Actuarial Society of India
EXAMINATIONS

3rd November 2006

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.30 – 13.30 pm)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*
5. *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1) You are given the following samples from an exponential distribution with mean 5: 10.6101, 2.7768, 11.8926, 0.1976, 6.6885, 6.4656. Transform these numbers suitably to produce six samples from the normal distribution with mean 2 and variance 4.

[5]

Q.2) An insurer considers covering a random loss which can assume the values X_1, X_2 and X_3, with probabilities p_1, p_2 and p_3, respectively ($X_1 < X_2 < X_3$, $p_1 + p_2 + p_3 = 1$). He has the options of
decision d_1: not covering the loss at all,
decision d_2: covering the entire risk with premium loading θ, and
decision d_3: covering the entire risk with premium loading θ and passing on loss in excess of X_2 to a reinsurer at the premium loading ξ ($\xi > \theta$).

(i) Write down the loss matrix for the direct insurer, indicating overall loss for different decisions under the three loss scenarios.

(3)

(ii) Describe the Bayes strategy for the direct insurer.

(2)

(iii) Which decision of the direct insurer minimizes the minimum loss?

(1)

(iv) Assume $p_1 = 0.2$, $p_2 = 0.7$, $X_1 = 0$, $X_2 = 10$, $X_3 = 100$, $\theta = 0.5$ and $\xi = 0.6$. Which decision of the direct insurer is minimax?

(1)

(v) How does the answer to part (iv) change when ξ is allowed to take any value larger than 0.5?

(1)

[Total 8]

Q.3) Data on recent claim sizes (in hundreds of rupees) arising from an insurance are: 35, 111, 201, 309, 442, 617, 843, 1330, 2368 and 4685.

(i) Assuming that the claims come from a lognormal distribution with parameters μ and σ, derive the expression for the maximum likelihood estimates of these parameters and evaluate them for the available data.

(5)

(ii) Assuming that the claims come from a Pareto distribution with parameters α and λ, use the method of moments to estimates these parameters.

(3)

(iii) If the insurance company engages a reinsurer to cover losses in excess of Rs. 3 lakhs, estimate the percentage of claims that will involve the reinsurer under each of the two models above.

(2)

[Total 10]

Q.4) Claims generated by a portfolio arrive regularly at the end of every year, starting from year 1. The claim size at the end of each year is either Rs. 2 lakhs (with probability $\frac{3}{4}$) or Rs. 10 lakhs (with probability $\frac{1}{4}$). Claims of different years are independent of one another. A fixed premium at the rate of Rs. 6 lakhs per year accumulates continuously. The initial surplus is Rs. 10 lakhs.

(i) Write an expression for the surplus process (in lakhs of rupees) as a function of time (in years), and simplify this expression for integer time points.

(2)

(ii) Sketch on plain paper the graph of the surplus function over the range $0 \leq t < 5$, if the claims at the ends of the the first four years are Rs. 2 lakhs, Rs. 2 lakhs, Rs. 10 lakhs and Rs. 2 lakhs, respectively.

(3)

(iii) Calculate the probability of ruin at the end of the first year.

(1)

(iv) Calculate the probability of ruin at the end of the second year.

(2)

(v) Calculate the probability that the first ruin occurs at the end of the fourth year.

(2)

[Total 10]

Q.5) Losses arising from a risk, $X_1, X_2, \ldots$, are independent and have the lognormal distribution with $E(\log(X_1)) = \mu$ and $Var(\log(X_1)) = 1$. You have to find the credibility estimate of α, the mean of X_1, using the above data and collateral information. According to the collateral information, the parameter μ has the normal prior distribution with mean 10 and variance 4. The credibility estimate of α is of the form

$$\hat{\alpha} = z\bar{X} + (1-z)E(\alpha),$$

where z is a fraction to be determined, $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$ and $E(\alpha)$ is the mean of α obtained from the prior distribution of μ.

(i) Express α in terms of μ.

(1)

(ii) Calculate $E(\alpha)$ from the prior distribution of μ.

(2)

(iii) Using the well-known identity $E[(\hat{\alpha}-\alpha)^2] = E[E\{(\hat{\alpha}-\alpha)^2|\alpha\}]$, show that

$$E[(\hat{\alpha}-\alpha)^2] = z^2 E[Var(\bar{X}|\alpha)] + (1-z)^2 Var(\alpha).$$

(2)

(iv) Derive the credibility factor, that is, the value of z that minimizes the expression given in part (iii).

(2)

(v) Calculate $Var(\alpha)$ from the prior distribution of μ.

(2)

(vi) Calculate $Var(\bar{X}|\alpha)$ from the distribution of X_1, and then obtain $E[Var(\bar{X}|\alpha)]$ from the prior distribution of μ.

(3)

(vii) Determine the credibility factor and the credibility premium explicitly as a function of n and $\bar{X}$.

(3)

[Total 15]

Q.6) The transition rules for moving between the three levels, 0%, 35% and 50%, of a No Claims Discount system are as follows:

If no claim is made in a year, the policyholder moves to the next higher level of discount or remains at 50%.
When at the zero or 35% level of discount, the policyholder moves to (or remains at) the zero level of discount when one or more claims are made in the year.
When at the maximum level of discount (50%), the policyholder moves to the 35% level of discount if exactly one claim is made during the year, and moves to the zero level of discount if two or more claims are made during the year.

It is assumed that the number of claims X made each year has a geometric distribution with parameter q such that

$$P(X = x) = q^x(1-q), \quad x = 0, 1, 2, \ldots.$$

The full premium is Rs. 350.

(i) Write down the transition matrix.

(3)

(ii) Verify that the equilibrium distribution (in increasing order of discount), is of the following form, for some constant k:

$$\left(kq^2(2-q),\ kq(1-q),\ k(1-q)^2\right),$$

and express k in terms of q.

(5)

(iii) The value of the expected premium in the stationary state, paid by "low risk" policyholders (with $q = 0.05$), is Rs. 178.51. Calculate the corresponding figure paid by high risk policyholders (with $q = 0.1$). Comment on the effectiveness of the No Claims Discount system.

(4)

[Total 12]

Q.7) The cumulative incurred claims (in thousand rupees) and number of reported claims, by accident year and development year are given below.

CUMULATIVE COST OF INCURRED CLAIMS

	Development Year				
Accident Year	*0*	*1*	*2*	*3*	*Ultimate*
2002	342	429	458	471	490
2003	481	689	701		
2004	584	800			
2005	665				

CUMULATIVE NUMBER OF REPORTED CLAIMS

	Development Year				
Accident Year	*0*	*1*	*2*	*3*	*Ultimate*
2002	41	46	48	49	50
2003	45	51	53		
2004	50	56			
2005	54				

(i) Estimate the ultimate number of claims, for each accident year, using chain-ladder development factors.

(4)

(ii) Estimate the ultimate average incurred cost per claim, for each accident year, using grossing-up factors.

(4)

(iii) Using the results from parts (i) and (ii), calculate the total reserve required, assuming that claims paid to date are Rs. 1,821,300.

(3)

[Total 11]

Q.8) The starting salary of an actuarial job is a random variable with density $f(y,\lambda) = \frac{27}{2}\mu^{-3}y^2e^{-3y/\mu}$.

(i) Show that μ is the mean of the distribution.

(1)

(ii) Show that the above density can be written in the form of an exponential family, and identify the natural parameter of this family.

(2)

(iii) Consider the model

$$g(\mu) = \alpha + \beta x,$$

where $x = 1$ if the new employee has passed at least three core technical subjects of ASI and $x = 0$ otherwise, α and β are unspecified parameters and $g(\cdot)$ is the canonical link function. You are given data on the pair (x, y) for twenty individuals recruited in actuarial jobs in 2005. Obtain an expression for the maximum likelihood estimator of β.

(6)

[Total 9]

Q.9) **(i)** Derive the autocovariance and autocorrelation functions of an MA(1) process

$$X_t = e_t + \theta e_{t-1},\quad E(e_t) = 0,\ Var(e_t) = \sigma^2,\ Cov(e_t, e_s) = 0 \text{ if } t \neq s.$$

(3)

(ii) A time series is believed to be ARIMA$(0, d, 1)$. The sample ACF $r(k)$ of the m-times differenced time series are given in the following table, for $m = 0, 1, 2, \ldots, 5$ and $k = 1, 2, \ldots, 10$.

	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
$r(1)$	0.998	0.758	–0.476	–0.653	–0.740	–0.793
$r(2)$	0.995	0.746	–0.022	0.144	0.283	0.387
$r(3)$	0.993	0.744	0.004	0.004	–0.045	–0.109
$r(4)$	0.991	0.741	0.017	0.012	0.009	0.022
$r(5)$	0.988	0.731	–0.006	–0.011	–0.013	–0.014
$r(6)$	0.986	0.723	0.004	0.008	0.010	0.012
$r(7)$	0.984	0.715	–0.008	–0.007	–0.007	–0.008
$r(8)$	0.981	0.708	0.002	0.003	0.006	0.008
$r(9)$	0.979	0.701	–0.001	–0.008	–0.011	–0.012
$r(10)$	0.977	0.694	0.020	0.021	0.019	0.016

Assuming that a suitably differenced version of the time series is invertible, examine this table and suggest appropriate values of d and θ, with justification.

(5)

[Total 8]

Q.10) The annual claim numbers associated with a particular risk are assumed to be samples from a Poisson distribution, with unknown parameter λ. The parameter λ has a gamma prior distribution with second moment 3/2 and variance 1/2. The claim size distribution is exponential, with unknown mean $1/\theta$. The parameter θ has a gamma prior distribution with mean 0.005 and standard deviation 0.001. A total of 5 claims have been made in the past 8 years, and the claim sizes have been Rs. 241, Rs., 382, Rs. 213, Rs. 205 and Rs. 88.

(i) Calculate the moment generating function (MGF) of the aggregate annual claims, as a function of λ and θ.

(3)

(ii) Estimate the MGF of part (i) by replacing the unknown parameters by their respective prior means.

(2)

(iii) Obtain the Bayes estimate of λ with respect to the squared error loss function.

(2)

(iv) Obtain the Bayes estimate of θ with respect to the squared error loss function.

(2)

(v) Estimate the MGF of part (i) by replacing the unknown parameters by their respective posterior means.

(1)

(vi) Compute and compare the variances of the aggregate annual claims from the MGFs of parts (ii) and (v).

(2)

[Total 12]