CONSTANT PROPORTION PORTFOLIO INSURANCE (CPPI) FOR IMPLEMENTATION OF DYNAMIC ASSET ALLOCATION OF IMMEDIATE ANNUITIES

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1. Introduction

In this paper, we present a strategy of managing assets of an annuities portfolio using Constant Proportion Portfolio Insurance (CPPI) strategy. While the methodology has been presented for an annuities portfolio, it can be extended to other products like Single premium bonds. We have presented a dynamic asset allocation strategy and we also discuss some pertinent issues with the immunization technique presented in the paper.

2. CPPI Strategy

CPPI for fixed-income instruments was introduced by Perold; and for equity, and other instruments, by Black and Jones of Goldman Sachs in 1986. It is a dynamic asset allocation strategy which increasingly participates in any upside movement of the performance assets (equity or other riskier assets) but at the same time ensures a minimum guaranteed return through safe assets (bonds or cash).

The strategy guarantees a minimum payout at some future date. The present value of this minimum payout is called 'floor'. In other words, the floor is the amount which if grown at the risk free rate will be able to meet the future liability. The difference between the value of the portfolio and the floor is called 'Cushion'. This cushion is the surplus of the assets over liability. The CPPI framework requires a positive cushion which means that at no time in the cycle should the value of portfolio fall below floor. Hence, cushion is the maximum possible hit that the portfolio can take.

The whole portfolio consists of two types of assets 1) Performance assets and 2) Safe assets. The amount invested in the performance assets is called 'Exposure'. Rest of the portfolio is expectedly invested in the safe assets. Under the CPPI framework the exposure is calculated as Cushion times the multiplier.

$$\text{Exposure} = \text{Cushion} \times \text{Multiplier}$$

where, multiplier is the reciprocal of the maximum value erosion of performance assets in a single time period (e.g., one day, one year, etc.). This single time period is called rebalancing cycle. For example, if the maximum possible fall in equity in a rebalancing cycle is 50% then the multiplier would be 2 (= 1 / 0.5). A multiplier of 2 implies that twice the cushion will be invested in the risky assets while the remaining fund goes in safe assets. This allocation ensures that maximum possible loss in the value of portfolio in a rebalancing cycle is equal to cushion only. Any loss over 'cushion' will leave the strategy with a 'Gap Risk'. This risk has been discussed in detail in next section. Figure1 provides an illustration for multiplier 2.

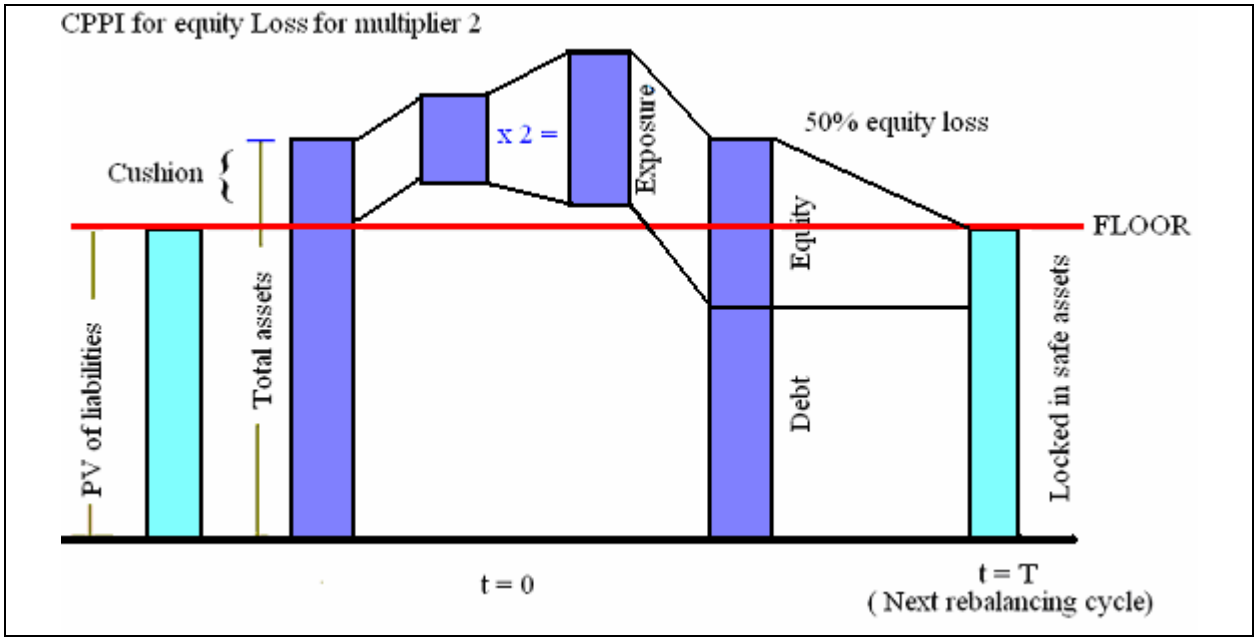

Figure1. CPPI for multiplier 2

An appreciation in the value of the performance assets results in an increase in 'cushion' which implies exposure in risky assets can be increased. Converse is also true i.e. if the performance assets lose their value then the cushion falls and, exposure in risky assets has to be reduced. This increase or decrease in exposure in the risky assets depending upon the past performance is called leveraging or de-leveraging. Leveraging is a measure of the risk appetite of the investor. If multiplier is very high then a bigger chunk is invested in the risky assets. A higher multiplier also means that cushion will erode faster for any adverse movement in the risky assets.

2.1 Difference between CPPI and Option based Portfolio Insurance (OBPI)

CPPI is different from other option based portfolio insurance techniques. In the option based techniques, the safe assets worth the floor are purchased and the rest, the cushion, is used to take enhanced exposure in the market. Though both schemes support the principal guarantee if implemented properly CPPI does provide a higher initial participation than with an equivalent option-based product.

2.2 Advantages of CPPI

- Simple to understand and Implement.
- Can be used for portfolios with all type of marketable securities.
- Higher exposure in the performance assets.
- Multiple and the cushion can be chosen as per the risk appetite of the investor.
- Unlike the option base strategies this does not have any sort of time constraints.

2.3 Limitations of CPPI

- All the assets can get locked in safe assets (bonds) portfolio if the floor is breached even once. This does not hold for assets like annuities where any future sales will again result in a cushion.
- Care has to be taken that in the case of any unexpected fall, the multiple should not be too high other wise the floor may be breached.
- It does not work very well for small cushions as in such cases exposure to the performing assets is very small.

- In the real markets, transactional costs and restrictions on leverage also become a constraint.

Apart from the above mentioned draw-backs, CPPI also faces following risks.

2.3.1 Gap Risk

A gap event occurs when the portfolio value falls below the floor. This happens when there is such a rapid fall in the value of riskier assets that portfolio value falls below the floor before the fund manager can rebalance it. In all other scenarios, the fund value remains above the floor. Between1999–2001 a lot of funds in Europe and Italy witnessed such cash-outs.

This risk can be mitigated by restricting the value of the 'multiplier'. A prudent estimation of the multiplier can be made by stress–testing the historical data of riskier assets.

2.3.2 Interest Rate risk

Interest rate changes can affect the strategy in two ways:

a) A sudden change in the interest rate can impact the value of the debt portfolio significantly.
b) A steady decline in the interest rate results in reinvestment risk.

Cushion provides a cover only against a fall in the performance assets. Even if multiplier is prudent and the performance assets don't fall beyond the floor still it can be breached if the safe assets are not immunized against interest rate risk. Conventional immunization and issues with respect to CPPI strategy have been discussed in next section.

2.3.3 Credit Risk

Credit risk for CPPI is same as that for other any other investment strategy. The risk can be mitigated only by investing in high quality credit instruments. Also the fund manger can build a margin in the cushion for this risk.

2.3.4 Float Risk

In some type of funds this risk arises when the debt instruments are not co-terminus with the scheme maturity. It can be taken care of by building cushion for float risk.

2.3.5 Liquidity Risk

Liquidity risk is another very common risk that this strategy faces. A margin in the cushion needs to be built in order to mitigate this risk.

2.3.6 Transaction costs

It results from the frequent churning of the portfolio. Presence of these frictional costs results in more then one preferred paths and scope of optimization.

2.3.7 Calculation of Cushion

A correct calculation of the cushion is subject to an accurate estimation of floor, which primarily depends upon the following quantities.

a). The uncertain future outflows.
b). Discount rate applicable.

For fund managers in the insurance industry, there is an element of uncertainty attached with the estimation of these two factors. This uncertainty makes it difficult to estimate the cushion accurately.

2.3.8 Estimation of Multiplier

Multiplier depends upon the assumption of the maximum possible fall in the riskier assets in a rebalancing cycle. A prudent multiplier is very critical to the success of the strategy.

3. Immunization

As mentioned in the previous section, in order to make the floor impervious, safe assets should be immunized against any interest rate risks. This section will describe the immunization in general while the next section will discuss the immunization for CPPI strategy.

Introduced by Frederick Macaulay way back in 1938, Duration still remains a critical tool for Asset Liability Management (ALM). Despite its wide usage immunization has some very pertinent limitations. For a perfectly immunized portfolio

PV of Assets = PV of Liability

Duration of Assets = Duration of Liability

Convexity of Assets > Convexity of Liabilities

In such a scenario the assets and liability vary with interest rate as shown in figure2.

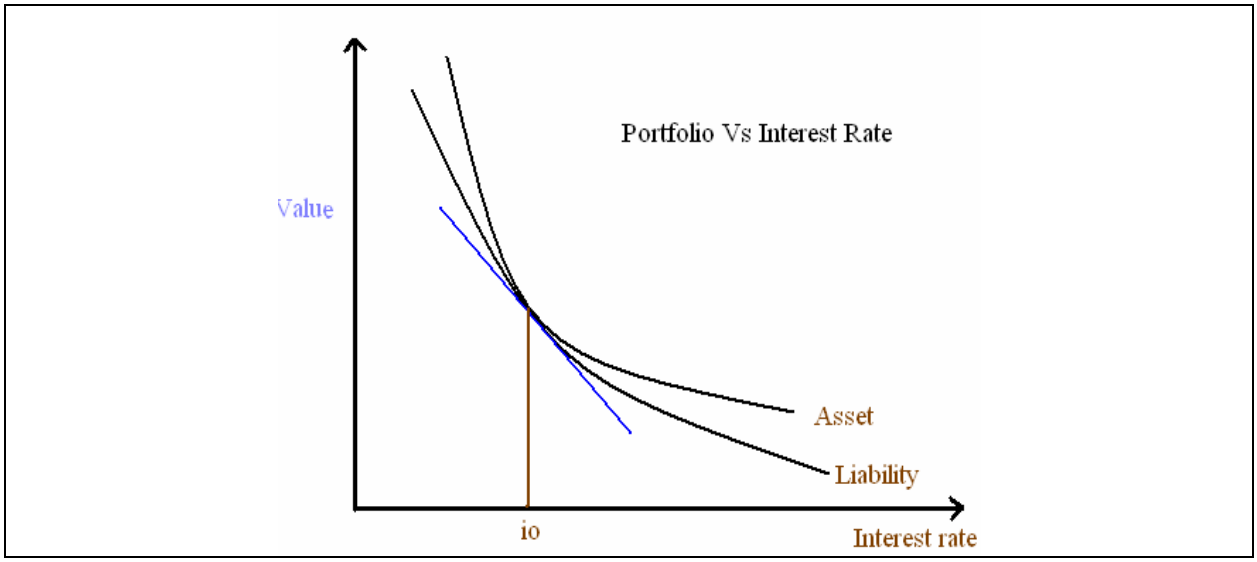

Figure 2: Assets & Liability vs interest rate for an immunized portfolio.

The above graph clearly indicates that for small changes in the interest rate the value of assets is always greater then the value of liabilities.
If the portfolio is immunized then the asset and the liability move in the same direction and also by same amount when the interest rates change (Figure 3).

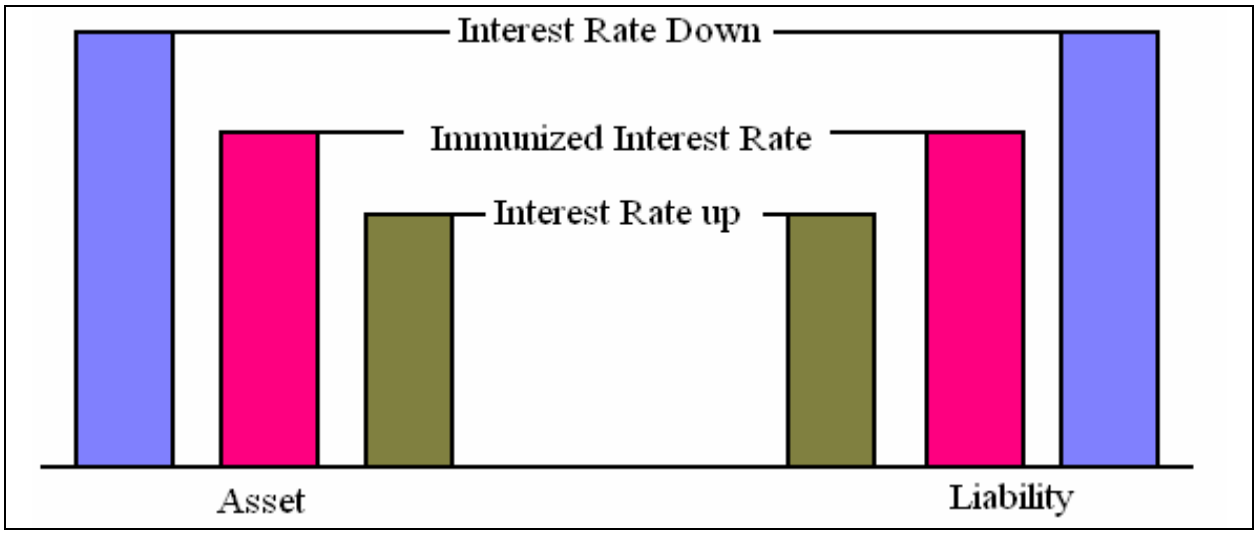

Figure 3: Movement in the asset and the liability when interest rates change

3.1. Limitations of Immunization

3.1.1. Uncertain Cash Flows/Future liabilities
There may be options or other uncertainties in the assets or in the liabilities, making the assessment of the cash flows approximate rather than known. At best, it is possible to make a good estimate of cash flows. An insurance company has to live with some uncertainty about the timing and magnitude of cash flows. Also, the duration of the liability depends upon the interest rate assumptions. In common practice a flat yield curve is used and same change in the interest rate is applied to all the terms, which may not be a case commonly.

3.1.2. Liquidity of assets
Assets may not exist to provide the necessary overall asset volatility to match the liability volatility. The assets that are available may have some sort of risk, say credit risk, associated with them. Immunization theory holds only for risk free assets.

3.1.3. Large interest rate changes
The theory relies upon small changes in interest rates. The fund may not be protected against large changes in interest rates.
3.1.4 Immunization removes the likelihood of making large profits.
As the assets are selected based upon their durations and not as per yields the immunization exercise is concerned with averting risk and not with profit building.

3.1.5 At the onset the assets should be equal to the liability.
This is the most preliminary of the Redington's immunization conditions and prohibits conventional immunization for CPPI strategy, where the value of safe assets is below the floor. Immunization for CPPI is slightly different and has been addressed in the next section.

Despite all the above-mentioned shortcomings, the immunization theory is still the most important guiding principle in asset selection.

4. Immunization in CPPI strategy.

As already mentioned, the CPPI strategy also faces an interest rate risk. It is possible that the floor is breached not due to rapid fall in the performing assets but due to changes in the interest rates. Figure 6 demonstrates the case where interest rate risk is also a present apart from the gap risk. As already discussed in the section three, for a portfolio to be immunized the present value of both assets and liabilities should be equal. In CPPI, assets more then cushion are invested in the risky assets and hence the value of debt portfolio is lower than the 'floor'. In such a case, the traditional immunization (figure 2 and 3) cannot be done.

The problem can be addressed by increasing the duration of the debt portfolio by the ratio of the liability (Floor) to debt portfolio. See figure 4.

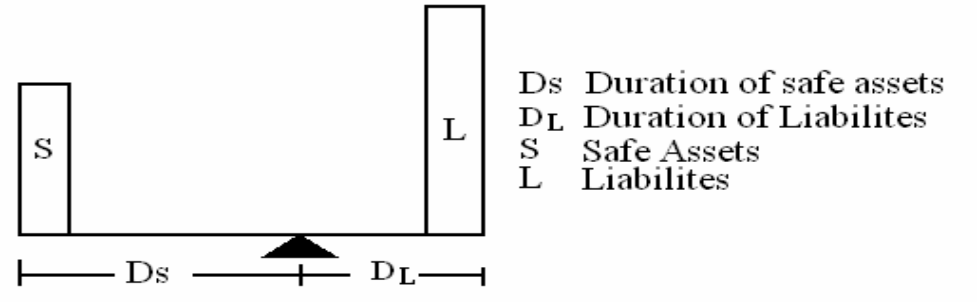

Figure 4: High duration safe assets can balance bigger liability of lower duration.

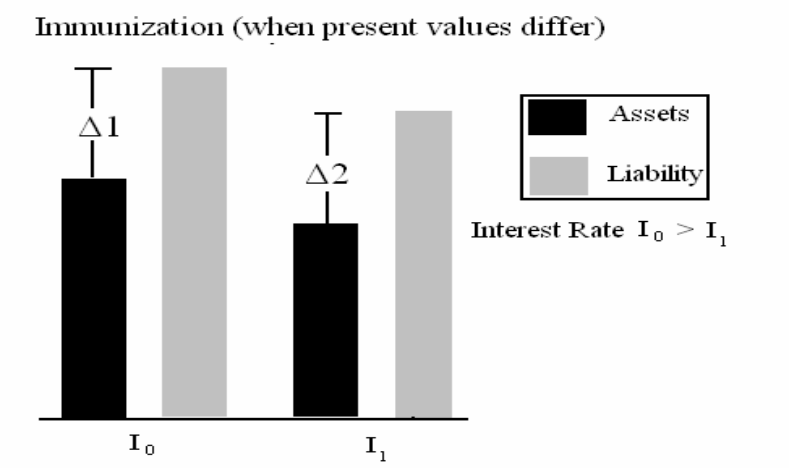

Figure 5: Change in debt assets and liability for proportional durations

In figure 5, if Δ1 is equal to Δ2 the portfolio is immune to any fluctuation in the interest rate movement though the debt assets are lower then liability. Changes in assets and liability can be matched if the duration of debt portfolio is proportionately higher then that of liability. We derive the relation between duration of asset and liability portfolio for immunization as follows:

$$\Delta D = \Delta L$$

$$\Rightarrow \lim_{\Delta i \to 0} \frac{\Delta D}{\Delta i} = \lim_{\Delta i \to 0} \frac{\Delta L}{\Delta i}$$

$$\Leftrightarrow \frac{dD}{di} = \frac{dL}{di}$$

$$\Leftrightarrow D_d D = D_l L$$

$$\Leftrightarrow \frac{D_d}{D_l} = \frac{L}{D} \quad \text{.....(1)}$$

Where,
D = Value of debt portfolio.
L = Value of liability.
D_d = Duration of debt portfolio.
D_l = Duration of Liability.
ΔD, ΔL = Changes in the value of debt portfolio and liabilities respectively.

The above relation allows us to implement the CPPI strategy for annuity portfolio. In case, there is a loss in the risky assets, the whole portfolio is locked in the form of safe assets. Before

locking this safe portfolio it is necessary to ensure that this portfolio is immunized. For this to happen, the fund manger will have to buy bonds of lower duration as debt portfolio already has duration higher than that of liability. As bonds of smaller duration are readily available in our markets, it is easy to immunize the portfolio. See figure 6.

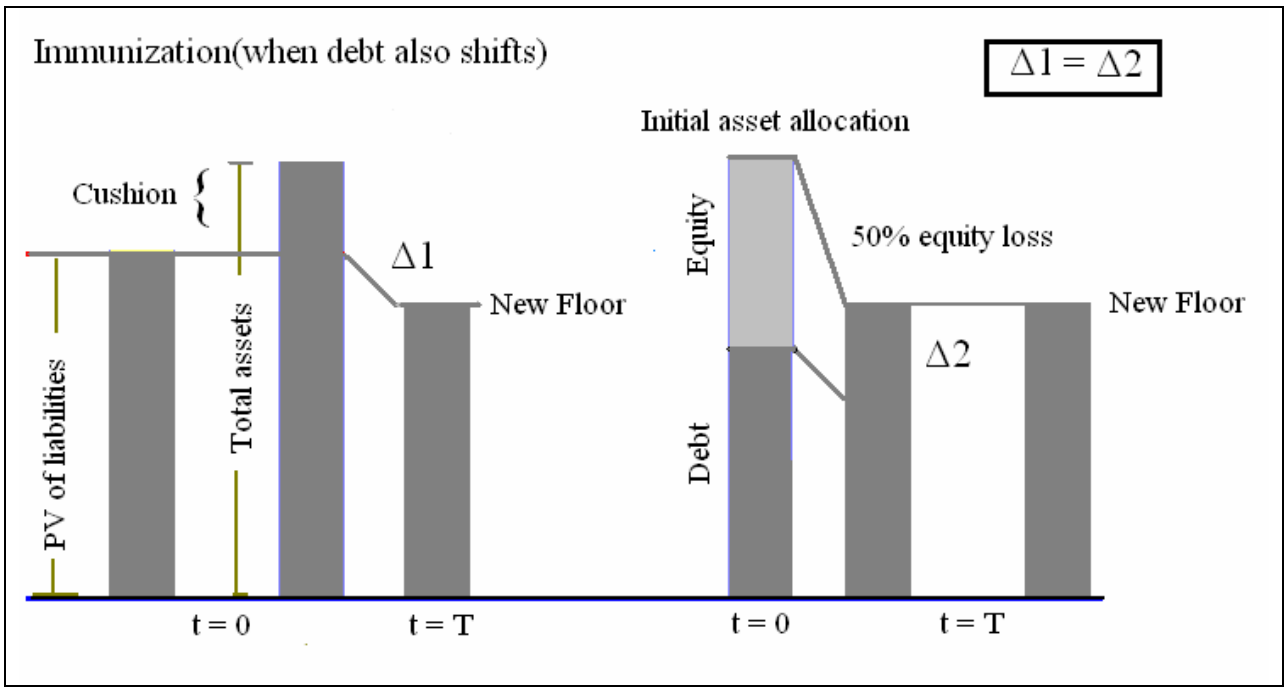

Figure 6: Implementation of CPPI strategy

5. Duration matching

The above-mentioned duration for the debt portfolio can be attained in lots of ways. Following steps were taken in the present exercise.

a. A bond universe was created such that it includes all the bonds present in the current holding statement as well as those that are available to the fund manager for investments.

b. Some of the bonds in the universe had duration higher than that of liability while others had duration lower than that of liability.

c. The *Closest Immunized Portfolio* (CIP) was created by changing the assets in just three of the bonds. One of these bonds had higher duration while the remaining two bonds had duration lower than that of the liability. The choice of 'two' is arbitrary and one can choose even one, three or more bonds for getting CIP Choosing one bond increases the concentration risk while choosing more then two defeat the purpose of CIP which was created to guide the fund manager as to how can the portfolio be immunized in least number of the transactions.

6. Optimization

The possible distribution of a debt portfolio, such that it has a given duration, has an infinite solution set; hence, a suitable optimization criterion is required to choose a suitable portfolio.

In this exercise, optimization was done so as to increase the Yield to maturity (YTM). Once CIP has been obtained other possible immunized portfolios were created by rebalancing the portfolio taking three bonds at a time.

In this way a universe of immunized portfolio with varying allocations in different bonds and with varying YTM and can be created. Fund manager can take care of risks such as concentration and liquidity by empirically choosing the portfolio he/she deems fit.

7. Conclusion

The strategy can be useful for managing assets of products like annuities or single premium bonds when a cushion is present. In the recent turbulent times structured products, many of whom were based on CPPI, have come under severe criticism. This has happened because the risks were under-estimated which lead to lower 'cushion'. Also, the recent turmoil in financial markets made many of the fixed income instruments illiquid. This made it difficult for fund managers to rebalance the portfolios.
We would like to highlight that any risk management strategy will back fire if the risks have not been correctly estimated. But if all the risk are identified and sufficient margins are built in the cushion, propped upon duration matched safe instruments, the CPPI strategy can ensure guaranteed returns, with a potential for better yields if risky assets deliver good returns.

8. References

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