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A CONSISTENT WAY TOWARDS INVESTMENT DECISIONS

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1. PRELUDE

The two widely used techniques for assessing the merit of any investment proposal are:

(a) Net Present Value method (NPV); and
(b) Internal Rate of Return method (IRR).

Although these two terms are quite common to any investor now-a-days and often feature in any analytical discussion on investment-related issues, there still seem to be considerable confusion about the exact nature of difference between the two methods as well as the domains of their applicability. In the subsequent paragraphs NPV and IRR will be defined and the scopes for the two methods will be gone into in some detail. The apparent inconsistencies between the two methods under certain situations will also be resolved and the possible role of IRR in leading towards an unambiguous investment decision indicated.

2. DEFINITIONS

A. NPV

Both NPV and IRR have the time value of money as their basic underlying concept. The net Present value (NPV) of any investment can be defined as the sum total of all discounted cash inflows less the aggregate of all discounted cash outflows. In other words,

$$NPV = \sum_{i=1}^{N} \frac{Ai}{Di} - \sum_{j=1}^{M} \frac{Bj}{Dj}$$

where the number of net inflows and net outflows during the tenure of an investment are N and M respectively. Also,

Ai = Net Cash Inflow at the ith sequence
Di = Discounting Denominator for the ith Inflow
Bj = Net Cash Outflow at the jth sequence
Dj = Discounting Denominator for the jth Outflow

Let us consider the undernoted simplified (at the same time practical) situations.

i) A single outflow followed by 'N' annual inflows: In this case

$$NPV = \sum_{i=1}^{N} \frac{A_i}{(1+K)^{*i}} - P_0$$

where $(1+K)^{*i}$ means $(1 + K)$ x $(1 + K)$ x $(1 + K)$ x … i times; Po represents the initial outflow and time is counted from the time of this outflow; K stands for the annual cost of money (annual rate of discount).

ii) A single outflow followed by N semi-annual inflows: In such a case;

$$NPV = \sum_{i=1}^{N} \frac{A_i}{(1+K)^{*i}} - P_0$$

Where K now is half of the annual cost of money.

It is quite obvious from the foregoing that an implicit assumption underlying the concept of NPV is that all the cash inflows during the life of the investment are reinvested at the same annual rate of return (i.e 'K' is constant over the entire life of the investment and does not vary over the sequence of inflows/outflows). Such a simplifying assumption is not strictly valid, particularly in view of the rapidly changing investment climate of recent times. Nevertheless, NPV continues to be a useful concept for the purpose of making a-priori appraisal of investment opportunities.

B. <u>IRR</u>

The same concept of time value of money is also at the heart of IRR, which can be defined as that particular value (more than one value in certain cases as we shall see in subsequent paragraphs) of discount at which the aggregate of all discounted inflows has the same value as the aggregate of all discounted outflows. If the Net Present Value (NPV) is considered as a function of the rate of discount (K), i.e. NPV = f(K), then IRR is that value of K for which NPV= 0. In other words f(r) = 0 where r =IRR.

As in the case of NPV, IRR also presumes reinvestment of all future inflows at the same rate of return, viz r. In spite of this deficiency, like NPV, IRR also continues to be a useful concept for assessing the merits of investment proposals. In the next paragraph (paragraph 3), IRR will be worked out for a few simple and illustrative case and its uniqueness indicated. Thereafter, a few more complicated situations will be dealt with, keeping in view the issues of uniqueness as well as the relative advantages of the NPV and IRR methods.

3. ILLUSTRATIONS

A. One Outflow followed by one Inflow after one year:
In such a situation

$$NPV = \frac{A_1}{1+K} - P_0$$

where Po = Amount of original outflow
A1 = Amount of Inflow after 1 year
K = Cost of money (Discount Rate)

NPV has therefore an inverse relationship with K.

Let the internal rate of return be r. In other words, r is that value of K for which NPV has a zero value.

Thus, A1 = Po (1+r)

The only interesting values of r are the positive ones. Mathematically, r can have all types of values – positive, negative or even complex. But, an Investment Banker would be interested in looking for only a sub-set of the entire range of possible values – viz., the positive real numbers. This restriction will immediately imply.

A1 > Po ; or a1 > 1(a1=A1/Po)

The amount of the inflow has always to be greater than that of the outflow. Besides, r will be directly proportional to a1 (figure 1).

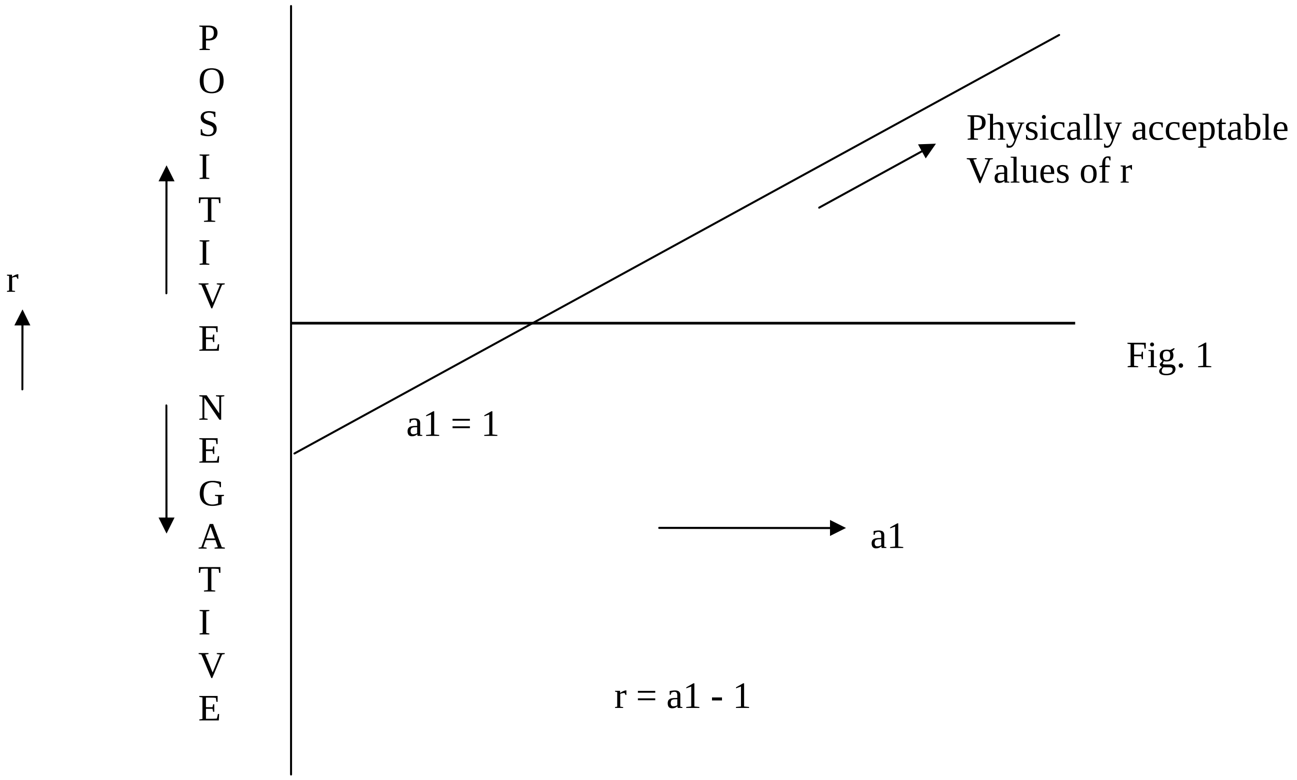


Fig. 1

For each value of a1 (a1>1), r has only one value. In other words, r is unique.

B. One outflow followed by one Inflow at the end of 2years:-

In this case,

$$NPV = \frac{A_2}{(1 + K)^{*2}} - P_0$$

where A2 = amount of Inflow at the end of 2years, and other terms have the same meaning as in the previous case.

Then, the Internal Rate of Return, r would be given by $(1+r)^{*2} = a2$

In order that r is real and not complex (a practical requirement), a2>0. Further, for r>0 (also a practical requirement), a2 > 1.In other words, the amount of the inflow in this case also has to exceed that of the outflow.

Taking realistic values of a2 (i.e. a2 >1),we get r = -1 $\pm \text{mod}(\sqrt{a2})$, where ”mod” stands for modulus (mod $\sqrt{a2}$ means the positive square root of a2). One root of the equation would thus yield a negative value for r, while the other would lead to a positive value. Since negative values of r are to be discarded from practical consideration, only the positive root of r would be of utility for assessing investment proposals. In other words, r continues to be unique in so far as investment decisions are concerned. The nature of the r - a2 curve is indicated in figure 2.

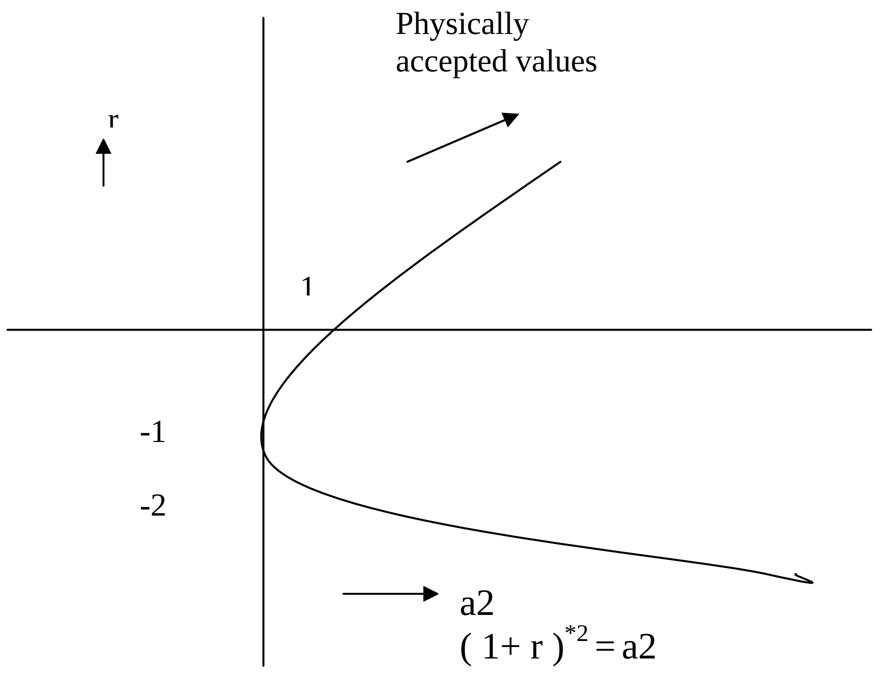


Fig.2

C. One outflow followed by one inflow at the end of 3 years:-

In such a situation,

$$\text{NPV} = \frac{A3}{(1+K)^{*3}} - Po = Po\left(\frac{a3}{(1+K)^{*3}} - 1\right)$$

Where a3 = $\frac{A3}{Po}$, and the other terms have the same connotations as in the above cases. The Internal Rate of Return will thus be determined by the equation

$(1+r)3 = a3$. For r to have a positive real value (this is demanded from practical considerations), $a3 > 1$. In other words, $A3 > Po$. The amount of the inflow must therefore exceed the amount of the initial outflow.

If α, β, γ are the roots of the above equation, then

$$(r-\alpha)(r-\beta)(r-\gamma) = 0 \quad \text{or}$$

$$r^*3 - r^*2(\alpha + \beta + \gamma) + r(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0.$$

Comparing with the equation

$$r^*3 + 3r^*2 + 3r + (1 - a3) = 0$$

One finds that :

$$\alpha + \beta + \gamma = -3$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3$$

$$\alpha\beta\gamma = a3 - 1$$

since $\alpha + \beta + \gamma =$ -ve, all the 3 roots can not obviously be positive. But $\alpha\beta\gamma$ = +ve ($a3 > 1$). Hence there would be either 2 negative roots or 2 complex (complex conjugate of each other) roots. Hence there is only one practically interesting value of r. In other words, r continues to be unique in this case also.

4. UNIQUNESS OF IRR

The uniqueness of IRR in the above 3 sample situations has already been demonstrated. Let us now try to analyse a more interesting and somewhat complicated situation as under.

Po = 10,000 units (outflow)
A1 = 22,500 units (inflow)
A2 = 12,650 units (outflow)

Where Po, A1and A2 represent respectively the amounts of inflow / outflow at zero times (time is counted from time of this initial cash flow) at the end of 1st year and at the end of 2nd year. For a discount rate of 10% ($r=0.1$), the aggregate of discounted outflows works out to 10,000 + $\frac{12,650}{1.21}$ = 20,454.55 units. For the same discount rate of 10%, the aggregate of discounted inflows turns out to be $\frac{22,500}{1.1}$= 20,454.55 units. In other words, the aggregate of net outflows equal the aggregate of net inflows at the discount rate of 10%, which is an IRR for the proposal in question (r=0.1). Similarly, the discount rate of 15% (r=0.15) is also an IRR for the proposal, because 10,000 + $\frac{12,650}{1.15^{*2}}$ = 19,565.22 = $\frac{22,500}{1.15}$.r can thus be 0.1 or 0.15, both of which are positive, real values for r and are thus physically acceptable. The question now arises which one of these 2 values is to be taken into account for the purpose of assessing the worth of the relative investment proposal and whether there is some way of commenting on the acceptability of such an investment proposal. The answer seems to lie in a more systematic analysis of the starting equation as under:-

NPV = $-Po+\frac{A1}{(1+k)}+\frac{A2}{(1+k)^{*2}}$ in the instant case, where Po, A1, A2and K have the same meanings as in the previous paragraph. Thus

$-1+\frac{a1}{(1+r)}+\frac{a2}{(1+r)^{*2}}=0$, where

a1 = $\frac{A1}{Po}$; a2=$\frac{A2}{Po}$ and r = IRR. Or

r*2 + r(2 –a1) + (1– a1 – a2) = 0.

If the roots of this equation are α and β, (r -α) (r-β) = 0. or,

r*2 – r($\alpha+\beta$) + $\alpha\ \beta$ = 0.

Thus, α + β = a1 – 2,
And $\alpha\ \beta$ = 1 – a1 – a2.

If both the roots are to be positive, $\alpha\ \beta$ = 1 – a1 – a2 > 0. or a1 + a2 <1.
In other words, A1 + A2 < Po, or the aggregate net inflow after the initial outflow has to fall short of the initial outflow. Further, for both the roots to be positive, $\alpha+\beta$ = a1 – 2 > 0; or a1 > 2 (A1 > 2Po). With a1 > 2, for a1 + a2

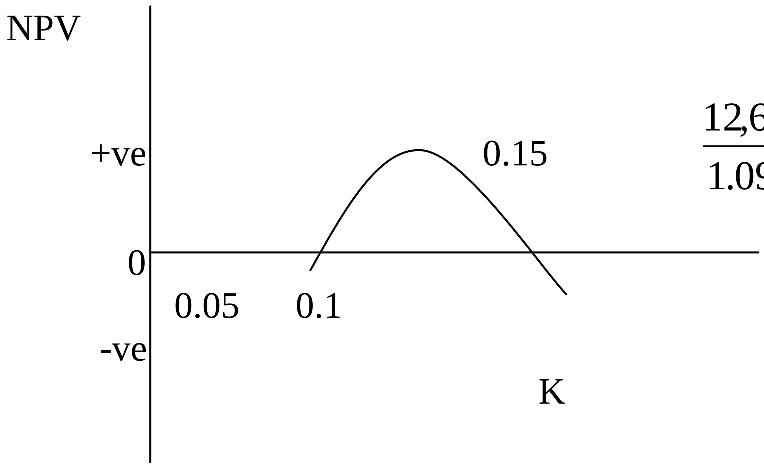
to be less than 1, a2 < -1 (A2 < - Po). In the above example, Po = 10,000, A1 = 22,500 and A2 = -12,650 so that A1+ A2 < Po with A1 > 2 Po and A2 < - Po. But, in case a1 + a2 > 1, i.e. A1 + A2 > Po, only one of the roots of r would be positive and r would thus be unique. The necessary and sufficient condition for uniqueness of r thus appears to be that the net aggregate cash inflow must exceed the initial outflow. Now, considering the first derivative of NPV (i.e. $\frac{d}{dk}(NPV)$ representing the rate of change of NPV with k), we find

$$\frac{d}{dk}(NPV) = \frac{-[A1(1+K)+2A2]}{(1+K)^{*3}}$$

for K = 0.1 in the example under consideration, $\frac{d}{dk}(NPV)$ is positive. However, for K = 0.15, $\frac{d}{dk}(NPV)$ is negative. This means that NPV has negative values for k < 0.1and K > 0.15 and has positive values for 0.1 < K < 0.15 (figure 3). This can also be checked by calculating NPV for K = 0.09, 0.11and other values as under: -

Figure – 3

$$\frac{22,500}{1.09} = 20,642.20$$



$$\frac{12,650}{1.09^{*2}} = 10,647.25$$

$$\text{so that } \frac{22,500}{1.09} - \frac{12,650}{(1.09)^{*2}} - 10,000 < 0.$$

Thus the NPV for the investment proposal in question would be acceptable for market discount rates lying between the 2 roots of r.

5. RESOLUTION OF APPARENT INCONSISTENCIES

Any investment proposal is considered acceptable from return angle if either its NPV is positive for the market discount rates or its IRR exceeds the market discount rate. However, NPV is an absolute number and IRR is a ratio (similar to the concepts of Net Working Capital and Current ratio in case of balance sheet analysis). Thus for the purpose of comparative analysis, i.e. while comparing between alternative opportunities for investment, IRR is a better tool. In most cases, r is unique. In cases such as the one considered in paragraph 4, the slopes (rate of change) of NPV versus the market discount rate at the positive values of IRR can be used to resolve the situation.

The example given below would illustrate what is an apparent inconsistency between the NPV and IRR approaches.

	Initial Outflow (Po)	Inflow After One Year (A1)	IRR	NPV @10% Discount
Case 1	100	121	0.21 (21%)	10
Case 2	200	236.5	0.1825 (.1825%)	15

At first sight it would appear that while the NPV method suggests a preference for the 2nd case to the 1st one, the IRR method suggests just the opposite. But, there is no real inconsistency once the difference of scale between the 2 cases is taken into account. The IRR method obviously gives the right solution. Even when the initial outflow is the same for 2 competing options (no difference of scale), such an apparent inconsistency may arise simply due to the difference in the temporal distribution of their inflows. For instance, let us consider the following example:

Case 1 Po = 100, A1 = X1, Ai for i $\neq$ 1 is zero,
The annual market rate of discount being 10%

Case 2 Po = 100, An = Xn, Ai for i $\neq$ n is zero,
The annual market rate of discount being 10%

For case 1, (NPV)1 = $\frac{X1}{1.1} - 100$ and (1 + r1) = $\frac{X1}{100}$

For case 2, (NPV)2 = $\frac{Xn}{(1.1)^{*n}} - 100$ and (1 + r2)*n = $\frac{Xn}{100}$

Thus, (NPV)1 – (NPV)2 = $\frac{X1}{1.1} - \frac{Xn}{(1.1)^{*n}}$

$$= \frac{X1}{(1+r1)}\left[\frac{(1+r1)}{1.1} - \frac{(1+r1)}{(1.1)^{*n}}\frac{Xn}{X1}\right]$$

$$= \frac{X1}{(1+r1)}\left[\frac{(1+r1)}{1.1} - \frac{(1+r2)^{*n}}{(1.1)^{*n}}\right]$$

Although r1 may be larger than r2, it is possible that (NPV)1 < (NPV)2 because of the large number of iterations arising out of a large value of n(r1 > r2 > 0.1). in such a situation r1 > r2 and (NPV)1 < (NPV)2. Thus , there may be a whole range of situations where the NPV and IRR methods lead to apparently inconsistent results. However, the reason for such apparent inconsistencies is quit easy to follow once the underlying equations are carefully kept under scrutiny. The IRR method is obviously the one to go by in such situations.

The above position can be clarified by a rather simple illustration as under: -

Let r1 = 0.20 (20% p.a)

And r2 = 0.15 (15% p.a)

Then $\frac{1+r1}{1.1}$ = 1.09091

$\frac{1+r2}{1.1}$ = 1.04545

$\left(\frac{1+r2}{1.1}\right)^{*2}$ = 1.09297

Thus, although r1 > r2, (NPV)1 < (NPV)2 even for n=2. for higher values of n the same will continue to be true a-fortiori.

Thus, we observe that non-linearity of discounting factors may at times lead to conflicting results in regard to the relative preference of two or more competing investment proposals. However, the systematic mathematical exploration that we have carried out so far in this paper reveals that NPV of a project having a lower IRR can exceed that of another project having a higher IRR and involving an identical quantum of initial investment, if and only if the inflows arising out of the former are deferred in time compared to those arising out of latter. This, though theoretically acceptable, is unlikely to find acceptability with potential investors. A project having a longer gestation period is perceived to have a higher degree of uncertainty and resultant risk associated with it, and any potential investor would accordingly look for a higher return from such a project.

In other words, it would not be proper to discount the inflows from these two projects using the same market rate of discount. Consequently, the very basis for comparing the NPVs of these two projects, in the manner indicated hereinabove, would not exist any longer. In the circumstances, IRR can be the only guiding factor for comparing two competing projects and selecting one of them in preference to the other.

6. Conclusions and Usefulness of the study

From the analyses and observations incorporated in the previous paragraphs, we can conclude as follows:-

a) NPV and IRR are the two standard yardsticks for evaluating competing investment proposals. There are a large number of situations under which both these criteria lead to the same conclusion and there is thus no confusion / controversy in decision making.

b) The IRR of a project satisfies in general a non-linear equation and may thus have more than one solution. However, all solutions may not be physically acceptable. IRR will have one real and positive value in most of the normal investment scenarios. In other words, IRR is unique for all practical purposes.

c) There may be some scenarios where NPV and IRR lead to conflicting decisions in regard to the relative superiority of two competing projects. Such situations may once again be subdivided into the following classes.

1) The two projects involve different investments; or in other words, the project costs are different.

2) The two projects involve identical investments; but the returns from them have different temporal distributions.

In case (i) above, since the scales of investment for the two projects are different, IRR, being a ratio, is the appropriate tool rather than NPV, which is an absolute number.

In case (ii) above also, we note that, from business considerations, IRR has an edge over NPV.

Despite their restrictive assumptions, NPV and IRR continue to be the time-tested tools for project evaluation. In the situation, if NPV and IRR lead to divergent decisions in regard to the relative superiority of competing projects, it is apt to lead to a good deal of managerial confusion/ dilemma. So, a critical analysis about the origin of such a confusion and working out a suitable way for its resolution are of great conceptual significance and practical interest.