Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

October/Nov 2007 Examination

INDICATIVE SOLUTION

INSTRUCTIONS TO MARKERS

1. MARKS ARE ALLOTTED FOR EACH BIT OF QUESTION IN THE MAIN EXAM PAPER, AS WELL AS IN THE MODEL/INDICATED SOLUTIONS.
2. MARKERS ARE REQUESTED TO ALLOT MARKS AS PER THE SCHEME INDICATED BELOW.
3. PLEASE USE AN EXCEL SHEET (SOFTCOPY) WHEN MARKER'S SHEET IS FURNISHED TO IOAI—[SO THAT TOTALLING MISTAKES DO NOT TAKE PLACE, AND ALSO FOR FURTHER EVALUATION/RESEARCH OF SCRIPTS AT IOAI'S OFFICE AND REVIEW EXAMINER]

1. The benefit amount on death will be 100000 for the first year, 102000 for the second year, 104000 for the third year, etc, and the benefit amount on maturity will be 140000.

   So the premium equation can be expressed as

$$P\,\ddot{a}_{[40]:\overline{20}|} = 98000A_{[40]:\overline{20}|} + 2000(IA)^{1}_{[40]:\overline{20}|} + 42000 + \frac{D_{60}}{D_{[40]}} + 500 + 3000A_{[40]:\overline{20}|}$$

   The increasing assurance function can be calculated as:

$$(IA)^{1}_{[40]:\overline{20}|} = (IA)_{40} - \frac{D_{60}}{D_{[40]}}\left[(IA)_{60} + 20A_{60}\right]$$

$$= 7.95835 - \frac{882.85}{2052.54}\left[8.36234 + 20 * .45640\right]$$

$$= 0.43531$$

   So the premium equation becomes:

   13.930P = 98000*0.46423 + 2000*0.43531 + 42000*0.43013
   + 500 + 3000*0.43013

   = 66323.31

   Hence the premium is:

$$P = \frac{66323.31}{13.930} = 4761.18$$

Total [7]

2. The policy starts when the policyholder is age 45. The 5th premium is paid on the policyholder's 49th birthday, when the remaining term will be 6 years.

   If death occurs at age 49 last birthday, the benefit amount will be Rs. 24000, which will increase by Rs.1000 each year. The maturity value is Rs.35000. So the prospective reserve is:

$${}_4V^{pro} = 23000 * A^{1}_{[49]:\overline{6}|} + 1000 * (IA)^{1}_{[49]:\overline{6}|} + 35000 * \frac{D_{55}}{D_{49}} - P * \ddot{a}_{[49]:\overline{6}|}$$

Where the premium is given by :

$$P * \ddot{a}_{[45]:\overline{10}|} = 19000 * A^{\,1}_{[45]:\overline{10}|} + 1000 * (IA)^{1}_{[45]:\overline{10}|} + 35000 * \frac{D_{55}}{D_{45}}$$

Note that, in order to specify the calculation of reserve precisely, it is necessary to state how the premium is calculated.

Total [5]

3. (i) Present value of random variable is:

$$\text{PVRV} = \begin{cases} 100000v & if \quad K_x = 0 \\ 200000v^2 & if \quad K_x = 1 \\ 0 & if \quad K_x = 2 \end{cases}$$

(ii) Standard deviation

The EPV of the benefit is:

EPV = $100000 * v * q_x + 200000 * v^2 * q_{x+1}$

$$= \frac{100000}{1.06} * 0.025 + \frac{200000}{1.06^2} * 0.975 * 0.030$$

$= 7564.97$

And

$$E(PV^2) = (100000 * v)^2 * q_x + (200000 * v^2)^2 * p_x * q_{x+1}$$

$$= \left(\frac{100000}{1.06}\right)^2 * 0.025 + \left(\frac{200000}{1.06^2}\right)^2 * 0.975 * 0.030$$

$= 1149248696$

So the variance of the present value random variable is:

$$Var(PV) = E(PV^2) - (EPV)^2 = 1092019925$$

And the standard deviation is $\sqrt{1092019925} = 33045.72$

Total - [7]

4. Crude death rate

- heavily influenced by mortality at older ages
- beware epidemics distorting figures
- OK if population structure reasonably stable
- hence beware mass immigration or emigration
- easy to calculate

Standardised Mortality Rate

- influenced again by mortality at older ages
- generally OK
- but practical constraints
- since need age/sex-specific mortality rates at each time point
- no problems caused by changing population structure

(ii)
A change in the level of underwriting may alter the effect of temporary initial selection, *eg* more rigorous underwriting often leads to increasing the intensity and duration of temporary initial selection.

This may give the impression that mortality rates are improving more quickly than they really are. In other words, the effect of time selection is being distorted, hence spurious selection is occurring.

Conversely, a relaxation of underwriting standards is likely to lessen the effect of temporary initial selection. This may give the impression that mortality rates are worsening or improving at a slower rate than they really are. This again distorts the effect of time selection.

Total - [7]

5.

$$50,000\int_0^9 e^{-\delta(t+s)} \ {}_tp_{50}^{aa}\ \sigma_{50+t}\left(\int_1^{10-t} {}_sp_{50+t}^{\overline{ii}}\ \upsilon_{50+t+s}\ ds\right)dt$$

Suppose that the life gets sick at time t. The notation for this is ${}_tp_{50}^{aa}\ \sigma_{50+t}$

The life could get sick at any time, but if this happens after time 9, it will not lead to any benefit. So we integrate t between the limits of 0 and 9.
He has to stay sick for a year before any benefit is paid. If he remains sick for s (>1) years, and dies from the sick state at age 50+t+s, then the benefit is paid at time t + s and must be discounted back to time 0. The “probability” of this happening is

$${}_sp_{50+t}^{\overline{ii}}\ \upsilon_{50+t+s}$$

Note that s must be at least 1 for any benefit to be paid, but the policy term is 10 years. However, given that the life falls sick at time t, the duration of sickness required for the payment of the benefit is between 1 and 10 - t . So we integrate s between these limits.

Total [5]

6.

(i) ***Contribution rate***

The formula for the value of the future service benefits is:

$$\frac{75,000}{60}\frac{{}^{z}\overline{R}_{35}^{ra}}{s_{34}D_{35}}-\frac{5\times 2,000}{60}\frac{\overline{R}_{35}^{ra}}{D_{35}} \qquad [2]$$

The benefit formula is based on the same definition of final average pay as is used in the *Tables*.

From the *Tables*:

$s_{34}=6.389$ (page 142) $D_{35}=4,781$ (page 143)

${}^{z}\overline{R}_{35}^{ra}=3,524,390$ (page 147) $\overline{R}_{35}^{ra}=327,244$ (page 146) [½]

So the value of the future service benefits is:

$$\frac{75,000}{60}\times\frac{3,524,390}{6.389\times 4,781}-\frac{5\times 2,000}{60}\times\frac{327,244}{4,781}=132,818 \qquad [1]$$

The formula for the value of the total future contributions (members and company combined) is:

$$k\left(75,000\frac{{}^{s}\overline{N}_{35}}{s_{34}D_{35}}-5\times 2,000\frac{\overline{N}_{35}}{D_{35}}\right) \qquad [2]$$

From the *Tables*:

${}^{s}\overline{N}_{35}=502,836$ (page 143) $\overline{N}_{35}=59,914$ (page 143) [½]

So the value of future contributions of $100k$ % of pensionable pay is:

$$k\left(75,000\times\frac{502,836}{6.389\times 4,781}-5\times 2,000\times\frac{59,914}{4,781}\right)=1,109,311k \qquad [½]$$

So, in order to meet the cost of the benefits (£132,818), the total contribution rate must be:

$$k = 132,818 / 1,109,311 = 11.97\% .$$ [1]

Since the members contribute 5%, the company must pay the remaining 6.97%. [½]

[Total 8]

(ii)(a) ***Modified starting salary***

The salary given in the data is now the starting salary on 1 May 2003. This is the same as the salary that would have been earned during the 2003 calendar year (*ie* between ages $34\frac{2}{3}$ and $35\frac{2}{3}$). So the salary scale factor in the denominator should be changed from s_{34} to $\frac{1}{3}s_{34} + \frac{2}{3}s_{35}$. [2]

(ii)(b) ***Limited period of accrual***

If service was limited to 20 years, the summation in the definition of ${}^{z}\overline{R}_{35}^{ra}$ and $\overline{R}_{35}^{ra}$ should only include ages 35, 36, ... , 54. [2]

Total [4]

Total for Qn [12]

7.

$${}_4p_{50} = e^{-(0.05)(4)} = 0.8187$$ [1]

$${}_{10}p_{50} = e^{-(0.05)(10)} = 0.6065$$ [½]

$${}_8p_{60} = e^{-(0.04)(8)} = 0.7261$$ [1]

$${}_{18}p_{50} = \left({}_{10}p_{50}\right)\left({}_8p_{60}\right) = 0.6065 \times 0.7261$$
$$= 0.4404$$ [1 ½]

$${}_{4|14}q_{50} = {}_4p_{50} - {}_{18}p_{50} = 0.8187 - 0.4404 = 0.3783$$ [1]

Total [5]

8.

$$1000A_{81} = \left(1000A_{80}\right)(1+i) - q_{80}\left(1000 - A_{81}\right)$$ [1 ½]

$$689.52 = (679.80)(1.06) - q_{80}(1000 - 689.52)$$ [1]

$$q_{80} = \frac{720.59 - 689.52}{310.48} = 0.10$$ [½]

$$q_{[80]} = 0.5 q_{80} = 0.05$$ [½]

$$1000 A_{[80]} = 1000 v q_{[80]} + v p_{[80]}\, 1000 A_{81}$$ [1 ½]

$$= 1000 \times \frac{0.05}{1.06} + 689.52 \times \frac{0.95}{1.06} = 665.14$$ [1]

Total [6]

9. During the first n years the annuity is payable if (x) is alive whether y is alive or not, so the value for these years is $a_{x:\overline{n}|}$ [1]

After the first n years the annuity is payable if (x) is alive and if (y) was alive n years earlier, with value

$$\sum_{t=n+1}^{\varpi} v^t * {}_t p_x \; {}_{t-n} p_y$$ [2]

And substituting t=n+s

$$= \sum_{s=1}^{\varpi} v^{n+s} * {}_{n+s} p_x * {}_s p_y$$ [1]

$$= v^n * {}_n p_x * \sum_{s=1}^{\varpi} v^s * {}_s p_{x+n} * {}_s p_y$$ [1]

$$= v^n * {}_n p_x * a_{x+n:y}$$ [1]

So total value $= a_{x:\overline{n}|} + v^n {}_n p_x a_{x+n:y}$ [1]

Total [7]

10. $$(al)_x = (al)_0 e^{-\int_0^x \left(\frac{1}{1000-t} + 1 \right) dt}$$ [1]

$$= 1000 * e^{[\log_e(1000-t)]_0^x} * e^{-x}$$ [½]

$$= 1000 * e^{\log_e(1000-x) - \log_e(1000)} * e^{-x}$$ [1]

$$= 1000 * \frac{1000-x}{1000} * e^{-x} = (1000-x) * e^{-x}$$ [1]

$$(ad)_x^1 = \int_0^1 (al)_{x+t} * (a\mu)_{x+t}^1 \, dt$$ [1 ½]

$$= \int_0^1 (1000-x-t) * e^{-x-t} * \frac{1}{(1000-x-t)} dt$$ [1]

$$= \left[-e^{-x-t} \right]_0^1$$ [½]

$$= e^{-x} - e^{-x-1}$$ [½]

Total [7]

11.

$$_t\bar{V}_x = \bar{A}_{x+t} - \bar{P}_x \times \bar{a}_{x+t} = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$ [1]

$$\frac{\partial}{\partial t} \bar{a}_{x+t} = \frac{\partial}{\partial t} \int_0^\infty e^{-\delta s} \, _s p_{x+t} ds = \int_0^\infty e^{-\delta s} \frac{\partial}{\partial t} \, _s p_{x+t} ds$$ [1]

where $\frac{\partial}{\partial t} \, _s p_{x+t} ds$ is derived as follows:

Multiplying throughout by ${}_s p_{x+t}$ gives ${}_s p_{x+t} * (\mu_{x+t} - \mu_{x+t+s})$

[1]

$$= \int_0^\infty e^{-\delta s}\ {}_s p_{x+t}(\mu_{x+t} - \mu_{x+t+s})ds = \mu_{x+t}\bar{a}_{x+t} - \bar{A}_{x+t}$$

[½]

$$\Rightarrow \frac{\partial}{\partial t}\,{}_t\bar{V}_x = \frac{-(\mu_{x+t}\bar{a}_{x+t} - \bar{A}_{x+t})}{\bar{a}_x} = -\mu_{x+t}(1 - {}_t\bar{V}_x) + \frac{(1 - \delta\bar{a}_{x+t})}{\bar{a}_x}$$

[1]

$$= -\mu_{x+t}(1 - {}_t\bar{V}_x) + \delta\left(1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}\right) - \delta + \frac{1}{\bar{a}_x}$$

[½]

$$= -(1 - {}_t\bar{V}_x)\mu_{x+t} + \delta\,{}_t\bar{V}_x + \left(\frac{1 - \delta\bar{a}_x}{\bar{a}_x}\right)$$

[½]

$$= -(1 - {}_t\bar{V}_x)\mu_{x+t} + \delta\,{}_t\bar{V}_x + \bar{P}_x$$

[½]

Total [6]

12.

The equation of value to solve is

P $\ddot{a}_{25:20}$ = 10 * P * $A_{25:30} + \frac{D_{55}}{D_{25}}$ * 48000 * $\ddot{a}_{55}^{(4)}$ [1]

$$\text{LHS} = P\left(\ddot{a}_{25} - v^{20} * \frac{l_{45}}{l_{25}}\right) = P\left(22.520 - 0.45639 * \frac{9801.3123}{9953.6144}\right)$$

$$= P\,(22.520 - 0.449406691)$$

$$= 22.07059331\ P \qquad [1]$$

1st term of the RHS $= 10 * P * \left(A_{25} - v^{30} * \frac{l_{55}}{l_{25}} * A_{55}\right)$

$$= 10 * P * \left(0.13886 - 0.30832 * \left(\frac{9557.8179}{9953.6144}\right) * 0.3895\right)$$

$$= 10 * P * (0.01854465)$$

$$= 0.1854465\ P \qquad [1]$$

2nd term of the RHS $= v^{30} * \frac{l_{55}}{l_{25}} * \left(48000 * \ddot{a}^{(4)}_{55}\right)$

$$= (0.30832)\left(\frac{9557.8179}{9953.6144}\right) * 48000 * \left(\ddot{a}_{55} - \frac{3}{8}\right)$$

$$= (0.30832)\,(0.960235901) * 48000 * (17.364 - 0.375)$$

$$= 241{,}428.5857 \qquad [1]$$

Solving for P we get

$$(22.07059331 - 0.1854465)\ P = 241{,}428.5857$$

$$\Rightarrow \quad P = 11{,}031.61831 \qquad [1]$$

Total [5]

13.
Unit fund

Policy year	Premium allocated	Cost of allocation	Plus Fund b/f	Fund before charge	Annual charge	Fund c/f
1	7500	7125.00	7125.00	7481.25	56.11	7425.14
2	10500	9975.00	17400.14	18270.15	137.03	18133.12
3	10500	9975.00	28108.12	29513.53	221.35	29292.18
4	10500	9975.00	39267.18	41230.53	309.23	40921.31

[5]

Non Unit Fund

Policy year	Profit on Allocation	Expenses	Non Unit interest	Annual charge	Non unit death cost	Profit in each year
1	2875.00	300	128.75	56.11	338.62	2421.24
2	25.00	75	-2.50	137.03	178.00	-93.48
3	25.00	25	0.00	221.35	10.62	210.73
4	25.00	25	0.00	309.23	0.00	309.23

[5]

The non-unit reserve required are

Start of year 3: 0.00
End of year 2: 93.48
Start of year 2: 89.03
End of year 1: 87.69 [2]

Profit emerging in each year is

Policy year	Non Unit reserve	Interest on reserve	Increase in reserve	Profit ignoring reserve	Profit allowing for reserve
1	0.00	0.00	87.69	2421.24	2333.55
2	89.03	4.45	-89.03	-93.48	0.00
3	0.00	0.00	0.00	210.73	210.73
4	0.00	0.00	0.00	309.23	309.23

[4]

Total [16]

14. For the first equation, a whole life benefit is equal to a term assurance for n years (which pays out on death in the first n years) plus a benefit covering the whole of the remainder of the policyholder life, provide (s)he survives for n years

[1 ½]

For the second equation, an endowment assurance is equal to a term assurance paid immediately on death if it occurs within n years plus a pure endowment benefit paid if the policyholder survives for the n year period

[1 ½]

For the third equation the deferred whole life assurance is paid on death, but only if death happens after n years. Therefore the benefit is equal to a whole life assurance paid to a life n years older, but only if the life aged x survives for n years (and discounted to allow for interest) [2]

Total [5]

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