Actuarial Society of India

Examinations

November 2006

ST5 – Finance and Investment A

Indicative Solution

Question 1
Country X is likely to be best suited to a mixed regime involving statutory regulation, self-regulation and codes of practices.

With a system is already in place, the new system should be a development of the old rather than a step change. This would minimise costs, avoid unnecessary confusion and maintain confidence in the system.

Part self-regulation is recommended because the country is a substantial nation with a mature stock market. It therefore has skilled and experienced market participants who can help to ensure good use of market knowledge, relatively rapid response to changing market needs and help to minimise the costs of the system - also because it should help to maintain public confidence and minimise abuse of the system.

Country Y may also adopt a mixed regime involving statutory regulation, self-regulation and codes of practice.

However, statutory regulation is likely to be a far more important component of the regime than for Country X. (A full statutory regime is also acceptable) - because market development relies on the confidence of both domestic and foreign investors, the promotion of effic ient and orderly markets and the protection of consumers. Market participants are likely to be relatively scarce and inexperienced. Self regulation could therefore lead to mistakes, fraud and consequent inefficient markets and loss of confidence.

Partly self-regulation as regulatory systems should develop over time and eventually the depth and experience of the market participants will grow. The system adopted now should allow for increasing emphasis on self-regulation in the future.

[6]

Question 2
The beta of a portfolio is a measure of the volatility of the portfolio relative to movements in the whole market as measured by an index.

It is defined as the covariance of the return on the portfolio with the return on the market index, divided by the market index return.

The risk adjusted measures which make use of beta are

Treynor Measure — measure of reward per unit of systematic risk defined as

$T = (R_p - r) / B_p$

where R_p is the return on the portfolio

r is the risk free rate of return over the period, and
B_p is the systematic risk in the portfolio

Jensen Measure — is a measure of return relative to a benchmark with the same degree of risk

$$J = R_{portfolio} - R_{benchmark}, \text{ where } R_{benchmark} = r + B_p (R_{market} - r)$$

[5]

Question 3

(i) In the classical system company profits are taxed twice- once in the hands of the company and once in the hands of the investor.

The investor may pay both income tax and capital gains tax. The tax rates paid on income and capital gains may be different.

Under the imputation system the investor will receive their distributions after the company has paid some or all of their tax liability on the distribution.

The sum paid by the company to the government is imputed to the investor. If the investor is not liable to tax they may be able to reclaim some or all of the tax paid. For some investors there may a further tax liability on the distribution.

(ii) The factors to be considered are:
The total rate of tax on an investment and how it is split between income and capital gains.
The timing of the tax payments e.g. whether the tax is deducted at source or has to be paid subsequently.
To what extent losses or gains can be aggregated over different investments and time scales.
The extent to which tax deducted at source can be reclaimed.

[6]

Question 4

(i)

Term - likely to be several years
Risk – high chance of total loss of investment. Say 80% failure rate on many new corporate ventures

Return – returns will be earned by dividends an/or profit share or eventually selling interest to another investor
Return – over the medium to long term expected returns are high (commensurate with the risk and poor marketability)
Marketability – very poor or nil
Minimum investment size – large or small depending on individual circumstances
Other – likely to involve heavy hands on involvement/may get board representation/influence over company's future

(ii)

The pension scheme needs to outsource investment management/oversight to a specialist in the venture capital area. It is unlikely that the investment expertise will reside with the scheme.
The valuation of such investments is difficult.
The scheme will need to consider the cashflow carefully – firms in which the scheme may invest may expect cash injections at times that are not convenient for the scheme.
It is much easier to invest in VC funds (eg access to skills)
The returns are probably matched somewhat to inflation hence ok to meet salary related liabilities
The investment is illiquid, and may not be suitable for certain circumstances (eg. During a bulk transfer situation)
Overall this would not be a suitable investment for a defined benefit pension scheme.

[6]

Question 5

(i) Inflation
Short-term interest rates
Fiscal deficit
The exchange rate
Institutional cash flow

(ii) (a)
Corporate bonds are unlikely to carry the same security as loans made to a government. The yield differential will be based on the differences between the borrowers.

The risk of default requires investors to demand high yield. The default risk (and hence the additional premium) increases with pressures on profits.

Liquidity will also be an issue. Corporate bonds are less liquid than gilts, again leading to investors demanding additional yield premium. Add to this, the potential impact of world events causing investors to move to ultra secure investments will lead to an increase in the yield premium over government bond yields.

(b)
Demand features dominate the drivers of the level of the equity market. Central to this will be investors'expectations for corporate profits. Investors will reflect the level of risk (over government securities) they are willing to take. They will accept a lower premium if they are confident about future corporate profitability.

(iii)

Investor views on economic growth, interest rates and inflation expectations as well as general market confidence will drive the equity market.

As growth starts to slow, equity markets start to fall in anticipation of corporate profitability being lower.

Demand shifts towards bond based securities and government bonds in particular.

Company insolvency has increased the risk of corporate bonds. The yield premium on corporates over government bonds increases.

Actual defaults also serve to drive underperformance relative to government bonds.

Short-term interest rates (expected and in time actual) reduce as the government looks to stimulate demand. As economic confidence begins to recover, investors are more willing to accept equity risk and corporate default risk. After a period of underperforming the bond markets, equity returns improve.

[5]

Question 6

(i) A discounted cash flow calculation could be used to determine the price to offer. Factors required for this calculation are:

. • term of the lease and the ground rent, if leasehold
. • current market price for the apartment
. • current rental level appropriate for the apartment + expected rental growth in order to calculate the income forgone.
. • the expected period until the apartment becomes vacant allowing for the health of the retired couple
. • the expected growth in market price over the period until the apartment becomes vacant

. • the transaction costs, both for the current purchase and the resale, other costs e.g. insurance, maintenance etc.
. • tax
. • the rate of return required from such an investment

The calculation could be carried out on alternative assumptions to test for sensitivity e.g. using different rates of growth in the market price, allowing for different periods until both parents dead etc.

Consideration should also be given to factors that could dramatically affect the value of the apartment e.g. the risk of development blight, changes in the taxes on property investments, the possibility of redevelopment etc.

(ii) This investment is real in nature, it is expected to be of medium to long term, its marketability would be poor, there is no income, it may entail a significant amount of management and there is a high level of uncertainty about the term and return of the investment. Are these characteristics consistent with the daughter's objectives for her investments?

Is the size of the investment convenient and is this opportunity competitive relative to alternatives available to her?

Would she like to live in the apartment herself – she may not have a home of her own, so this maybe a genuine need – would this be agreeable to her parents?

Note : In the suggested answers given below, the symbols "x" and "*" stand for multiplication.

[10]

Question 7(a)

An interest rate floorlet provides a payoff at time t_{k+1} of

L* ? $_k$*max ($R_x - R_k$,0)

Where

L is the principal amount

? $_k$ is the tenor of the contract

R_x is the floor interest rate cap

R_k is the variable interest rate, upon which the floor is based

R_k is the floating rate which is compounded with a frequency corresponding to the length of the tenor. Hence the effective interest rate from t_k to t_{k+1} is R_k ? $_{k.}$

Therefore the discounted value of the above payoff at time t_k is:

$$? \frac{L ?_k}{(1 ? R_k ?_k} ? * Max ? R_x ? R_k, 0 ?$$

$$= \text{Max } [\frac{(R_x ? R_k) L ?_k}{(1 ? R_k ?_k}, 0\,]$$

$$= \text{Max } ? ? \frac{(R_x L ?_k ? R_k L ?_k ? L ? L)}{(1 ? R_k ?_k)}, 0 ?$$

$$= \text{Max } ? \frac{L(1 ? R_x ?_k ?}{(1 ? R_k ?_k ?} ? L, 0 ? ....(A)$$

Clearly $\frac{L(1 ? R_x ?_k)}{(1 ? R_k ?_k)}$ is the value at time t_k of a Zero Coupon bond that pays

$L(1+R_x$? $_k)$ at time $t_{k+1.}$

RHS of (A) represents the value of a payoff from a call option with an exercise date and price of t_k and L respectively, on a Zero Coupon bond with a principal of $L(1+R_x\, t_k)$ payable at time t_{k+1}

It therefore follows that an interest rate floor can be viewed as a portfolio of call options on Zero Coupon bonds

[5]

Question 7(b) (i)

Flat and Spot Volatilities:

- ? In the context of an interest rate floor, volatility refers to the annualized standard deviation of the changes in the short term interest rate on which the possible floor payments are based over the life time of the particular floorlet
- ? The Black's model assumes that the log of the short term interest rate at the start of any floorlet (i.e., at time t_k) has a variance given by ? $^2{}_k\, t_k$

- If a different volatility is used to value all floorlets in any particular floor, then each volatility is referred to as the <u>spot volatility</u> for the term t_k
- If the same volatility is used to value all of the floorlets in any particular floor, then this is referred to as flat volatility. It represents an average of the spot volatilities over the entire term of the floor.

[2]

Question 7(b)(ii)

Step 1: Payoff for the floorlet due for payment at the end of the first year.

$$100{,}000{,}000 \times \frac{1}{4} \times Max\left(0.0510 - R_{0.75}^{(4)}, 0\right)$$

Where $R_{0.75}^{(4)}$ is the value of the 3 month LIBOR rate per annum convertible quarterly at the start of the fourth quarter

Step 2: Using Black's model, the value of this floorlet can be found using the following formula

$$100{,}000{,}000 \times \frac{1}{4} \times P(0,1) \times \left[R_x \times N(-d_2) - F_{0.75}^{(4)} N(-d_1)\right]$$

Where $F_{0.75}^{(4)}$ is the 3 month forward LIBOR rate per annum convertible quarterly for the fourth quarter of the first year

Step 3: The continuously compounded forward rate over the fourth quarter can be estimated through linear interpolation as follows:

$$\frac{(R_2T_2 - R_1T_1)}{(T_2 - T_1)}$$

$$= \frac{(1 \times 0.0487 - 0.75 \times 0.0485)}{0.25} = 0.0493$$

The equivalent rate pa convertible quarterly can be found from the equation

$$\left(1 + \frac{F_{0.75}^{(4)}}{4}\right)^4 = \exp(0.0493)$$

$$i.e., F_{0.75}^{(4)} = 0.049604$$

Substituting this value in the Black's formula we get

100 x 0.25 x exp (-0.0487) x (10 ^ 6) x

[0.0510 $N(-d_2)$ – 0.049604 N $(-d_1)$

=23,811,671 x [0.0510 $N(-d_2)$ – 0.049604 N $(-d_1)$

d_1 = [ln [0.049604/0.0510] + (0.5 x 0.1 x 0.1 x 0.75)] divided by 0.1 * 0.75 ^ 0.5

= [-0.027754 + 0.00375] /0.086603

=-0.277173

$d_2 = d_1$ -0.1 * 0.75 ^ 0.5

=-0.363776

$N(-d_1)$ = N(+0.28) =0.61026

$N(-d_2)$ = N(+0.36) =0.64058

Hence value of the floor let

= 23,811,671x{0.0510 x 0.64058- 0.049604x0.61026}

=$ 57,106

Note : The above answer is extremely sensitive to rounding. **[6]**

Question (8)(a)

1. If V_T, the value of the company at time T exceeds F{he face value of the bonds} then the company will choose to repay the bonds in full and the default size will be zero.

2. If V_T, is less than F the company can afford to repay only V_T and the magnitude of default will be F - V_T

3. Combining (1) and (2), we get

$?_T$ = Max [F-V_T, 0]

[2]

Question (8)(b)

The formula for $?_T$ from (2)(a) exactly matches the payoff from a option at expiration date on the stochastic quantity V_t ith strike price F

Since V_t follows Geometric Brownian motion, it fits in with the Black Scholes formula for the value of a put option i.e.,

$?_t$= [F x exp (r(T-t)] x N(-d_2)]-[V_t x N(-d_1)]

Where

d_1 = [ln[V_t/F]+[r_X(T- t)] x [0.5 x $?^2$ x (T-t)]] divided by ? x [(T – t) ^ 0.5]

and = d_2 = d_1- [(T – t) ^ 0.5] x ?

[2]

Question(8)(c)(i)

Allowing for the possibility of default, the value of the bond

= F exp[- r (T - t)] - $?_t$

= F exp [- r (T - t)]

-[F *exp(r (T - t)xN (-d_2)] – V_t ** N(-d_1)]

= [F *exp r (T - t_] x N (-d_2)] + V_t N(-d_1)]

Given F = 460 r =0.06

T-t = 5 ? = 0.2

V_t = 1,150

d_1 = [ln(1150/460)x(0.06x5)+(0.2x0.2x0.5x5)]divided by(0.2x5^0.5)

= [0.916291 + 0.3 +0.1] divided by 0.447214 = 2.9433

d_2 = 2.9433 – 0.4472 = 2.4961

N ($-d_1$) = N(-2.94)

= 0.00164

N ($-d_2$) = N(2.50)

= 0.99379

Hence the value of the bond after allowing for the possibility of default

= 460 x exp (- 5 x 0.06) x 0.99379

+ 1150 x 0.00164

= Rs. 340.55 mln

[5]

Question(8)(c)(ii)

Change in the value of the bond for a given change in the value of the company is given by the expression $N(-d_1)* dV_t$ where $N(-d_1)$ is the delta of the bond when viewed as a derivative instrument.

$N(-d_1)$=0.00164 and dV_t= -0.02

Hence the fall in the value of the bond = 0.00164* 0.02 =0.0000328 or 0.00328 %. (0.5 mark)

[1]

Question No (9)(a)

Define

? S = change in spot price S, during a period of time equal to the life of the hedge

? F = change in futures price F, during a period of time equal to the life of the hedge

? S = standard deviation of ? S

? F = standard deviation of ? F

? = correlation coefficient between ? S and ? F

h = hedge ratio

For a long hedge, the change in the value of the hedger's position during the life of the hedge

= h * ? F - ? S

Likewise for a short hedge, the change in the value of the hedger's position during the life of the hedge

= ? S – h * ? F

In either case the variance 'v' which denotes the variance of the changes in the value of the hedged position is given by

$v = ?^2_S + h^2 ?^2_F - 2 h ?_S ?_F ?$

Setting ? V/ ? h = 0, we get

$2 h ?^2_F - 2 ?_S ?_F ? = 0$

$h = ? ?_S / ?_F$

The second order partial derivative of V with respect to $h = 2 ?^2_F > 0$

This implies that the value ;of 'h' which minimizes the variance is $h = ? ?_S / ?_F$

In other words the optimal hedge ratio = $h = ? ?_S / ?_F$

[4]

Question (9) (b) (i)

Theoretical futures price

= 1250 x exp [(0.06 – 0.03) x 0.25]

= 1250 x 1.00753

= 1259.41

[1]

Question (9)(b) (ii)

Number of contracts the fund manager should short [using the minimum variance hedge ratio formula]

= beta of the portfolio x 50,000,000/(1250 x 250)

= 139.2

Rounding to the nearest whole number, 139 contracts should be shorted

[2]

Question (10)(a)

Limitations of Using a Deterministic Investment Model

- ? A deterministic model is based on a single set of assumptions – for example with respect to the estimates of the expected return from each asset class.
- ? This fails to take into account the variability of the asset returns and the correlated variability of the liability values
- ? This is a problem because it is difficult to test whether the nature of assets (i.e., “fixed” or “real”) is suitable to match the liabilities
- ? We therefore need to run the deterministic model a number of times considering different scenarios (e.g., low inflation/high growth, high inflation/low growth, etc) in order to investigate how surplus might vary under different scenarios.
- ? However identifying the scenarios to be investigated is a fairly subjective exercise

- ? If there is a lot of variability in the parameters, then insolvency may have a

  Non- negligible probability even if the deterministic approach suggests that there is a large excess of assets over liabilities. Even scenario testing may not uncover this problem. A stochastic model is really needed.

  Some apparently innocuous scenarios can infact lead to financial difficulties. These can be assessed only if these scenarios are actually modeled and investigated, which may or may not happen when a deterministic model is used, Again a stochastic model is really needed.

Question (10)(b)

The alternative courses of action are as follows:

- ? Move atleast some part of the portfolio into short dated government bonds. The result is that the capital loss will be smaller if inflation and yields rise leading to a drop in bond prices.
- ? Buy real assets such as index linked government bonds or equities
- ? Sell government bond futures or buy put options on a government bond future. Then if inflation and yields rise, the profit on the derivatives position will compensate for the loss on government bonds.
- ? Move into cash [money market instruments] completely to avoid any capital loss which can arise with a rise in inflation and yields.

[4]

Question (10) (c)

Risks Inherent in the Portfolio Construction of the Pension Fund

- ? The fund's liabilities are deemed to be equivalent to a 60% equity/20% property/20% undated bond portfolio. There is a certain element of doubt in this equivalence because of the assumptions involved in the modeling (eg., the assumption that equity share prices will move in line with salary inflation in the long run)

- ? The strategic risk of the fund is measured by the difference between the total strategic benchmark chosen by the trustees for the fund (85% in equities and 15% in bonds) and the liability equivalent split of 60%/20%/20%.

  The downside is that the strategic benchmark can under-perform relative to the value of the liabilities.
- ? There is a significant structural risk inherent in the portfolio due to the fact that the bond manager's benchmark is over the 10 year Government bond index (not undated bonds). Hence the sum of the aggregate benchmarks given to the managers will not equal the strategic benchmark set by the trustees.

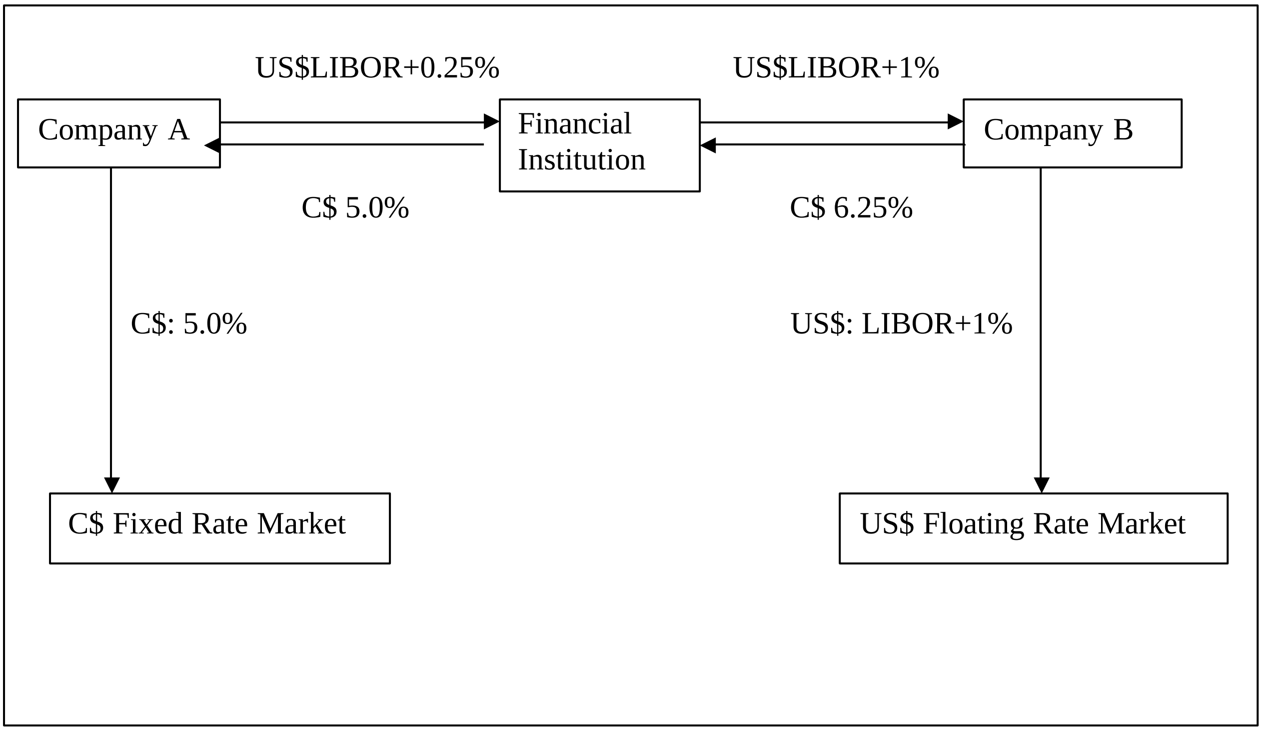
The downside is that the aggregate of all the various benchmarks given to the individual managers can under-perform relative to the strategic benchmark asset allocation

? The active risk of this fund stems from the fact that the equity manager may not perform in line with the domestic equity market index. We are told that the equity manager follows a growth style. Hence he will select only from a subset of stocks (classified as "growth stocks") and not from all stocks comprising the equity market index

The downside is that individual investment managers can in aggregate under- perform the aggregate of their benchmarks.

Question (11) (a)

- Company A has a comparative advantage in the Canadian Dollar (C$) fixed rate market Company B has a comparative advantage in the US $ floating rate market. However Company A wants to borrow in the US$ floating – rate market and Company B wants to borrow in the C$ fixed rate market. This gives rise to the swap opportunity.
- Differential between US$ floating rates is 0.5% pa and the differential between C$ fixed rates is 1.5%pa. The difference between the differentials is 1%. Therefore the total potential gain to all parties from the swap = 100 BP. If the financial institution requires 50 BP, then A and B can be made 25 BP better off each.
- The following swap structure shows that A's net borrowing cost is US$: LIBOR + 0.25% pa while B's net borrowing cost is C$ 6.25% pa



? Principal payments flow in the opposite directions to the arrows at the start of the swap and in the same direction as the arrows at the end of the swap

? The financial institution would be exposed to some FX risk which could be hedged using forward contracts.

Questions 11 (b)

? At the start of the swap, both contracts have a value of approximately zero

? As time passes, it is likely that the swap values will change, so that one swap has a positive value to the bank and the other has a negative value to the bank

? If the counterparty on the other side of the positive value swap defaults, the bank still has to honor its contract with the counterparty. It is liable to lose an amount equal to the positive value of the swap

? Thus the bank is subject to credit risk when it enters into two offsetting interest rate swap contracts.

Question (11) (c)

? At the end of year 3, the bank was due to receive $500,000 (=10 x (10 ^ 6) x 0.5 x 10%)

? And pay $450,000 (10 x (10 ^ 6) x 0.5 x 9%) Thus the immediate loss is $50,000

? To value the remaining swap, we assume that the forward rates are realized. All forward rates are at 8% pa.

? Floating payment on every interest payment date after 3 years is $ 400,000 (=10 x (10 ^ 6) x 0.5 x 8%) and the fixed rate payment is $ 500,000.

Thus the net payment that should have been received by the bank is $ 100,000 on each of these interest payment dates, which will not be received as a consequence of default.

? Therefore the cost of default can be calculated by discounting the following cash flows to end of year 3@ 4% per six months

Year	Net Cash Flow	Discounted Value at end of Year 3
3	50000	50000
3.5	100000	96154
4	100000	92456
4.5	100000	88900
5	100000	85480

Total cost of default = sum of the amounts in column 3 of the above table

= \$412,990

= \$413,000

[4]

Question (12)(a)

The three methods of assessing portfolio performance are:

- Against a published market index
- Against other portfolios eg., those of competitors, peer groups, etc
- Against a pre determined bench mark portfolio (notional fund)

Question (12)(b)

- Actual fund return over the two year period
  = (1+0.5 x 0.095 + 0.5 x 0.06) (1 + 0.6 x 0.145 + 0.4 x 0.055) – 1
  = (1.0775) x (1.109) -1
  = 0.195 or 19.5%pa …………(A)

- Index Return over the two year period using index returns and actual asset allocation
  =(1+0.5 x 0.1 + 0.5 x 0.055) (1 + 0.6 x 0.125 + 0.4 x 0.045) – 1
  = (1.0775) x (1.093) -1
  = 0.1777 or 17.77%pa ………….(B)

- Both the above calculations assume that the fund is rebalanced at the end of the first year.
- Similar Funds Return
  = (1.07)(1.08) – 1
  =15.56%pa ……………(C)

- Bench mark return (assuming benchmark asset allocation and index returns)
- = (1+0.6 x 0.1 + 0.4 x 0.055) (1 + 0.4 x 0.125 + 0.6 x 0.045) – 1
  = (1.082 x 1.077) -1

= 16.53%pa ………(D)

Comments:

- ? Actual fund has outperformed the index (based on actual asset allocation) by 1.73% (=A-B)
- ? The actual fund has outperformed similar funds by 3.94% (= A- C)
- ? The actual fund has outperformed the benchmark fund by 2.97% (=A –D)

  Relative to the benchmark fund, both stock selection (=A –B =1.73%) and asset allocation (=B – D = 1.24%) have contributed to out performance.

[5]

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