

# **Institute of Actuaries of India**

## **Subject ST5 – Finance and Investment A**

**October/November 2007 Examination**

### **INDICATIVE SOLUTION**

#### **Introduction**

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

- Q.1 a)** Treasury Bills  
Commercial Paper  
Repos  
Government agency securities  
Bank time deposits and certificates of deposit  
Bankers' acceptances and eligible bills.
- b)** The company can issue commercial paper .  
The main features are :
- It is a bearer document
  - Terms at issue vary from a few days to up to several months with terms up to two being the most common.
  - It is a single name instrument, the security is provided only by the company issuing the paper , i.e. the borrowing company
  - Companies who wish to raise finance by issuing commercial paper have to meet certain minimum standards
  - The effective rate of interest paid will be slightly higher than the equivalent rate on a risk-free investment. The size of the margin over the risk-free rate of interest will depend upon the company's credit rating.
  - Rating agencies such as CRISIL, ICRA publish ratings for commercial paper.
- c)** The company wishes to reduce the cost of funds for the next 9 months by exploring alternatives to the current working capital arrangements as the interest rates are declining in the market while the company can only negotiate the interest revision at the end of 9 months.
- Since the yields on treasury bills indicate short term risk-free interest rates , the yields on t-bills will be lower than the yield on commercial paper.
  - The spread between the T-bills and the commercial paper depends upon the rating of the instrument issued by the company.
  - Since the company is AAA rated one, it may be assumed that the commercial paper also is expected to get the highest credit rating thus minimizing the spread over the risk free rate of interest.
  - Given that the company is toying with the idea of alternative instruments, the interest paid on commercial paper plus the cost of issuing and administering this instrument must be lower than the working capital interest.

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- Q.2 a)** Standard Definition  
Starting point is to take the market prices of conventional bonds (e.g. gilts) for a range of possible maturities.  
Starting at the shortest maturity, T1 say, use the observed market price and solve for the yield. This yield is an approximation for the zero coupon rate for maturity T1, called R1, say.  
Using the next shortest maturity conventional bond maturing at T2, again take the observed price and using R1 solve for the forward rate starting at T1 for the period T2 T1. Now solve for the spot rate R2.  
Repeat using the next maturity conventional bond until the longest maturity bond has been used. This fixes the longest spot rate at the maturity of the longest bond.  
Plot the spot rates R(T) against T to arrive at the zero coupon yield curve.

- b)  $S1: 102.01 * (1 + S1) = 100 + 6.75$   $S1 = 4.65\%$   
 No price given for 2 year gilt so need to calculate:  
 $P2: P2 * (1 + 0.049)^2 = 9.5 * (1 + 0.049) + 100 + 9.5$   $P2 = 108.56$   
 Now solve for S2  
 $108.56 * (1 + S2)^2 = 9.5 * (1 + 1f1) + 109.5$  where 1f1 is the 1 year forward rate starting in 1 years time  
 $(1 + S2)^2 = (1 + S1) * (1 + 1f1)$   
 $S2 = 4.91\%$   
 $S3: 110.41 * (1 + S3)^3 = 7.75 (1 + 2f1)^2 + 7.75 * (1 + 1f2) + 107.75$   
 $(1 + 2f1)^2 = (1 + S3)^3 / (1 + S1) (1 + 1f2) = (1 + S3)^3 / (1 + S2)^2$   
 $S3 = 3.94\%$   
 If continuous rates used the answers are  $S1=4.54\%$ ,  $S2=4.80\%$  and  $S3=3.86\%$ .  
 $P2=108.32$ (using continuous rates) is not correct but we do not penalise in other calculations.

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## Q. 3

- a) (i)
- The main risks the Bank C will face are credit risk and market risk.
  - Credit risk arises from the possibility of a default by the counterparty when the value of the contract is positive to the bank.
  - The market risk arises from the possibility that market variables such as interest rates and exchange rates will move in such a way that the value of a contract to the bank becomes negative
  - Market risks can be hedged by entering into offsetting contracts
  - Credit risks cannot be hedged.
- (ii) Company A has a comparative advantage in fixed interest market while Company B has comparative advantage in floating rate market as it pays 1.2% more in fixed interest rate market than Company A while it pays only 0.7% more in floating interest rate market
- Company A pays Mibor to the bank.  
 Company A gets 9.90% from the bank  
 Company B gets Mibor from the Bank  
 Company B pays 10.00% to the Bank.  
 The net interest paid by Company A is Libor +0.1% . Thus gains 0.2%  
 The net interest paid by Company B is 11% fixed . Thus gains 0.2%  
 The Bank C gains 0.1% to meet its expenses and profit margins.
- (iii) Company A:  $-2.5e^{-0.25*0.0675} - 100e^{-0.25*0.0675} + 4.95e^{-0.25*0.0675} + 104.95e^{-0.75*0.07}$   
 Company B:  $+2.5e^{-0.25*0.07} + 100e^{-0.25*0.07} - 5e^{-0.25*0.7} - 105e^{-0.75*0.0725}$
- b) Payoffs from the swaption is a series of five cash flows equal to  $\text{Max} [0.076 - S(t), 0]$  in million of dollars where  $S(t)$  is the five year swap rate in four years

The value of an annuity that provides \$ 1 per year at the end of years 5,6,7,8 and 9 is  $[(1.08)^{-5} + (1.08)^{-6} + (1.08)^{-7} + (1.08)^{-8} + (1.08)^{-9}] = 2.9348$

The value of the swaption is :  
 $2.9348 [0.076 N(-d_2) - 0.08 N(-d_1)]$

Where  $d_1 = [1_n (0.08/0.076) + (0.25^2) \times 2] / (0.25 \times 2) = 0.3526$   
 $d_2 = d_1 - (0.25 \times 2) = -0.1474$   
 $N(-d_1) = N(-0.3526) = 0.3622$

$$N(-d_2) = N(0.1474) = 0.5586$$

Hence value of the swaption

$$= 2.9348 [(0.076 \times 0.5586) - (0.08 \times 0.3622)]$$

$$= \$0.039554 \text{ mln (or) } \$39,554$$

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**Q. 4 a)** If the economy is moderately buoyant and profits are fairly stable, both defensive and cyclical companies might be similarly rated in terms of the P/E ratios. As the economy starts to move into recession P/E ratios for cyclical companies are likely to fall while those of defensive companies will remain stable or may even rise slightly. At the bottom of the cycle P/E ratios of cyclical companies will probably have risen from their low point as earnings have fallen, but defensive stocks will still be more highly rated. As the economy starts to recover, the P/E ratios of cyclical companies will rise in anticipation of future earnings growth. P/E ratios of defensive companies may now be lower than those of cyclical stocks.

As growth continues, the earnings of cyclical companies will catch up with the share price and P/E ratios will fall back towards their long-term average level.

**b) i)** The share price is likely to fall possibly very substantially. The fall in value could be very steep given the fact that the company's value is entirely derived from the expectation of success of this product.

- ii)**
- Is the drug effective against any other conditions?
  - Even if it is not, the research done may have other applications
  - If not, can the company survive or will it inevitably fail?
  - Is there any possibility of company being taken over? Does it have any research projects going on which may be attractive for other more successful companies?
  - Any venture capital funding available?
  - What are the assets of the company? Are they realizable?
  - What is the cash flow position?

Does the company still have credibility in the market place? Having spent substantial time and money developing an ineffective drug, the company may have difficulty in persuading investors that its other activities (if any) will subsequently bear fruit.

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**Q. 5 a) Three Reasons for Not Hedging:**

- If the company's competitors are not hedging, the CFO might feel that the company will experience less risk if it does not hedge.
- The CFO may be of the view that the company's shareholders have diversified the risk away.
- If there is a loss on the hedge and a gain from the company's exposure to the underlying assets the CFO might feel that he will have difficulty justifying the "decision to hedge" to other executives within the organization
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**b) Deriving the Optimal Hedge Ratio**

Define

$\delta S$  : Change in spot price S during the period of time equal to the life of the hedge

$\delta F$  : Change in future price F during the period of time equal to the life of the hedge

$\sigma(S)$  : Standard deviation of  $\delta S$

$\sigma(F)$  : Standard deviation of  $\delta F$

$\rho$  : Coefficient of correlation between  $\delta S$  and  $\delta F$

$h$  : hedge ratio

For a short hedge, the change in the value of the hedge's position over the life of the hedge is :  $\delta S - h \delta F$  for each unit of asset held.

For a long hedge the change is

$$h \delta F - \delta S$$

In either case the variance of the change in the value of the hedged position is given by

$$v = \sigma^2(S) + h^2 \sigma^2(F) - 2 \rho h \sigma(S) \sigma(F)$$

Differentiating w.r.t. 'h' we get

$$\frac{\partial v}{\partial h} = 2h \sigma^2(F) - 2 \rho \sigma(S) \sigma(F)$$

$$\text{Setting } \frac{\partial v}{\partial h} = 0, \text{ we get } h = \frac{\rho \sigma(S)}{\sigma(F)}$$

We find that the second order derivative w.r.t. h is equal to  $2 \sigma^2(F) > 0$

Therefore the minimum variance (optional) hedge ratio is  $h = \frac{\rho \sigma(S)}{\sigma(F)}$

c) i) This statement is not true

The minimum variance hedge ratio is  $\frac{\rho \sigma(S)}{\sigma(F)}$

It is equal to 1.0 when  $\rho = 0.5$  and  $\sigma(S) = 2 \sigma(F)$ . Since  $\rho < 1.0$ , the hedge is clearly not perfect.

ii) This statement is true.

If the hedge ratio is 1.0, the hedger locks in a price of  $F_1 + b_2$  where  $F_1$  is the futures price at time  $t_1$  and  $b_2 = S_2 - F_2$  is the basis risk [ $S_2$  denotes the spot price at time  $t_2$  and  $F_2$  denotes the futures price at time  $t_2$ . Since the given statement states that there is no basis risk,  $b_2$  is equal to zero and  $F_1$  is known. Therefore  $F_1 + b_2$  has a variance of zero. In other words, the hedge ratio of 1.0 turns out to be the minimum variance hedge ratio.

iii) This statement is not true

A forward (future) contract locks in a forward (futures) price which is usually different from the current spot price of the asset. For example, the forward exchange rate and the spot exchange rate for a foreign currency are usually different.

(12)

Q. 6 a) (i) Money Weighted Rate of Return (MWRoR) is given by the equation

$$2000(1+i) + 400(1+i) + 360(1+i)^{0.75} - 200(1+i)^{0.5} - 300(1+i)^{0.25} = 2850$$

Using the relationship  $(1+i)^t = 1 + it$ , we get

$$i = 0.2365 \text{ or } 23.65\%$$

(ii) The quarterly wealth ratios are as follows:

$$\begin{aligned} \text{Q1: } & (1770 + 460) / (1600 + 400 + 400) \\ & = 0.92916 \text{ i.e., } 7.08\% \end{aligned}$$

$$\begin{aligned} \text{Q2: } & (2060 + 490) / (1770 + 460 + 360) \\ & = 0.98456 \text{ i.e., } 1.54\% \\ \text{Q3: } & (2000 + 460) / (2060 + 490 - 200) \\ & = 1.04681 \text{ i.e., } 4.68\% \\ \text{Q4: } & (2300 + 550) / (2000 + 460 - 300) \\ & = 1.31944 \text{ i.e., } 31.944\% \end{aligned}$$

Time weighted rate of return (TWRoR) is given by the equation  
 $(0.92917 \times 0.98456 \times 1.04681 \times 1.31944) - 1 = 0.2636$  or 26.36%

#### Assumptions

- Fund values at the end of each quarter do not include any of the contributions received in respect of the following quarter.
- Both the benchmark bond index and the benchmark equity index have income reinvested
- No taxes or other expenses are deducted from the fund

**b)**  
**i) & ii)**

- Bench mark fund:

- Q1:  

$$\frac{(1200 + 240) \times 1.1 + (800 + 160) \times 0.91}{(2000 + 400)} = \frac{2457.6}{2400} = 1.024 \text{ i.e., } 2.4\%$$
- Q2:  

$$\frac{(1584 + 216) \times 0.98 + (873.6 + 144) \times 0.85}{(2457.6 + 360)} = \frac{(1764 + 864.96)}{(2457.6 + 360)} = \frac{2,628.96}{2817.6} = 0.93305 \text{ i.e., } 6.695\%$$
- Q3:  

$$\frac{1764 - 120) \times 1.057 + (864.96 - 80) \times 0.97}{(2628.96 - 200)} = \frac{(1737.71 + 761.41)}{2,428.96} = \frac{2,499.12}{2,428.96} = 1.02888 \text{ i.e., } 2.888\%$$
- Q4:  

$$((1,737.71 - 180) \times 1.035 + (761.41 - 120) \times 1.02) / (2,499.12 - 300) = 1.03063 \text{ i.e., } 3.063\%$$

Notional Fund : Actual Asset Allocation, Bench mark equity and Bond Stock Selection
- Q1:  

$$((1600 + 320) \times 1.1 + (400 + 80) \times 0.91) / 2400 = 1.0620 \text{ i.e., } 6.2\%$$
- Q2:  

$$((2112 + 288) \times 0.98 + (436.8 + 72) \times 0.85) / 2548.8 = 0.9573 \text{ i.e., } (-) 4.27\%$$
- Q3:  

$$((2352 - 160) \times 1.057 + (432.48 - 40) \times 0.97) / 2584.48 = 1.0438 \text{ i.e., } 4.38\%$$
- Q4:  

$$((2316.94 - 240) \times 1.035 + (380.71 - 60) \times 1.02) / 2397.65 = 1.0330 \text{ i.e., } 3.3\%$$

## Comparative Returns :

Quarter	Benchmark	Notional	Actual	Asset Allocation	Stock selection	Overall performance
A	B	C	D	E =C-B	F=D-C	G=D-B
Q1	2.4%	6.2%	-7.08%	3.8 %	-13.28 %	-9.48 %
Q2	-6.7%	-4.27%	-1.54%	2.43 %	2.73 %	5.16 %
Q3	2.89%	4.38%	4.68%	1.49 %	0.38 %	1.79 %
Q4	3.06%	3.3%	31.94%	0.24 %	28.64 %	28.8 %
Aggregate	1.31%	9.62%	26.36%	8.31 %	16.74 %	25.05 %

Note: Full marks can be awarded even if the candidate has just added the quarterly performance to determine the annual overall performance. In this case, the answers would be :

Out performance due to asset allocation : 7.96%  
 Out performance due to stock selection : 18.47%  
 Overall Out performance : 26.35%

**iii) Bond Duration Attribution:**

Here we need two funds : the bench mark fund and a fund which has benchmark asset allocation, benchmark equity stock selection and <5 year bond performance.

The returns on the second fund are calculated as follows:

- Q1:  
 $((1200+240) \times 1.1 + (800+ 160) \times 0.95) / 2400$   
 =1.04 i.e., 4 %
- Q2:  
 $((1584+216) \times 0.98 + (912+ 144) \times 0.92) / 2856$   
 =0.95782 i.e., (-)4.22 %
- Q3:  
 $((1764-120) \times 1.057 + (971.52- 80) \times 1.02) / 2535.32$   
 =1.0441 i.e., 4.41 %
- Q4:  
 $((1737.71-180) \times 1.035 + (909.35- 120) \times 1.06) / 2347.06$   
 =1.0434 i.e., 4.34 %

The quarterly out performances as compared to the bench mark index are as follows :

- Q1:  
 4 % - 2.4% = 1.6%
- Q2:  
 - 4.22 % + 6.7% = 2.48%
- Q3:  
 4.41% - 2.89% = 1.52 %
- Q4:  
 4.34% - 3.06% = 1.28 %

Aggregate out performance = 6.88%

This is contained in the 18.4% overall stock selection profit.

c) **Performance of the Fund Manager :**

- The fund manager attained a good performance in three of the four quarters. Indeed it was extremely good in Q4.
- Stock selection has been the main driver of out-performance. The stock selection was only negative in Q1 which was due to very poor stock selection.
- The asset allocation added to the performance. The decision to allocate more funds to equities through out was a good one as can be seen from the overall positive benchmark equity performance over the year and the overall negative benchmark bond performance over the year.
- The bench mark bond performance was particularly bad in Q1 and Q2 which was when most of the positive new moneys were received. This is reflected by the lower MWROR as compared to the TWROR. The MWROR is sensitive to the timing of the cash flows into the fund which is generally outside the fund manager's control.
- The < 5 year bond index out-performed the All- Stocks Bond Index in all quarters. Therefore the decision to invest in short bonds resulted in positive out performance in all quarters.
- Performance in Q4 was outstanding both in equity stock selection and in bond stock selection as compared to the bench marks and even as compared to the < 5 year bench mark. This quarter added a substantial amount to the overall performance.

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- Q. 7 a) ➤ The four key risks faced by an institutional investor are [a] Market Risk' [b] Credit Risk, [c] Operational Risk and [d] Relative Performance Risk

**Monitoring and Controlling Market Risk :**

- Market Risk is the risk related to changes in the value of the portfolio due to movements in than market value of assets held.
- The risk control guideline for this risk can be based on a "Value at Risk" measure. For example ,a reduction in statutory free assets of 10% over a one-year period must have a probability of less than x%.
- The VaR can be set relative to some suitable bench mark such as an index ,an industry median fund or the value of the liabilities.
- Permitted ranges need to be established within which the managers have the freedom to depart from the bench mark. These could be set either as a load difference [for example limiting domestic equities between 30% and 50% of the portfolio] or as a load ratio [for example, limiting domestic equities to 40% plus/minus 25% [of the 40%].

**Monitoring and Controlling Credit Risk :**

- Credit risk refers to the risk that a counter party to an agreement will be unable or unwilling to fulfill their obligations.
- The credit worthiness of the counter parties may be controlled by placing limits on credit ratings of the counterparties [as published by the major rating agencies]. An example is " No cash deposits with banks with a "BBB" rating or lower".
- It can be controlled in derivative transactions by dealing on recognized derivative exchanges with a central clearing house which stands as counter party to all deals rather than over-the-counter.
- Limits need to be placed on the credit exposure to any single counter party. For example, no more than Rs.15 mln will be deposited with any single "A" rated bank. While applying this type of guideline the investor needs to allow for relationships between different counter parties within the same group- setting group exposure limits.

- Credit Derivatives like credit default swaps can be used to limit credit exposure to particular borrowers.

**Monitoring and Controlling Operational Risk :**

- Operational Risk is the risk of loss due to fraud or mismanagement within the organization itself.
- The guidelines for controlling operational risk will typically relate to :
  - The separation of “front office”[dealing and recording] and “back office” [accounting and settlement ] functions
  - Ensuring that all individuals have the appropriate experience and qualifications required to perform their roles.
  - Ensuring that the roles and responsibilities of all individuals and departments are clearly divided and known.
  - Monitoring closely the actions of the derivative traders and the resulting derivative exposures.

**Monitoring and Controlling Relative Performance Risk :**

- This risk refers to the risk of under-performing comparable institutional investors in which case it might lose new and/or existing business to those competitors.
- This risk can be controlled in a way similar to controlling market risk, except that in this case the asset selection guidelines [loss ratios, etc] will relate to competitors’ portfolios [for example, the median fund] rather than to a bench mark portfolio.

**b) Suitable Mix of Coupon Bearing Government Bonds :**

- If the Government bonds under consideration can be stripped then it may be possible to select appropriate combinations of Government bond strips such that the cash flows from these combinations precisely match the liability outgo both in terms of magnitude and timing.
- If the Government bonds under consideration cannot be stripped, then we cannot match precisely as we will have coupon payments from the Government bonds that need to be reinvested during the first two years.
- In this case where the Government bonds cannot be stripped, we can adopt an immunization strategy which will work as follows:
- Find the present value of the liabilities by discounting at the gross redemption yield appropriate to each term. We need to invest this value in the three given Government bonds.
- Find the discounted mean term of the liabilities.
- Find the combinations of investment in the three Government bonds that have the same discounted mean term. We need as much as possible in the shortest and longest stocks so that the spread about the discounted mean term [DMT} is greater for the asset proceeds than for the liability outgo –thereby maximizing the convexity of the assets.

**c) Ways of enhancing the total return on the fixed interest portfolio**

**Buying Corporate Bonds :**

- Higher fixed interest returns might be achieved by buying bonds like the corporate bonds [or international bonds like Euro bonds if the option is available] with a higher credit risk than Government Bonds in order to receive the credit spread.
- This is because historical evidence suggests that on average such bonds provide higher actual returns even after allowing for the higher level of actual defaults.
- The risk in this strategy is that the price of such bonds may fall if credit quality is perceived to deteriorate and ultimately such bonds may default.

**Buying Callable Bonds :**

- Callable bonds tend to provide higher returns but with the risk of being called before maturity. In addition, if they are corporate callable bonds, then the credit risk issues as discussed above will also apply.

**Adjusting the Duration [Policy Switching] :**

- Another method of achieving higher returns is to adjust the duration of the fixed interest portfolio in anticipation of general movements in interest rates i.e., policy switching.
- For example, if the fund manager believes that the interest rates are going to fall then he would increase the duration of the bond portfolio relative to that of the annuity portfolio and vice-versa if he believed that interest rates are going to rise.
- The risk in this strategy is that the fund manager can get the call wrong and hence lose money. He would need excellent market timing skills to repeatedly call the market correctly
- In addition, active pursuit of this strategy will increase stock turnover and the associated dealing costs i.e., bid-offer spreads, commissions, settlement costs, etc.

**Anomaly Switching :**

- It may be possible to increase returns by anomaly switching between similar bonds that are temporarily mis-priced.
- Anomaly switching is most appropriate in the less efficient segments of the bond market. An example can be the small size corporate bond segment
- The advantage of anomaly switching is that it will not involve any mis-matching of the liabilities. However, the incremental returns from anomaly switching are likely to be very small [marginal] after allowing for transaction costs.

**Riding the Yield Curve :**

- The fund manager could buy longer-term bonds when the yield curve is upward-sloping and shorter-term bonds when the yield curve is down-ward sloping.
- The risk in this strategy is the duration mis-match between the asset and the liability portfolio.

**Use of Derivatives:**

- It may be possible to carryout the above switching strategies using traded derivatives so as to reduce the transaction costs involved.
- Purchasing and/or writing (if allowed) credit derivatives can also help to increase the returns particularly if the prices of such derivatives are thought not reflect the true risk/cost of default on the reference bonds

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Note to the Examiner: The above answer is more comprehensive than an answer which can be produced under exam conditions. Hence full marks can be awarded even if some of the above strategies are fully explained.

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