Actuarial Society of India

Examinations

November 2006

ST6 – Finance and Investment B

Indicative Solution

[1]
The range that we want is :

$S_T - K > 0$

i.e. $S_T - K$

Substituting the formula for S_T:

$$S_t e^{(r-\frac{1}{2}\sigma^2)(T-t)+\sigma(X_T - X_t)} > K$$

Dividing by S_t and taking logs:

$$(r-\frac{1}{2}\sigma^2)(T-t)+\sigma(X_T - X_t) > K \log \frac{K}{S_t}$$

$$\Leftrightarrow \quad \sigma(X_T - X_t) > K \log \frac{K}{S_t} - r - \frac{1}{2}\sigma^2 (T-t)$$

Dividing through by $\sigma\sqrt{T-t}$:

$$\frac{X_T - X_t}{\sqrt{T-t}} > \frac{1}{\sigma\sqrt{T-t}}\left(\log \frac{K}{S_t} - (r - \frac{1}{2}\sigma^2)(T-t)\right)$$

or $$X > \frac{1}{\sigma\sqrt{T-t}}\left(\log \frac{K}{S_t} - (r - \frac{1}{2}\sigma^2)(T-t)\right)$$

where $X = \frac{X_T - X_t}{\sigma\sqrt{T-t}}\left(\log \frac{K}{S_t} - (r - \frac{1}{2}\sigma^2)(T-t)\right)$

This gives us:

$$-d_2 \frac{1}{\sigma\sqrt{T-t}}\left(\log \frac{K}{S_t} - (r - \frac{1}{2}\sigma^2)(T-t)\right)$$

[5]

[2]

(i) No Loss

IF an investor buys a bond then the savings bank can invest the money in the S&P 500 so that they are not exposed to movements in the S&P 500. However, the bank is guaranteeing that investors will receive at least *x*% of their initial investment back. It can hedge against loses greater than this by buying a put option on the index with a strike price of *x*% of the current index value. This put option will cost a certain amount of money – *p*, say.

The bank is also limiting the investor's return to 1`25% of their initial investment. Because of this they can afford to sell call option with a strike price of 125% of the current index value. This call option will be priced at *c*, say Provided that $c \geq p$, the savings bank will not make a loss.

(ii) No profit or loss

If $c = p$ then the savings bank will not make a profit or loss. So the problem remains to work out the price of the call option *c* and then work out the value of *x* such that $c = p$

We will be using Black-Scholes analysis to price the option and because the numbers are all relative, we can assume that the underlying price is 100, say. Using the formulae on page 47 of the Tables, we can calculate the price of a call option.

$$d1 = \frac{In(S_t / K) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$= \frac{In\left(\frac{100}{125}\right) + \left(0.025 + \frac{0.25^2}{2}\right)}{0.25\sqrt{1}} = -0.6676$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = -0.6676 - 0.25\sqrt{1} = -0.9176$$

$$\begin{aligned} c &= S_1\Phi(d_1) - K\Phi(d_2)e^{-r(T-t)} \\ &= 100\Phi(-0.6676) - 125\Phi(-0.9176)e^{-0.025} \\ &= 100x0.252 - 125e^{-0.025}x0.179 \\ &= 3.35 \end{aligned}$$

We now need to work out the value of x so that p = 3.35. We will try K = 90 to begin with:

$$d1 = \frac{In(S_t / K) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$= \frac{In\left(\frac{100}{90}\right) + \left(0.025 + \frac{0.25^2}{2}\right)}{0.25\sqrt{1}} = -0.6464$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = -0.6676 - 0.25\sqrt{1} = -0.9176$$

$$\begin{aligned} p &= K\Phi(-d_2)e^{-r(T-t)} - S_1\Phi(-d_1) \\ &= 90\Phi(-0.3964)e^{-0.025} - 100\Phi(-0.6464) \\ &= 90e^{-0.025}x0.346 - 100x0.259 \\ &= 4.46 \end{aligned}$$

This is higher than the required value and so we try a lower value for the strike price K = 80:

$$d1 = \frac{In(S_t / K) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$= \frac{In\left(\frac{100}{80}\right) + \left(0.025 + \frac{0.25^2}{2}\right)}{0.25\sqrt{1}} = -1.1176$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = -1.1176 - 0.25\sqrt{1} = -0.8676$$

$$\begin{aligned} p &= K\Phi(-d_2)e^{-r(T-t)} - S_1\Phi(-d_1) \\ &= 80\Phi(-0.8696)e^{-0.025} - 100\Phi(-1.1176) \\ &= 80e^{-0.025}x0.196 - 100x0.132 \\ &= 1.86 \end{aligned}$$

We require a put option with a premium of p = 3.35. We can use linear interpolation to find the value of K that will give this premium:

$$K = 80 + (90 - 80)x\left(\frac{3.35 - 1.86}{4.46 - 1.86}\right) = 85.73$$

So the savings bank can set the guarantee level at 86% and use the hedging portfolio described to avoid making a loss.

[8]

[3]

This question is adopted from Question 2 of the 1999 CiD paper.

(i) Price of a European call option

We can calculate the price of a European call option using the Black-Scholes formula from page 47 of the Tables. Here we have:

S_t*=30, K=29, r=0.05,* σ*=0.25, q=0 and T-t= 1/3*

So we get:

$$d1 = \frac{\log(S_t / K) + (r - q + 1/2\sigma^2\,(T - t)}{\sigma\sqrt{T - t}}$$

$$= \frac{\log(30/29) + [0.05 - 0 + 1/2(0.25)^2](1/3)}{0.25\sqrt{1/3}} = 0.423$$

and $d_2 = d_1 - \sigma\sqrt{T - t} = - 0.423 - 0.25\sqrt{1/3} = - 0.278$

The option price is then:

$$\begin{aligned} C_1 &= S_1 e^{-q(T-t)}\Phi(-d_1) - Ke^{-r(T-t)} - \Phi(-d_2) \\ &= 30\Phi(0.423) - 29\,e^{-0.05(1/3)} - \Phi(0.278) \\ &= 30(0.664) - 28.53(0.610) = 2.52 \end{aligned}$$

So the price of the call option is 2.52.

Price of an American call option

Since this is a non-dividend-paying share, it would never be optimal to exercise it early. It, therefore, behaves like a European option and has the same value, namely 2.52.

Price of a European put option

Using the formula from page 47 of the Tables for the price of a put option (with the same parameter values), we get:

$$p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_1e^{-q(T-t)}\Phi(-d_1)$$
$$= 29e^{-0.05(1/3)}\Phi(-0.278) - 30\Phi(-0.423)$$
$$= 28.53(0.390) - 30(0.336) = 1.05$$

(ii) If the stock is dividend-paying

To calculate the price of the European call option when the share pays a dividend, we need to use a revised value of S_t in the Black-Scholes formula. This is calculated as the current share price minus the discounted value of the dividend, *i.e.,* $30\text{-}0.5e^{-0.05/1½/12)}$.

Because the payment of the dividend can be expected to reduce the future price of the share and hence the intrinsic value of the call option (and because the holder of the call option would not receive the dividend), this will result in a lower price than before.

In normal circumstances, the only time when it might be optimal to exercise an American call option early is on the final ex-dividend date. So the approximate value can be calculated as the maximum of the value of the corresponding European option and the value of the European option expiring on the final ex-dividend date.

When there is a dividend, this will increase the price for the European put option for the same reasons as for the European call option.

To calculate the price of the European put option when the share pays a dividend, we make the same adjustment to the value of S_t as for the call option.

As before, the payment of the dividend can be expected to reduce the future price of the share. This will increase the intrinsic value of the option at the expiry date, and so the revised price will be higher than before.

[10]

[4]

(a) The ratio of the size of the position taken in future contracts to the size of the exposure in the underlying asset is called the hedge ratio.

(b) Let Δs and Δf denote the (random) changes in the spot price and the futures price over the life of the hedge, so that var(Δs) = σ^2 and var(Δf) = σ^2_f.

If the hedge ration is h, the portfolio will consist of a long position in the underlying asset and a short position in the future, with h future for each unit of the underlying.

If we let *s* denote the spot price and *f* the futures prices, then the change in the portfolio value will be $\Delta(s\text{-}hf) = \Delta s\text{-}h\Delta f$, which has variance V=var$(\Delta s\text{-}h\Delta f)$.

So: V = var$(\Delta s\text{-}h\Delta f)$ = var $(\Delta s)+h^2$ var$(h\Delta f)$-2$h$cov$(\Delta s, \Delta f)$

$$= \sigma^2_f + h^2\sigma^2_f - 2h\rho s\sigma_f$$

Differentiating with respect to *h* to find the minimum:

$$\frac{dv}{dh} = 2h^2\sigma_f - 2\rho\sigma_f = 0 \Rightarrow h = \rho\frac{\sigma_s}{\sigma_f}$$

The second derivative is $2\sigma^2_f > 0$, which confirms that it is a minimum.

(c) Rolling the hedge forward is a technique for hedging long-term exposures using short-term futures contracts either because the futures contracts available are not sufficiently liquid or because the maturity of the futures contracts available is not long enough for the hedger's purposes.

The technique is not without risk.

- If *n* futures contracts are entered into and closed out over the hedging period, there are n-1 rollover basis risks and one final basis risk when the *n* th futures contract is closed out and the hedge unwound.

  We'will see shortly that "basis risk" refers to the risk involved in a hedge that arises as a result of a mismatch between the values of the securities involved.

- Rolling the hedge forward tends to work well in situations where there is a very high correlation (>0.8) between changes in the price of the asset being hedged and changes in the futures prices of the contract used.

- Also it is difficult to rollover large numbers of futures contracts over a long period of time without being selected against by other market participants.

- Finally, exchanges may become concerned about their exposure to market participants using the technique, especially where they account for a sizeable amount of the open interest in the contract.

  "Open interest" refers to the total number of contracts currently in force.

  In such circumstances, exchange clearing houses may impose higher margin requirements on the hedger, causing it significant cashflow problems.

(d) "Basis" can be defined as the spot price of the asset to be hedged minus the futures price of the contract used.

The uncertainty associated with the future basis is called "basis risk".

The three main causes of basis risk are:

1. The asset whose price risk is being hedged may not be the same as the asset underlying the futures contract.

   The futures contracts available are based on a standarised form of the underlying asset *e.g.*, Brent oil. In practice the price of the asset a producer or consumer is dealing with may be different, *e.g.*, it may be of a higher or lower quality.

2. As the spot and figures prices are guaranteed to coverage only at maturity of the futures contract, if the hedge is closed out before the expiry of the futures contract it is likely to be less than satisfactory.

3. Very often the hedger is unsure of the exact date that the asset will be bought or sold.

[7]

5(a) The bond can be regarded as the sum of four zero coupon bonds
Current price of the bond is:
50B(0, 0.5) + 50B(0, 1) + 50B(0, 1.5) + 1050B(0, 2)
Price of zero-coupon bond as per Vasicek model is given as:

$B(t,T) = e^{a(\tau)-b(\tau)r(t)}$ where $\tau = T - t$

$b(\tau) = \dfrac{1-e^{-\alpha\tau}}{\alpha}$ and $a(\tau) = (b(\tau)-\tau)\left\langle \mu - \dfrac{\sigma^2}{2\alpha^2}\right\rangle - \dfrac{\sigma^2}{4\alpha} b(\tau)^2$

$$b(0.5) = \frac{1-e^{-0.10x0.5}}{0.10} = 0.4877$$

$$b(1) = \frac{1-e^{-0.10x1}}{0.10} = 0.9516$$

$$b(1.5) = \frac{1-e^{-0.10x1.5}}{0.10} = 1.3929$$

$$b(2) = \frac{1-e^{-0.10x2}}{0.10} = 1.8127$$

$$a(0.5) = (0.4877-0.5)(0.1-\frac{0.02^2}{2x0.1^2}) - \frac{0.02^2}{4x0.1}x0.4877^2 = -0.00122$$

$$a(1) = (0.9516-1)(0.1-\frac{0.02^2}{2x0.1^2}) - \frac{0.02^2}{4x0.1}x0.9516^2 = -0.00478$$

$$a(1.5) = (1.3929-1.5)(0.1-\frac{0.02^2}{2x0.1^2}) - \frac{0.02^2}{4x0.1}x1.3929^2 = -0.0105$$

$$a(2) = (1.8127-2)(0.1-\frac{0.02^2}{2x0.1^2}) - \frac{0.02^2}{4x0.1}x1.8127^2 = -0.0183$$

$B(0,0.5) = e^{-0.00122-0.4877x0.10} = 0.9536$
$B(0,1) = e^{-0.00478-0.9516x0.10} = 0.9136$
$B(0,1.5) = e^{-0.0105-1.3929x0.10} = 0.8792$
$B(0,2) = e^{-0.0183-1.8127x0.10} = 0.8496$

Current Price of the bond is:
50x0.9536 + 50x0.9136 + 50x0.8792 + 1050x0.8496 = Rs. 1029.39

5(b) At the start of the swap, both contracts have a value of approximately zero. As time passes, it is likely that the swap value will change, so that one swap has a positive value to the bank and the other has a negative value to the bank. If the counterparty on the other side of the positive-value swap defaults, the bank still has to honour its contact with the other counterparty. It is liable to lose an amount equal to the positive value of the swap.

5(c) (i) Aggregation Risk: Aggregation risk is where this occurs because similar positions are held in different markets eg call options on bank shares in the UK and the US.

(ii) Concentration Risk: Concentration risk is where this occurs because large position are held in a small number of similar derivatives with in the same market eg Rs. 1000 crores in options on bank shares in India.

(iii) Operational Risk: Operational risk refers to potential losses from: (a) the use of inadequate or inappropriate pricing models; and/or (b) problems caused by "back office" administration systems.

[11]

6(a) On the first day of the delivery month, the bond has 15 years, 6 months and 8 days to maturity. Rounding down to the next 3 months then gives a term of 15 years and 6 months to maturity. Using half-year time period and an interest rate of 3% per half year, the present value of the bond is equal to:

$$\sum_{t=1}^{31} \frac{5}{1.03^t} + \frac{100}{1.03^{31}} = 140$$

The conversion factor is therefore 1.4000.

6(b) On the first day of the delivery month, the bond has 21 years, 3 months and 7 days to maturity. Rounding down to the next 3 months then gives a term of 21 years and 3 months to maturity. Using half-year time period and an interest rate of 3% per half year, the present value of the bond is equal to:

$$\frac{1}{\sqrt{1.03}}[3.5 + \sum_{t=1}^{42} \frac{3.5}{1.03^t} + \frac{100}{1.03^{42}}] = 113.66$$

Subtracting the accrued interest of 1.75, this becomes 111.91. The conversion factor is therefore 1.1191.

6(c) Cost of delivering the bond 1 = 170 – 118.75x1.4000 = Rs. 3.75
Cost of delivering the bond 2 = 140 – 118.75x1.1191 = Rs. 7.11
The cheaper bond to deliver is bond 1.

[7]

7. If the swapation gives the holder the right to receive a fixed rate R_X, the value of the swapation is:

$$LA[R_X\Phi(-d_2)-F_0\Phi(-d_1)]$$

$$A=\frac{1}{m}\sum_{i=1}^{mn}P(0,t_i)$$

$$A=\frac{1}{2}[e^{-0.06x4.5}+e^{-0.06x5}+e^{-0.06x5.5}+e^{-0.06x6}+e^{-0.06x6}+e^{-0.06x6.5}+e^{-0.06x7}]=2.1275$$

A rate of 6% per annum with continuous compounding translates into 6.09% with semi-annual compounding.
F_0 = 0.0609, R_X = 0.06, T = 4 and σ = 0.2

$$d_1=\frac{\ln(0.0609/0.06)+0.2^2x4/2}{0.2\sqrt{4}}=0.2372$$

$$d_2=d_1-0.2\sqrt{4}=-0.1628$$

$$\Phi(-d_1)=0.4062$$

$$\Phi(-d_2)=0.5647$$

Value of the Swapation is:
10,000,000x2.1275[0.06x0.5647 – 0.0609x0.4062] = Rs. 194,428.74

[12]

8(a) 1 year zero-rate

$$1050e^{-R_1}=1000$$

$$R_1=4.88\%$$

2 years zero-rate

$$52e^{-0.0488}+1052e^{-2R_2}=1000$$

$$R_2=5.07\%$$

3 years zero-rate

$$60e^{-0.0488}+60e^{-2x0.0507}+1060e^{-3R_3}=1000$$

$$R_3=5.88\%$$

4 years zero-rate

$$70e^{-0.0488}+70e^{-2x0.0507}+70e^{-3x0.0588}+1070e^{-4R_4}=1000$$

$$R_4=6.92\%$$

5 years zero rate

$$70e^{-0.0488} + 70e^{-2x0.0507} + 70e^{-3x0.0588} + 70e^{-4x0.0692} + 1070e^{-5R_5} = 1000$$

$$R_5 = 6.89\%$$

8(b) Forward rate for 1 year to 2 years:

$$\frac{5.07x2 - 4.88x1}{2-1} = 5.27\%$$

Forward rate for 2 years to 3 years:

$$\frac{5.88x3 - 5.07x2}{3-2} = 7.48\%$$

Forward rate for 3 years to 4 years:

$$\frac{6.92x4 - 5.88x3}{4-3} = 10.03\%$$

Forward rate for 4 years to 5 years:

$$\frac{6.89x5 - 6.92x4}{5-4} = 6.77\%$$

8(c) Price of 4-year zero = $1000e^{-4x0.0692} = 758.32$

Price of 5-year zero = $1000e^{-5x0.0689} = 708.71$

For each 4-year zero issued today, use the proceeds to buy:

758.32/708.71 = 1.07 five-year zeros

Your cash flows are thus as follows:

Time	Cash Flow	
0	$ 0	
4	-Rs. 1,000	The 4-year zero issued at time 0 matures; the issuer pays out $1,000 face value
5	+Rs. 1070	The 5-year zeros purchased at time 0 mature; receive face value

This is a synthetic one-year loan originating at the end of year 3 (beginning of year 4). The rate on the synthetic loan is $1000e^{r} = 1070; r = 6.77\%$ precisely the forward rate for 4 years to 5 years..

8(d) $60e^{-0.0488} + 60e^{-2x0.0507} + 1060e^{-3x0.0588} = Rs.1000$

8(e) Price after one year will be:

$$60e^{-0.07} + 1060e^{-2x0.07} = Rs.977.46$$

1 year holding period return is:

$$\frac{977.46 + 60 - 1000}{1000} = 3.75\%$$

[12]

[9]

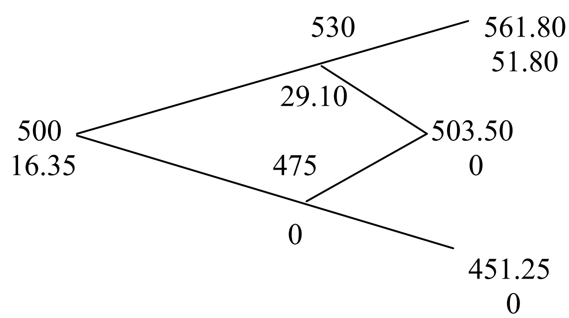
a) In the simple one-period binomial model, a replication strategy for any derivative portfolio can be constructed at the start of the period which will reproduce the payoff from the derivative portfolio at the need of the time period.

In order to price a derivative security with pay-off H = $h(S_t)$ at time δ^t, we construct a replicating portfolio.

This consists of ϕ unite of stock and ψ units of bonds. The value of this portfolio at time zero is Vo. $Vo = \phi * S_o + \psi * 1$
$= \phi S_o + \psi$

b)

i) A tree describing the behaviour of the stock price is as below. The upper number of each mode is the stock price, the lower number is the option price.



The risk neutral probability of an up-move, p, is given by

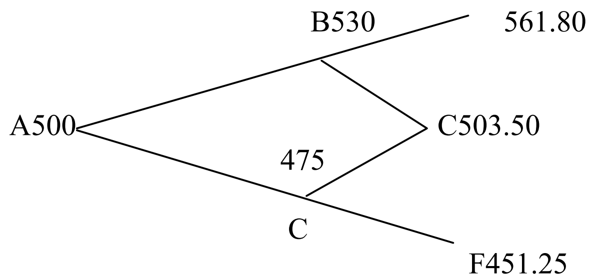
$$1.06\,p + (I\text{-}p)\,0.95 = 1 * e^{0.05*0.25}$$
$$P = 0.568895$$

There is a payoff from the option of 561.80-510 = 51.80

For the highest final mode. The payoff is zero in other cases. The value of the option is therefore,

$$51.80 * 0.568895 * e^{0.05*0.25} * 0.568895 * e^{0.05*0.25} = 16.35$$

ii) The tree for valuing the put option is as below:



The value of put option at C is

$(510\text{-}503.50)\ p\ e^{0.05*0.25} + (510\text{-}451.25)\ (1\text{-}p)\ e^{0.05*0.25}$

Where the value of *p* is as in (i) above.
= 28.6646

Value of put option at B is

$(510\text{-}503.50)(1\text{-}p)\ e^{0.05*0.25} = 2.76737$

The value of the option is therefore

$[2.76737p + 28.6646(1\text{-}p)\ e^{0.05*0.25} = 13.76$

iii) The value of the put plus the stock price is 13.76 + 500 = 513.76

The value of the call plus the present value of the stock price is
$16.35 + 510\ e\ e^{0.05*0.25} = 513.76$

This verifies that the put-call parity holds.

iv) In order to test whether it is worth exercising the option early we have to compare the value calculated for the option t each mode with the payoff from immediate exercise.

At mode C the payoff from immediate exercise is 510-475 = 35. Since this sis greater than the value calculated for the option at their mode (28.6646), the option should be exercised at this mode.

The option should not be exercised either at mode A or mode B.

v) A filtration (F*i*) is the history of the stock up until tick-time *i* on the tree.

The filtration starts at time zero with F_o equal to the path consisting of the single mode 1, that is F_o {A}from the tree in (ii) above. By time 1, the filtration will either be $F_1 = \{A, C\}$ if the first jump was down, or $F_1 = \{A, B\}$if the first jump was up.

If the stock price at t=1 is t30 then
$F_1 = \{500, 530\}$. The filtration thus fixes a history of choices, and thus fixes a mode.

[15]

[10]

a) Silver a held for investment by some investors. If the futures price is too high, investors will find it profitable to increase then holdings of silver and short futures contracts.

If the futures price is too low they will find it profitable to decrease their holding of silver rand go long in the futures market.

Sugar is a consumption asset. If the futures price is too high, a "buy sugar and short future" contract works.

However, since investors in general do not hold the asset, the "sell sugar and buy futures strategy" is not widely used when the futures price is low. There is, therefore, an upper bound but no lower bound to the futures price.

b) Convenience yield measures the extent to which there are benefits obtained from ownership of the physical asset that are not obtained by owners of long futures contracts.

The cost of carry is the interest plus storage cost less the income earned.

The futures prices, F_o, and spot price, so, are related by $F_o = S_o\, e^{(c-y)T}$ where C in the cost of carry, Y is the convenience yield, and T is the time to maturity of the futures contract.

c) Suppose that the futures contract lasts for *n* days and that F_i is the futures price at the end of day *i* ($0<i<n$). Define δ as the risk-free rate per day (assumed constant).

Consider the following strategy:

1. Take a long futures position of e^{δ} at the end of day 0 (*i.e.,* at the beginning of the contract).

2. Increase the long position to $e^{2\delta}$ at the end of day 1.

3. Increase the long position to $e^{3\delta}$ at the end of day 2.

And so on.

This strategy is summarized in the Table below:

Day	**0**	**1**	**2**	***n*-1**	***n***
Future Price	F_o	F_1	F_2	F_{n-1}	Fn
Futures Position	$e\delta$	$e^{2\delta}$	$e3^{\delta}$	$e^{n\delta}$	0
Gain (Loss)	0	$(F_1-f_o)\ e^{2\delta}$	$(F_2-f_1)\ e^{2\delta}$	$(F_{n-1}-f_{n-2})\ e^{(n-1)\delta}$	$(F_n-f_{n-1})\ e^{n\delta}$
Gain (Loss) compounded to day *n*	0	$(F_1-f_o)\ e^{n\delta}$	$(F_2-f_1)\ e^{n\delta}$	$(F_{n-1}-f_{n-2})\ e^{n\delta}$	$(F_n-f_{n-1})\ e^{n\delta}$

By the beginning of day *i*, the investor has a long position of $e^{\delta i}$.

The profit (possibly negative) from the position on day *i* is $(F_i-f_{i-1})\ e^{\delta i}$

The value at the end of day *n*, accumulated at the risk free rate is,

$$(F_i-f_{i-1})\ e^{\delta i}\ e^{(n-i)\ \delta} = (F_i-f_{i-1})\ e^{ni}$$

The value at the end of day *n* of the entire investment strategy is, therefore

$$\sum_{i=1}^{n} (Fi - F_{i-1})e^{n\delta} = (F_n - F_0)e^{n\delta}$$

Because F_n in same as the terminal asset price, S_T, the terminal value of the investment strategy can be written $(S_T - F_0)\ e^{ni}$

An investment of F_0 is a risk-free bond combined with the strategy just given yields

$F_0 e^{n\delta} + (S_T F_0)e^{n\delta} = S_T e^{n\delta}$ at time T.

No investment is required for all the long futures positions described. It follows that an amount F_0 can be invested to give an amount $S_T e^{n\delta}$ at time T.

Suppose next that the forward price at end of day 0 is G_0. investing G_0 in a risk less bond and taking a long forward position of $e^{n\delta}$ is also guaranteed at time T.

It follows that, in the absence of arbitrage opportunities $F_0 = G_0$, *i.e.,* the futures price and forward price are equal

[13]

[11]

a) The process W = (Wt : t ≥0) is a P-Brownian motion if and only if

i) W_t is continuous, and $W_0 = 0$

ii) The value of W_t is distributed, under P, as a normal random variable N(0, t)

iii) The increment $W_{s+t} - Ws$ is distributed as a normal N(O, t), under F, and is independent of F_s, the history of what the process did up to time s.

b) X is not a Brownian motion.

The increments are wrong, in equivalently the conditional distribution are wrong. The increment $X_{s+t} - X_s$ is a normal with variance $t - 25(\sqrt{1 + t/s - 1})$, which is not t, and the increment is not independent of X_s.

c) The Martingale Representation Theorem states that, if X_t is Brownian motion with respect to a probability measure Q, then any continuous Martingale M_t (with respect to the same probability measure Q) can be written in a unique way in the form.

$$M_t = M_0 + \int_0^1 \phi_s dX_s$$

For some previsible process ϕ

[7]

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