(8)

[10]

Q. 5) Based on 91 days Government of India Treasury bill, a benchmark 3-month interest rate had the following values (compounded quarterly) during 2006:

From 1 January	:	5.00%
From 5 March	:	5.255%
From 2 July	:	5.75%
From 6 August	:	6.00%
From 5 November	:	6.25%

a) Interest rate caps, floors and collars are available on this benchmark rate, with resets on 1^{st} April, 1^{st} July, 1^{st} October and 1^{st} January.

Sketch a graph of the benchmark interest rate over this period. (2)

b) Ram & Company has a 5.5% cap on the benchmark rate with a principal of Rs. 100 crore, whereas Gopal & Company has a 5.5% floor on the benchmark rate with a principal of Rs. 100 crore.. Both caps were taken out in 2005.

What cash flows will each organization receive from these arrangements in respect of 2006? (4)

c) An interest rate “pay-fixed-for-floating” swap based on 5% and the same interest payment dates is also available on this benchmark interest rate.

What can you say about the price of the swap relative to the price of the cap and the floor? (2)

d) Expressed in terms of the principal, the current price of a 5-year 6% cap is 0.8%and the current price of a 5-year 4% floor is 0.6%

What can you say about the price of a 4% / 6% collar (2)

[10]

Q. 6) You are a consulting actuary of ABC Bank Ltd., The bank has vanilla caps, floors and swaptions which are currently used using black model. You have been asked to propose a (single currency) term structure yield curve model for pricing and hedging exotic interest – rate swaps and options.

a) Discuss why you might need a fully yield curve model, and describe the features which would be desirable in your model (5)

b) Comment on the suitability (or otherwise) of the following in the context of a full yield curve model:

i. One-factor equilibrium models
ii. No arbitrage models

Define all the terms you use (5)

[10]

Q. 7 The current zero coupon yield curve in a given fixed income market is as follows (all rates assume annual compounding):

Term	Rate % p.a.
1 year	3.5 %
2 year	3.6%
3 year	3.9 %
4 year	4.2 %
5 year	4.45%
6 year	4.65%
7 year	4.75 %

a. Calculate the continuously compounded 5-year zero coupon rate (1)

b. Calculate the value of a zero coupon bond of maturity 3 ½ years (1)

c. Calculate the fixed leg coupon of a 5-year par value annual to annual interest rate swap (2)

d. Calculate the fixed leg coupon of a forward-starting 5-year par value annual to annual interest rate swap commencing in two years time. (2)

e. A constant maturity swap (CMS) is a swap where each payment on the floating leg is reset to be equal to the fixed leg coupon of a par value interest rate swap of the given maturity that is being traded in the market at each reset date.

i. Calculate the fixed leg coupon of annual to annual par value 1-year CMS which is based on the 5-year swap rate. (2)

ii. Explain, without giving detailed algebra, how you would value a 2-year CMS based on the 5-year swap rate. What further information would you require? (2)

[10]

Q. 8)

a) **(i)** Define kappa (vega). (1)

(ii) Derive a formula for kappa for a call option on a dividend-paying share, assuming the Garman-Kohlhagen price model. (3)

You may assume the lemma $S_t e^{-q(T-t)}\phi(d_1) = Ke^{-r(T-t)}\phi(d_2)$ without proof.

b) Evaluate kappa for each of the following options on a share currently priced at 212, with dividends paid continuously at a rate of 4% per annum and an assumed volatility of 30% per annum.

- **i.** 90-day call option, strike 200
- **ii.** 90-day call option, strike 250
- **iii.** 15-day call option, strike 200
- **iv.** 90-day put option, strike 200

Assume that the risk-free continuously-compounded rate of interest is 5% per annum and that time periods are based on a 360-day year. (4)

c) Comment on the signs and relative magnitudes of your answers to (b). (2)

[10]

Q. 9) A fund manager has a well diversified portfolio that tracks the performance of the S&P 500 and is worth 275 crore. The current value of the S&P 500 is 1,100. The manager would like to buy portfolio insurance against a reduction of more than 5% in the value of the portfolio over the next year. The risk free rate is 5% p.a. The dividend yield on both the portfolio and the S&P 500 is 3%. The market implied volatility for the S&P 500 is currently 25% p.a.

- **a.** Calculate the cost of hedging the portfolio using European put options. (3)
- **b.** Describe alternative strategies involving European call options which would have the same effect as the options in (a). (3)
- **c.** Calculate the initial (delta) position if the manager sought to replicate the effect of the put options by investing part of the portfolio in risk-free securities. (2)
- **d.** Calculate the initial number of futures contracts required if, instead of risk-free securities in (c), the manager decided to use 9-month index futures contracts, each contract nominal being 250 times the index. (2)

[10]

Q. 10) You are using the Black- Derman-Toy (BDT) model of the discrete short-term rate ***r*** to value interest-rate swaps and options.

Under BDT the process for ***R***, the continuous-time equivalent of ***r***, is given by the formula:

$$d(\ln R) = \left(\theta(t) + \frac{\partial \ln \sigma(t)}{\partial t} \ln R \right) dt + \sigma(t) dz$$

Where ***σ(t)*** is the short-rate volatility and ***z(t)*** is a standard Brownian motion.

You have been given the following term structures for the year t=0 to 4 of a Euro-zone swap curve:

Time t In years	Zero coupon bond prices	Caplet volatility in %
0	100.00	0
1	97.023	22.0
2	94.124	22.5
3	91.008	21.5
4	87.738	20.0

You have then obtained the binomial tree for the annual short rate ***r(t)*** in %, for t=0 to 3 and steps of one year, as follows:

0	1	2	3
			6.695
		5.132	
	3.752		4.355
3.069		3.272	
	2.416		2.833
		2.086	
			1.843

a. Calculate, using the tree, the value of a four-year 3.5% cap and floor with annual resets and explain how you would calculate their deltas with respect to the short rate. Demonstrate that Put-Call parity is satisfied for the option prices. (5)

b. Explain how your answers might differ from the values which would be obtained from the usual Black futures model with the same input term structures. (2)

c. Describe without calculation how the BDT model above might be used to value a European two-year option on a four-year bond, and discuss briefly how accurate the answer might be. (3)

[Note: a four year annual cap or floor has three annual options expiring at the ends of years 1, 2 and 3 respectively. The strike rate may be assumed to have the same day-count convention as ***r(t)***]

[10]

Q. 11) A bull spread is the simultaneous purchase of a call option of a given strike and maturity and a sale of a second call option of the same maturity but higher strike.

A bull spread is purchased on a commodity priced at 250, with the two strikes of 230 and 270, and expiry date six months from today. Implied volatility is at 35% and the risk-free rate may be assumed to be zero.

a. Discuss the conditions under which a trader or hedger might wish to purchase a bull spread on a commodity. (2)

b. Sketch the value of the bull spread against a range of commodity prices, showing the situation as at today and just before expiry. (2)

c. Sketch the value of the Delta and Gamma against the commodity price as at today (2)

d. Describe (separately for each case) how the value of the bull spread at a given commodity price would change:

i. If volatility falls to 25%
ii. A month from today (2)

[8]

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