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VARIABLE ANNUITY GUARANTEES AND THE RISK MANAGEMENT 'OPTIONS'

[Subject Code (A) – Life Insurance]

Rajasekhar Mallela, FIII,FLMI,ARA
Purnananda Kumar Divakaruni, FIII, FLMI

Abstract

The objective of this paper is to first provide an overview of Non-Traditional Guarantees on Variable Annuity Contracts and then to present the Risk Management Framework that explains Risks involved in managing these Guarantees, Risk Assessment and the Risk Management Options.

Contact Address: CSC India Pvt Ltd, Electronics Complex, Indore-452010 email: rmallela@csc.com, pdivakaruni@csc.com Phone: +91-731-2570717

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Part-1 Non-Traditional Guarantees on Variable Annuity Contracts

1.1 Overview

In the US Insurance Market, one of the major product innovations during the last decade was offering Variable Annuities (VA) with embedded Derivatives. Now, Variable Annuity Products are available with exotic Options - Guaranteed Death and Living Benefits. Non-Traditional Guarantees, as they are called, provide gain on investment to the customers while protecting their investment from fall in equity market. This 'Downside' Risk provides market space to the insurers, but poses a great challenge too.

This section presents the features of the minimum guarantee benefits on VAs currently in the market place. These guarantees are either payable on death - Guaranteed Minimum Death Benefits (GMDB) or on survival in the form of Guaranteed Living Benefits - Guaranteed Minimum Accumulation Benefit (GMAB), Guaranteed Minimum Income Benefit (GMIB) and Guaranteed Minimum Withdrawal Benefit (GMWB). We shall also find various combinations of these four types of guarantees added to one VA Contract – for example, a VA contract with both GMDB and GMIB riders protects contract holder against decline in equity market in case of death or annuitization.

1.2 Guaranteed Minimum Death Benefit (GMDB)

This benefit guarantees that the death benefit will never fall below a given level independent of the fund performance. The guarantee can be based either on the Premium or the Contract value.

(a) Premium-based guarantees pay either the premium or the premium accumulated with interest.

Examples:

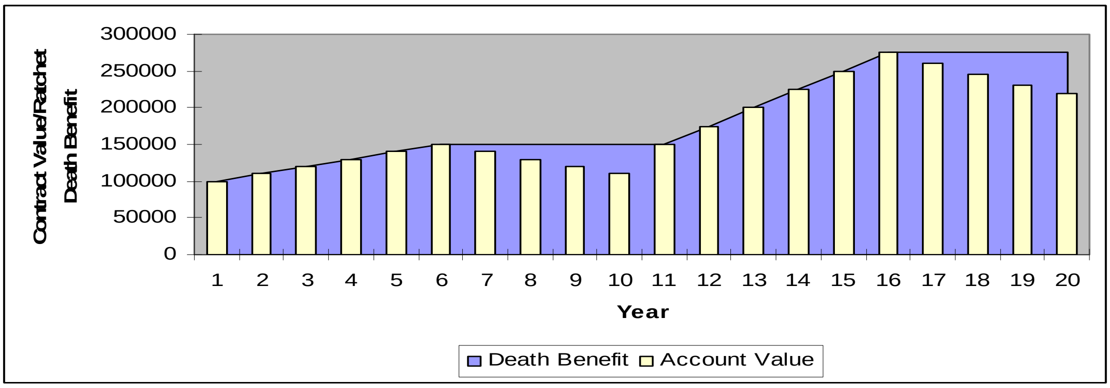
- **Return of Premium (ROP):** Guarantees that at least the premiums paid into the contract will be paid out on death.
- **Roll-up Benefit**: The death benefit increases at a given rate each year. The Rate ranges from 1 to 5% of Premiums paid.

(b) Contract Value based guarantees pay the death benefit based on the contract values at set times.

Examples:

- **Reset**: Death Benefit at any given time will be linked to the contract value at the end of a certain period called Reset Period. At each Reset Period, the benefit is reset up or down to the current contract value. Reset periods may be from 1 to 7 years.
- **Ratchet:** Same as Reset in terms of revising the Death Benefit after certain period based on Contract value. But, with a ratchet benefit, the death benefit is reset to 'the maximum Contract Value' at the end of all previous ratchet periods. Figure-1 explains how the Death Benefit was reset at higher value while the Contract Value declined in the years 7 thru 11 and 16 thru 20 in an Annual Ratchet.

Figure-1: Annual Ratchet Death Benefit



Within the minimum guarantee allowed, the Death Benefit can also go down by the amount of Partial Withdrawals during the Benefit Period. The adjustment will be on $ to $ reduction or on a proportionate basis to the Contract Value.

Spousal Continuation Benefit is an additional feature that allows the spouse to take over and continue the contract after the contract owner's death – with a contract value that equals the death benefit.

1.3 Guaranteed Minimum Withdrawal Benefit (GMWB)

This benefit guarantees return of capital in the form of annual withdrawals regardless of investment performance. The benefit provides for locking in the gains on one hand, while guaranteeing the principal on the other. This is attached to Variable Annuities during their Accumulation phase and the Contract owner will typically choose this rider at the time of issue of contract itself, though it may be elected at a later stage. The Guarantee will have a Withdrawal Benefit Base (WBB) and specifies Withdrawal Allowance per annum. The Guaranteed Withdrawal Amount (GWA) is calculated as Withdrawal Allowance percentage times WBB. In any given contract year, systematic withdrawals can be made up to this GWA till WBB is depleted to Zero. There will be provisions to Reset or Step-up at the predefined intervals. Purchase payments (premiums) will increase WBB and Withdrawals will decrease the same.

This is an elective benefit with a separate charge which is typically a percentage of contract value. This rider is normally offered for Life Time, or for 20 years with 5% Withdrawal Allowance, or for 14.2 years with 7% Withdrawal Allowance.

While typically we find these systematic withdrawals starting immediately after taking the contract, some companies require a minimum waiting period of 10 years or annuitant reaching 55 years of age.

1.3.1 GMWB Features

Table-1 has GMWB features in greater detail.

Table-1: GMWB Features

Feature	GMWB Description
Election of the Guarantee	Can be elected at the purchase of the contract or can be added as a rider later. Minimum initial premium/ Maximum age at entry may be specified.
Exercising Option	Contract Owner may start the Systematic Withdrawals immediately after taking the benefit. However, if withdrawals are not made immediately, there could be Bonus additions.
Rider Benefit Base	If rider is added after issue of contract. WBB = Contract Value as of Rider issue date and if the rider is added at issue of contract, WBB = Initial Purchase Payment. Another way of deciding WBB is - the greatest of the following four values 1) Contract value as of first withdrawal 2) Sum of - • Contract value on rider issue date accumulated at *Benefit Base Accumulation Rate*[(1)] until *Benefit Base Accumulation Cease Date*[(2)]/date of first withdrawal whichever is earlier; and • Each purchase payment prior to first withdrawal accumulated at Benefit Base Accumulation Rate until Benefit Base Accumulation Cease Date/date of first withdrawal whichever is earlier 3) Highest contract value as of contract anniversary date until Benefit Base Accumulation Cease Date/date of first withdrawal whichever is earlier 4) Contract value on the last *Step-up date*[(3)] [(1)] *Benefit Base Accumulation Rate: Accumulation Rate specified in the contract to calculate WBB which is typically 5%.* [(2)] *Benefit Base Accumulation Cease Date: Date on which the Accumulations will cease which is typically 10 years from rider issue. This is also called Roll-up.* [(3)] *Step-up Date: The date on which Step-up can be exercised.*
Additional Purchase Payments	Increases WBB \$ to \$. If Contract Value becomes zero, then no purchase payments will be allowed on the contract. Therefore, WBB will not have impact in such situations.
Withdrawals/ Annuity Payments	If cumulative withdrawals in the contract year are up to GWA, WBB decreases \$-\$. **Example 1:** Last WBB = 100000, Last GWA = 5000 Cumulative withdrawals in the current contract year 5000 Revised WBB = (100000 – 5000) = 95000 **Example 2:** Last WBB = 100000, Last GWA = 5000 Cumulative withdrawals in the current contract year 3000 Revised WBB = (100000 – 3000) = 97000. In this example, withdrawals are less than GWA. However, the remaining 2000 can not be carried forward to the following year.

Feature	GMWB Description
Withdrawals greater than GWA	In any given Contract year, if cumulative withdrawals are in excess of GWA, then revised WBB will be lesser of – (WBB less withdrawal amount) or (Contract Value prior to withdrawal less withdrawal amount) **Example 1:** WBB = 100000, GWA = 5000 Contract Value = 150000 Cumulative withdrawals in the current contract year 7000 Revised WBB = Lesser of (100000 – 7000) or (150000 – 7000) = 93000 **Example 2**: WBB = 100000, GWA = 5000 Contract Value = 75000 Cumulative withdrawals in the current contract year 7000 Revised WBB = Lesser of (100000 – 7000) or (75000 – 7000) = 68000 Another way is to reduce WBB proportionately to contract value of the contract, typically when contract value is less than WBB. Revised WBB = WBB - (Withdrawal Amount/Contract value) * WBB. **Example 3**: Contract Value = 1000 WBB = 1100. Withdrawal = 999 If it is $ to $ reduction, then revised WBB = 1100 – 999 = 101. If it is proportional, then revised WBB = 1100 - (999/1000) * 1100 = 1.1 Obviously, risk to the insurer is less in the second method.
Bonus	This will increase WBB by a flat percentage for each year before the first withdrawal subject to a maximum. There could be a one-time Bonus too if there are no withdrawals for first 'n' number of specified years. **Table-3** illustrates this.
Step-up/Reset	At predefined Reset periods, WBB will be Reset to the Contract Value. In case of Step-up, WBB will only increase but in case of Reset, WBB may go down if Contract Value is less than WBB.
Ratchet/MAV	WBB will be the Maximum Anniversary Value (MAV) – i.e. the maximum contract value on any anniversary.
If Contract Value equals zero and WBB is greater than zero	Contract Owner will start getting benefit of this GMWB Rider. All other privileges on the contract will terminate but for the Guaranteed withdrawals. This will continue till WBB becomes zero. Additional premiums may not be allowed into the contract so as to freeze the risk to the insurer.
In Case of Death of Contract Owner	Joint Owner/Spouse can continue till WBB depletes to zero. Alternatively, present value of the future withdrawals can be paid in lump sum.
Charges	Typically charges will be specified percentage times contract value. The percentage varies with age of contract owner. Some companies use the specified percentage on WBB. Charges are typically deducted until termination of rider but charges will not be deducted if contract value is zero. Reset may increase the charge rate.
Investment choices	It is possible that some companies may require contract owner to choose among specific funds for Premium/Rebalancing allocations. Contract owner may not transfer contract value into or opt for Rebalancing/Dollar Cost Average or Interest Sweeps with funds that are outside the specified funds.
Termination/ Cancellation	There may be restrictions in place to terminate this benefit once chosen. It could be irrevocable or can be terminated after specified number of years.

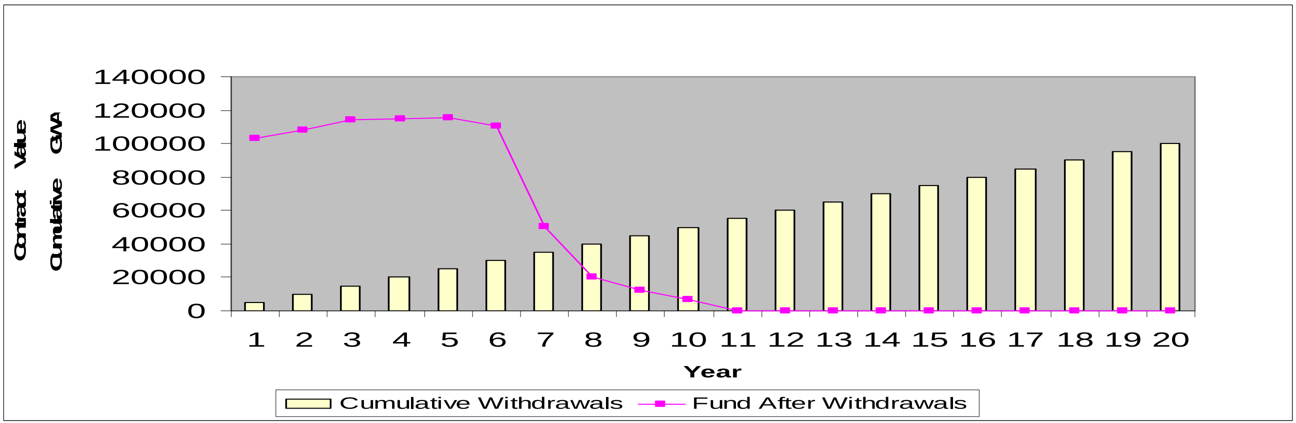
1.3.2 GMWB Benefit Illustration

This illustration explains how GMWB works when the Contract Value becomes zero or negative. We consider a 20 year GMWB where 5% is the GWA and the initial purchase payment is 100000. There are no further payments. With the hypothetical Rate earned during each year, we can see in 11th year that Contract Value is not sufficient to disburse withdrawal amount, but still allows the systematic withdrawal to the extent of GWA i.e. 5000. Table-2 gives year-wise values and Figure-2 plots these values in a graph.

Table-2: GMWB Benefit Illustration

Year	Interest Rate	Fund before withdrawals	GWA	Fund After Withdrawals	WBB	Cumulative Withdrawals
1	8%	108,000	5000	103000	95000	5000
2	10%	113,300	5000	108,300	90000	10000
3	10%	119,130	5000	114,130	85000	15000
4	5%	119,837	5000	114,837	80000	20000
5	5%	120,578	5000	115,578	75000	25000
6	0%	115,578	5000	110,578	70000	30000
7	-50%	55,289	5000	50,289	65000	35000
8	-50%	25,145	5000	20,145	60000	40000
9	-15%	17,123	5000	12,123	55000	45000
10	-5%	11,517	5000	6,517	50000	50000
11	-30%	4,562	5000	-438	45000	55000
12	-10%	-394	5000	-5,394	40000	60000
13	-10%	-4,855	5000	-9,855	35000	65000
14	-10%	-8,870	5000	-13,870	30000	70000
15	-10%	-12,483	5000	-17,483	25000	75000
16	-10%	-15,734	5000	-20,734	20000	80000
17	-10%	-18,661	5000	-23,661	15000	85000
18	-10%	-21,295	5000	-26,295	10000	90000
19	-10%	-23,665	5000	-28,665	5000	95000
20	-10%	-25,799	5000	-30,799	0	100000

Figure-2: GMWB Benefit Illustration



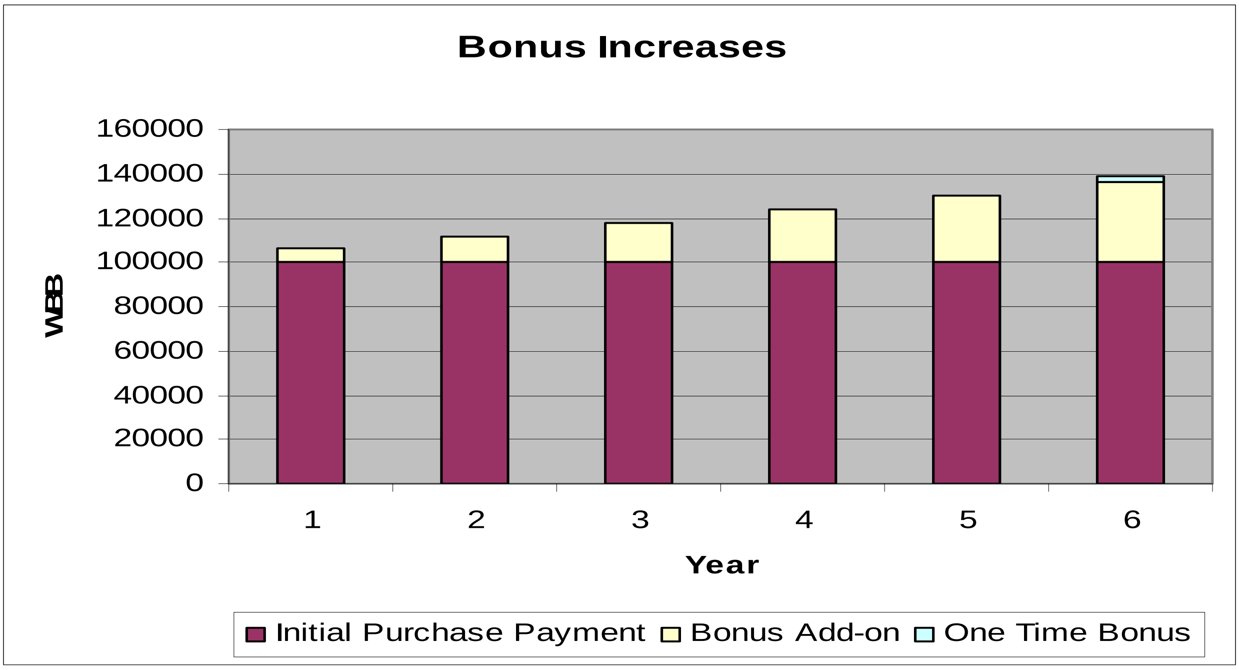
1.3.3 GMWB Bonus Increases (Roll-up) Illustration

This illustration is to explain how WBB will increase due to Bonus (Roll-up). Let us take an Initial Purchase Payment of 100000 and assume that there are neither additional purchase payments nor withdrawals in first six years. Let us assume a Bonus of 6% on purchase payments for each year and a one time bonus of 4% of initial purchase payment if there are no withdrawals during first six years. Table-3 gives the increase in WBB due to Bonus Additions and Figure-3 depicts these values in a Graph.

Table- 3: Bonus Increases for WBB

Year	WBB	Cumulative Bonus Additions	One-Time Bonus
1	106000	6000	
2	112000	12000	
3	118000	18000	
4	124000	24000	
5	130000	30000	
6	140000	36000	4000

Figure-3: Bonus Increases



1.4 Guaranteed Minimum Accumulation Benefit (GMAB)

The Guarantee will have a Base that equals the Contract Value at the start of the Benefit period. The Base will increase with purchase payments made during the initial few contract years as specified by the Product feature. The Base will be reduced by a proportional amount for any partial withdrawals of the Contract Value during the benefit period. At the end of the Benefit period, maximum of Contract value or Benefit Base will be set up as the Contract Value.

1.*5 Guaranteed Minimum Income Benefit (GMIB)*

This benefit guarantees a certain minimum value on annuitization. At annuitization, the typical benefit is to get the higher of the following two:

- Guaranteed Purchase Rates applied to GMIB Base
- Current Purchase Rates applied to Contract Value.

There are a number of different ways of determining the GMIB Base, such as a rollup of premium with or without a cap, the highest contract value achieved, or a Step-up with contract value.

Since the frame work for GMAB or GMIB is almost similar to that of GMWB we had discussed, the details of GMAB and GMIB are presented in **Appendix A** and **Appendix B** respectively. Table-4 presents a summary of the Guarantees we discussed.

Table-4: Summary of Guarantees in Variable Annuity Contracts

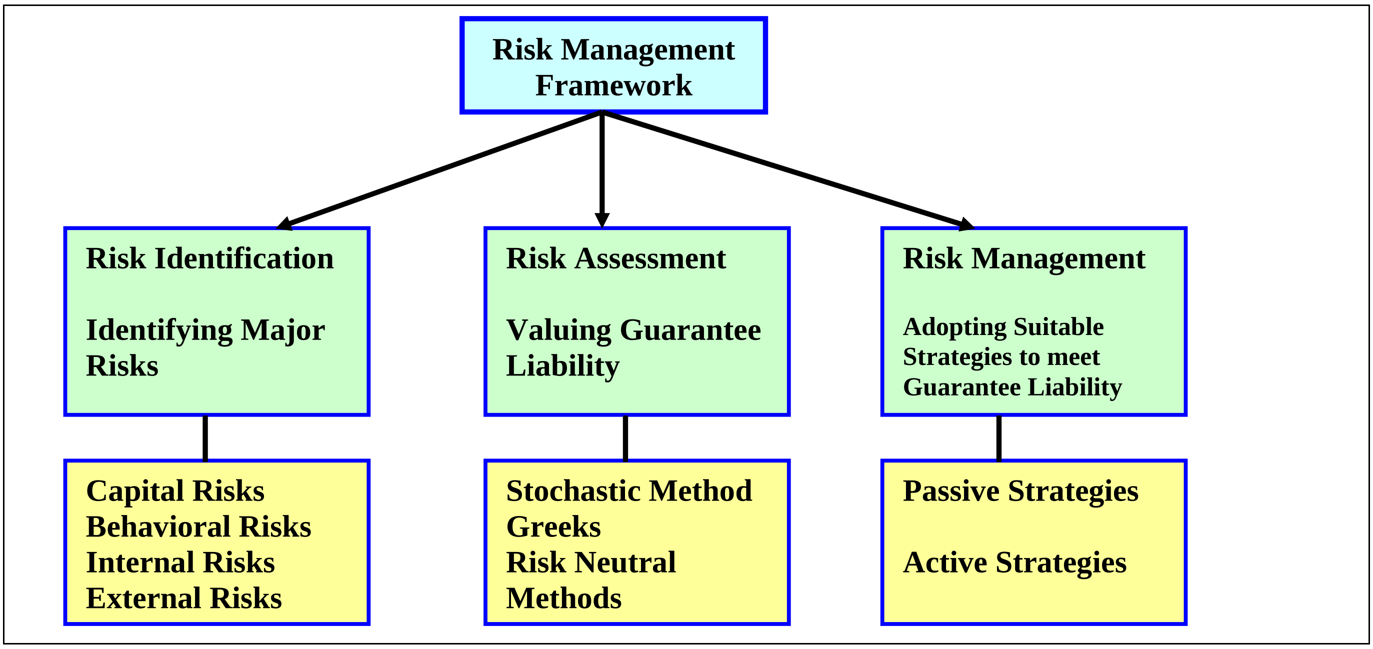
Benefit	Guarantee	Standard Features	Rich Features
GMDB	A Minimum Death benefit regardless of performance of underlying funds	Return of Premium, Ratchet, Roll-up	Combination of Roll-up and Ratchet
GMWB	Return of Principal through systematic withdrawals	7% withdrawal for 14.2 years or 5% for 20 years.	Reset, Step-up, Maximum Anniversary Value (MAV)
GMAB	Minimum Accumulation Value by the end of a specified period	Return of Premium	Step-up
GMIB	Minimum level of annuity payments on annuitization of contract regardless of market conditions	Roll-up Ratchet	Combination of Roll-up and Ratchet

Part -2 Risk Management Framework

2.1 Overview – Risk Management Framework

The Guarantee features embedded in the VA Products that we discussed pose several risks to the insurers. Some of the risks can be measured quantitatively and some others are not. To handle these risks and to remain financially stable, insurance companies need to adopt a comprehensive Risk Management Framework for administering these products. The goal of Risk Management Framework is to maximize insurer's financial objectives subject to given risk tolerances. Figure 4 presents the Risk Management Framework. In this section, we discuss Risk identification, while the other aspects of the Framework are dealt in next two sections.

Figure-4: Risk Management Framework



2.2 Risk Identification

We can classify the Risks under these VA Guarantees as shown in Figure-5. Table-5 describes these Risks and the impact on VA Guarantees.

Figure-5: Risk Identification

Table–5: Risk Identification

Risk	Impact on Guarantees
Capital Market Risks	
Equity Risk Risk of changes in levels of Equity Market.	Contract values may fall below Benefit Base. Higher benefit base for Ratchet – if there is sudden rise in the market followed by deep crash. Roll-up rates may be higher than market rate of return in case of prolonged bear markets.
Earning Volatility Risk – Risk of changes in Earning/Revenue for the insurers due to changes in Capital Market	Reduction or loss of Fee income due to market underperformance. Higher volatility of earnings due to volatility of market levels.
Interest Rate Risk	Impacts, if actual interest rates are lower than rates used in the projections of Contract Value, Discounted Claims and Cash Flows.
Basis Risk Risk of loss due to imperfect correlation between liability & asset portfolios.	Guarantee liabilities may be under-hedged or over-hedged.
Fund Distribution Risk Risk of Fund Transfers.	The loss is locked up for insurer, if fund transfers are made to low-yielding safe funds when markets go down. This could partly be a behavior risk too.
Contract Owner Behavior Risk	
Benefit Utilization Risk	Impacts heavily since all contract owners will exercise benefit when contract values deplete – that is when the Guarantee is in-the-money.
Fund Choice Risk Risk of contract owner choosing a risky portfolio	The presence of Guarantee may lead to adventuristic tendency among contract owners to choose a risky portfolio. This finds insurers searching for suitable hedging portfolio - hedging costs may shoot up.
Persistency Risk	If guarantee is in-the-money, contract that would have otherwise been lapsed, will now continue.
Fund Switches Risk	Increases liability if contract owners switch the funds between 'from' and 'to' funds, depending on degree of yield differences.
Ratchet Risk Risk of higher Benefit Bases with bull market immediately followed by long bear markets.	Higher liability is retained by Insurer if market declines since Ratchets are exercised when guarantee is out-of-the-money.
Internal Risks	
Model Risk Risk of using inappropriate model or model assumptions.	Impacts valuation/pricing of guarantee liabilities if the parameters that go into the Model do not represent the reality.

Risk	Impact on Guarantees
Operational Risk Risk of loss resulting from lack of adequate processes, people and systems.	Difficulty in administering guarantees without • adequate computer systems • Effective controls, procedures, reporting systems
Counterparty Credit Risks Risk of failure of meeting obligations by counterparty.	Credit Risk exists where counterparties like Reinsurers and Investment Bankers are involved as a part of Risk Management of these guarantees.
External Risks	
Regulatory/Capital Risk Risk of potential loss due to change in Regulations pertaining to Capital Requirements.	Design of these guarantees assumes certain level of capital requirements. Any changes to these assumptions due to intervention of regulators will lead to this risk. Capital Volatility may also exist
Financial Reporting Risks Risk of Volatility of reported earnings	This results because Assets are 'valued at book' while Liabilities are 'marked to market'. This artificial accounting treatment leads to volatility of reported earnings which impacts investor's perception of the insurer.
Legal Risk Risk of legal actions or uncertainty in the applicability or interpretation of laws or regulations	This risk may arise from Contract Owners due to complexity of these Guarantee Features itself. Various third party agreements involved in Hedging or Reinsurance Treaties may be source of this Risk.

Part –3 Risk Assessment

3.1 Overview

The assessment of VA Guarantee Risks is very different from that of traditional insurance risks. The management of insurance risk relies heavily on diversification while risks of investment guarantees are non-diversifiable. Traditional Deterministic methods can not capture the risk profiles of the VA Guarantees and meet the liability modeling needs. In order to better understand the risk/return tradeoffs we need to use Stochastic Assumptions versus Deterministic (Table-6 in 3.2 presents more details about these two methods). These Stochastic Assumptions will be fed to Real World or Risk Neutral Models for valuation of Liabilities. We shall now discuss Risk Neutral Models.

3.2 Deterministic & Stochastic Methods

Table-6: Deterministic and Stochastic Methods

Deterministic	Stochastic
Deterministic methods provide the 'Expected' values. The results are 'point' estimates. If we have to use this for investment return assumption, then, high, medium and low scenarios will be generated and an average of them will be assumed in the model.	Stochastic models provide 'Variance' of the values. The results suggest a 'Range' of values at a given 'Confidence' level. For investment return assumptions, a large range of possible investment returns will be generated. This gives better understanding of how a broad range of future investment results might affect its guarantee liability.
Uses Historical Data	Uses statistical sampling to evaluate results of repeated simulations for the same model.
Diversifiable – Independent Risks Mortality, Withdrawal, Lapse, Interest Rate & Contract Owner Behavior can be modeled using Deterministic Approach	Non-diversifiable - Dependent Risks With VA Guarantees, Risks are a combination of capital market risk with contract owner control of the liabilities. Capital Market Risks have to be modeled using Stochastic Methods.
Limited Volatility in underlying variables	Significant Volatility
Deals with Symmetric Risks. Traditional risks will converge to Mean/Median and does not deal with fat-tails	Deals with fat-tails

3.3 Risk Neutral Valuation of Guarantee Liabilities

The payoffs associated with VA Guarantees are identical to the pay offs under a Put Option. This leads to the application of '*Option Valuation*' methods for valuing the VA guarantees. These methods assume that Option values are independent of risk preferences of contract owners. Therefore, these are called Risk Neutral Valuation methods. In this section, we shall illustrate measuring the Guarantee Liability using Black Scholes Risk Neutral Method. An illustration of measuring the Guarantee Liability using another Risk Neutral Method - the Binomial Approach – is presented in **Appendix-C.** Table-7 relates VA Guarantees to 'Put Option' terminology.

Table-7: VA Guarantees in Option Terminology

Option	**VA Guarantee**
Put Option Writer	Insurance Company selling VA Guarantee
Put Option Holder	Contract Owner
Strike Price	Benefit Base of the Guarantee
Stock Price	Underlying Contract Value
Right	To Contract Owner to exercise the Benefit (Long Position)
Obligation	To Insurance company (Short Position)
Option Expiry date	Maturity date or End of term for Guarantees
Holder exercises Option	If Stock Price (Contract Value) is less than Strike Price (Benefit Base)
Liability to Writer	Max of [(Strike Price less Stock Price), zero)] i.e. Max of [(Benefit Base less Contract Value), zero)]

3.3.1 Black Scholes Approach - Illustration

The price (or value) of put Option under Black-Scholes Model is computed using the following formula:

$$P(S,t) = [K * e^{-rt} * N(-d2)] - [S * N(-d1)]$$

Where

N(x) = Standard Normal Distribution Function

$d1 = [\ln(S/K) + ((r+\sigma^2/2)\,t)]/\,\sigma\sqrt{t}$ ln is Natural Logarithm

$d2 = d1 - \sigma\sqrt{t}$

Assumptions for key parameters:

Contract Value at time t=0 (S) = 100000
Guarantee Amount – (K) = 100000
Risk Free Rate (r) = 6%
Volatility (σ) = 15%
Time period (t) =20 Years

Guarantee Value Calculation:

$$d1 = \frac{\ln(100000.00/100000.00) + (0.06+0.15^2/2)20}{0.15\sqrt{(20)}}$$
$$= 2.1242645786248$$

$$d2 = 2.1242645786248 - 0.15\sqrt{(20)}$$
$$= 1.4534441853749$$

N (-d1) = 0.0168240129374
N (-d2) = 0.0730502329817

Value at t=0 is
$= ((100000.00 * e^{(-0.06*20)} * 0.0730502329817) - (100000.00 * 0.0168240129374))$
= 517.8294415525890
= 517.83

The Guarantee Value at t=0 is $517.83.

3.4 Greeks

Option Price depends on the following key variables. "Greeks" are the Sensitivity Measures to calculate change in Option price (value) due to change in any of these variables. Black Scholes formulae to calculate the Greeks are given in **Appendix-D.** The Greeks will be used in Hedging Process that we illustrate later in Part-4.

a) Stock Price
b) Strike Price
c) Volatility of Stock Price
d) Time to expiration of Option
e) Risk Free Rate.

Table-8 provides more details about Greeks.

Table-8: Greeks

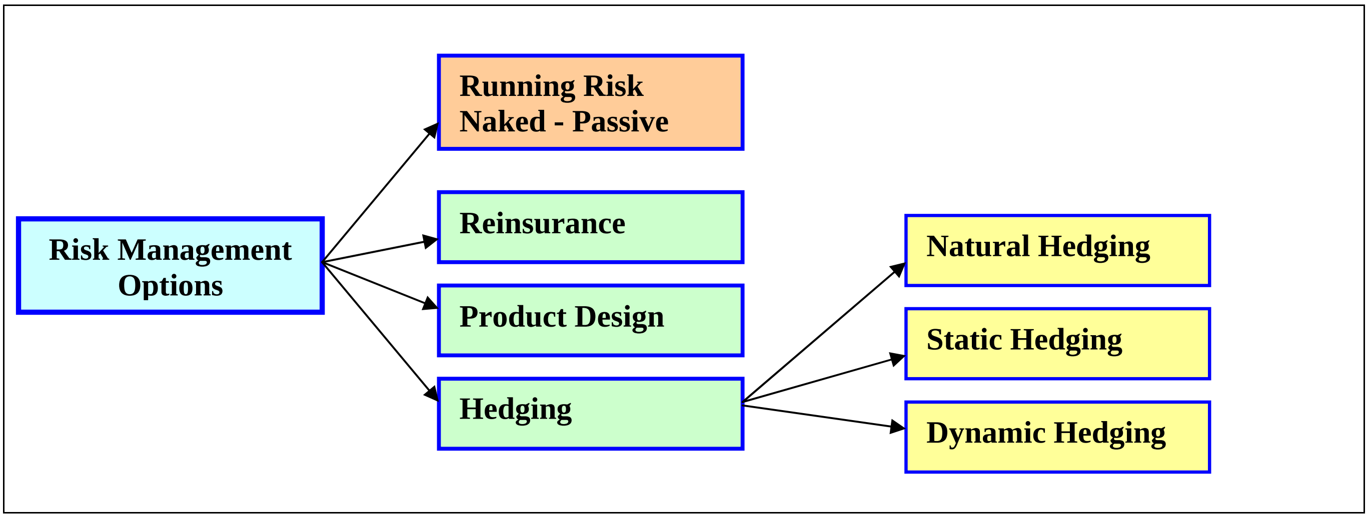
Greek	Measurement	Greek Property
Delta (Δ)	Change in Option price for a given change in the underlying stock price (Per $)	Call Options: If stock price increases, Option price will increase. Delta is positive (0 thru 1) Put Option: If stock price increases, Option price decreases. Delta is negative (-1 thru 0). Delta is close to 1, 0.5 and zero if Option is in-the-money, at-the-money and out-of-the-money respectively.
Gamma (Γ)	How fast the Delta changes for small change in underlying stock price. Delta of Delta (Per $ per $)	Call Options: If Risk Free Rate increases, Option price will increase. Put Option: If Risk Free Rate increases, Option price will decrease Always positive for both Call and Put Options Gamma is larger if Option is at-the-money and progressively decreases if Option is in-the-money or out-of-the-money. .
Vega	Change in Option Price given a one percentage point change in volatility. (Per %)	Increase in volatility increases Option price – for both Call and Put Options. Vega decreases as Option is close to its expiry. More at at-the-money and less impact at in-the-money or out-of-the-money.
Theta (Θ)	Change in Option Price given a one day decrease in the time to expiration. Refers to $ amount an Option will lose each day due to the passage of time. (Per Day)	Option price increases with increase in 'time to expiration'. At-the-money: Increases as Option approaches expiration date In-the-money/Out-of-the-money: Decreases as it approaches to expiry date.
Rho (ρ)	Change in Option Price given a one percentage point change in the risk free interest rate (Per %)	Call Options: If Risk Free Rate increases, Option price will increase. Put Option: If Risk Free Rate decreases, Option price will decrease.

Part-4 Risk Management

4.1 Overview

VA Guarantees are typically long term risks. Further, we have discussed various types of risks challenging the Actuaries in modeling them. In this section, we shall dwell on Risk Management Options presented in Figure-6 - while the focus is to explain Hedging in detail.

Figure-6: Risk Management Options



4.2 Running Risk Naked

Running the risk naked refers to holding securities which are not hedged against market risks. This is a 'do nothing' or 'default' approach for Risk Management. If the company is constantly reviewing & quantifying its risk exposure and if it's within its risk tolerances, then running the risk naked can be a perfectly justifiable risk management strategy. However on the other side of this, this will attract increased attention from Regulators and Rating Agencies. In the present day context, insurers offering VA Guarantees consider this a very risky proposition, though.

4.3 Reinsurance

Some VA guarantee writers consider Reinsurance as a Risk Management Option. Reinsurers normally cover all risks and not just the financial risks. Usual types of reinsurance available for VA guarantees are Modified Coinsurance (Modco), Non-Proportional Reinsurance, and the most common form - Risk Premium Reinsurance. Under Risk Premium Reinsurance, only the guarantee amount is reinsured rather than the total benefit under VA contract. The reinsurance premium is based on the account value or the guaranteed amount. The reinsurer pays the excess, if any, of the guarantee over the variable account. Under Non-Proportional treaties, ceding company may like to retain the claims up to a certain level and then the reinsurer pays the rest. Another type could be to have 'per policy' claim limits and 'aggregate' claim limits. Yet another type is a simple form of 'stop-loss' on this product in which the reinsurer takes only a catastrophic risk – say a Stock Market Crash!

4.4 Risk Management through Product Design

Some of the risks are being minimized or eliminated by bringing in some restrictions in exercise of rich features of the Guarantees. Few of these to mention - Minimum Reset period, Extension of Term after Reset, Maximum Number of Resets, Intervals between two Resets, Possible increase in Charge rate for the benefit after each Reset, Restrictions in Fund Allocations & Transfers, No Premium infusion when Contract Value becomes zero, reserving right to charge fees for Resets/Ratchet Benefits though there is no charge now, Maximum Age restriction for Reset and Roll-up, Reducing Benefit Base in case of withdrawals on Proportionate basis to Contract Value rather than $ to $ basis and irrevocability of Benefit.

4.5 Hedging

Hedging is an investment made in order to reduce the risk of adverse price movements in a security by taking an offsetting position in a related security. This is a Trading or Investment Strategy used to limit investment loss by effecting a transaction which offsets an existing position. This is looked on favorably by Rating Agencies. However, running a hedging operation is potentially a complex activity requiring a detailed knowledge of the trading instruments and markets. We shall discuss three types of Hedging – Natural, Static and Dynamic Hedging.

4.5.1 Natural Hedging

Some products may offer natural hedges between each other. One example frequently cited is the variable annuity Enhanced Earnings Benefit (EEB) and the other variable annuity guaranteed benefits. The EEB increases with positive market performance and the other benefits are guarantees against market drops. At the right sales mix, these benefits may partially hedge each other.

4.5.2 Static Hedging

A Hedge that will not be changed once initiated is called Static Hedging. This means 'buy-and-hold' strategy where there will be no rebalancing of Hedge Portfolio. The investments under Static hedging are typically made on long-term basis. There may not be 'over-the-counter' instruments to suit to Static Hedge Portfolio because of which insurance companies will look for counter parties that will offer tailor made instruments.

4.5.3 Dynamic Hedging

Dynamic Hedging involves active trading of publicly-traded instruments such as short-term 'Futures' and 'Options' to maintain desired balance between the liability and asset portfolios. The objective of Dynamic Hedging is to replicate the Option that has been sold in the liabilities. In the case of any product that has an investment guarantee, the insurer has basically written an Option or sold a derivative that is embedded in the liabilities. Then, the goal is to buy an Option or create a hedge portfolio that replicates that Option and whose value is going to increase or decrease in the opposite direction for given changes in market variables.

4.6 Hedging Illustrations

We shall now illustrate how we use Binomial and Black Scholes methods to hedge the guarantee liabilities. We shall also discuss how Greeks are useful in deciding the hedge portfolios.

4.6.1 Hedging Using Binomial Approach – No Arbitrage Principle

We will demonstrate the crucial concepts of No-arbitrage pricing with a simple Binomial Model that would also discuss the idea of valuation through replication.

The No-Arbitrage Principle states that two identical cash flows must have the same value at any time't'. Replication is the process of finding a portfolio that exactly replicates the Option liability—that is, the market value of the Replicating Portfolio at maturity exactly matches the Option liability at maturity, regardless of the outcome for the risky asset. So, if it is possible to construct a Replicating Portfolio, then the value of that portfolio at any time *t* must equal the value of the Option at time *t*, because there can only be one value (price) for the same cash flows.

Let us illustrate this with an example using One-period Binomial Approach. The following are assumptions under a VA Guarantee.

Guarantee Base (K) =100000; Contract Value (S_0) =100000; Risk Free Interest Rate(r) =6%; Term=1 year

Let our Hedge Portfolio (or Asset portfolio) consist of *Risk Free Asset* of 'a' and *Risky Asset* units – say 'b'. The objective of this portfolio is that it should be able to pay off the guarantee liability at any time't'. We shall find out the unknowns 'a' and 'b' in the Hedge Portfolio as per following steps:

1) At t=0, we shall construct the hedge portfolio with a Risky Asset and a Risk Free Asset.
2) At t=1, we shall assume that Contract Value goes up and find out the Guarantee Liability and Portfolio Value.
3) At t=1, we shall assume that Contract Value goes down and find out the Guarantee Liability and Portfolio Value.

As per No Arbitrage principle, the two cash flows (Liability and Hedge Portfolio) at time t should be same under both scenarios of Contract Value going up and down. By solving the equations at (2) and (3), we shall find out the unknowns in our Hedge Portfolio – 'a' and 'b'. With this, we shall arrive at the Option Value as well.

<u>Step-1: (t=0)</u>

Contract Value = S_0 = 100000
Risky Asset = b Units
Let us say value of this Risk Free Asset is 'a' at t=1. Then, assuming continuous compounding, its present value at t=0 will be ae^{-r} where r is the risk free interest rate per time unit.
Risk Free Asset = ae^{-r}
Port folio = $ae^{-r} + b\,S_0 = ae^{-0.06} + b\,(100000)$
Let P_0 be the Guarantee Price/ Value at t=0 and the Portfolio is constructed for this value.
Then $P_0 = ae^{-0.06} + b\,(100000)$ - (1)

<u>Step – 2 (t=1, Contract Value Goes Up</u>)

Let S_u be the contract value at t=1 which is 125000
Guarantee Liability in this case = 0 since the guarantee will not be in effect as long as Contract Value is greater than Guarantee Base.
Portfolio Value in this scenario will be = a + bS_u (125000) = a + b (125000)
As per no-arbitrage principle, the value of portfolio should be value of liability at t=1 too.
Therefore, we have a + b (125000) = 0 - (2)

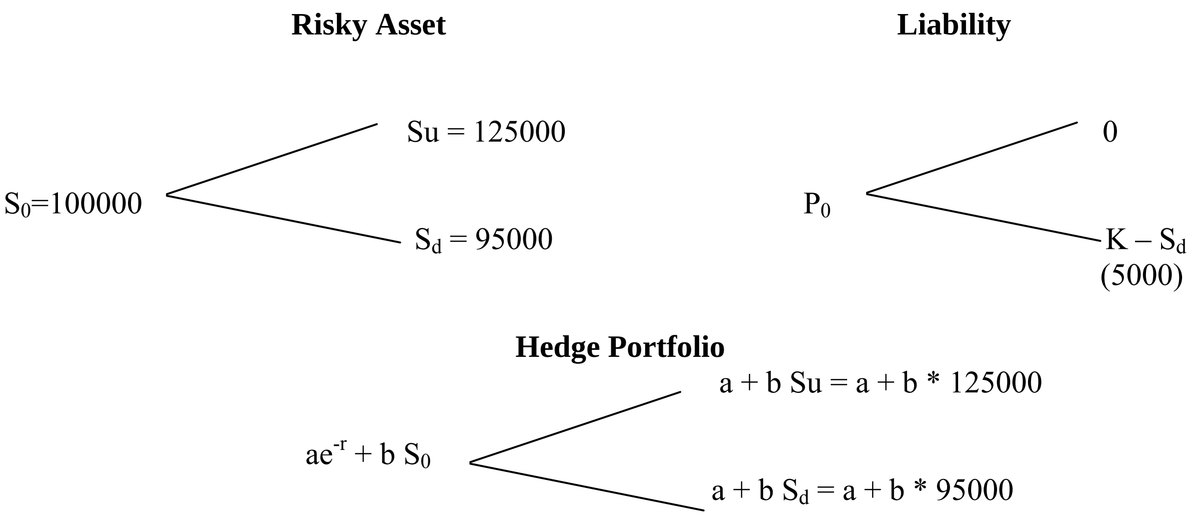
<u>Step – 3 (t=1, Contract Value Goes Down)</u>

Let S_d be the contract value at t=1 which is 95000
Guarantee Liability in this case = 5000 which is (100000 – 95000). Here the guarantee will be in effect since Contract Value is less than Guarantee Base.
Portfolio Value in this scenario will be = a + bS_d = a + b (95000).
As per no-arbitrage principle, the value of portfolio should be value of liability at t=1 too.
Therefore, we have a + b (95000) = 5000 - (3)

Solving (2) and (3), we get a = 20837.50 b = - 0.16667

This solution means that if the insurer buys the portfolio at time t=0 that consists of a short holding of units (with price −$16667, since S_0 = 100000) and a holding of ae^{-r} = 19624.02 in the risk-free asset, then whether the contract value goes up or down, the portfolio will exactly meet the Guarantee Liability. Therefore, the Guarantee is perfectly hedged by this portfolio.

Since the portfolio and the Guarantee have the same payout at time t = 1, then they must, by the no-arbitrage principle, also have the same price at time t = 0. Hence the price of the Guarantee at t = 0 must be the same as the price of the matching portfolio at t = 0. Substituting values of 'a' and 'b' in the equation (1), we get the Guarantee Price/Value P_0 = 2954.018.



4.6.2 Hedging Using Black Scholes Approach

We shall discuss here how to create a Replicating Portfolio – using Black Scholes Method - so that the liability is exactly hedged. We shall also demonstrate the idea of rebalancing the Replicating Portfolio without additional cost. This is also called 'self-financing' hedging meaning thereby the change in value of one Asset in hedge portfolio, at each time step, must precisely be sufficient to finance the change in other Asset of the hedging portfolio.

The Black Scholes model will tell us the composition of Risk Free and Risky Assets in Hedge Portfolio which are detailed below. The Black Scholes results prove that a portfolio constructed using the parameters below will exactly replicate the liability portfolio and hence is a perfect hedging.

Hedge Portfolio at t=0 is given by $\mathbf{a + bS_0}$

Risk Free Asset (a) = N (-d2) * K * e^{-rt}
Risky Asset (b) = - N (-d1)
Stock Price of Risky Asset = S_0

Where

N(x) = Standard Normal Distribution Function

d1 = [ln (S/K) + ((r+ σ2/2)T)]/ σ √t (ln is Natural Logarithm)

d2 = d1 - σ √t

Let us take the same illustration in 3.3.2 (Part-2) to hedge the liability already calculated using Black-Scholes formulae.

Stock Price (S_0) = 100000
Guarantee Value (P_0) = 517.83
Term (t) = 20 years
Risk free interest rate (r) = 6%
Volatility (σ) = 15%

<u>a) Hedge Portfolio Construction</u>

$$d1 = \frac{\ln(100000.00/100000.00) + (0.06+0.15*0.15/2)20}{0.15\sqrt{(20)}}$$
$$= 2.1242645786248$$

$$d2 = 2.1242645786248 - 0.15\sqrt{(20)}$$
$$= 1.4534441853749$$

N (-d1) = 0.0168240129374
N (-d2) = 0.0730502329817

Given these, the Hedge Portfolio ($a+bS_0$) to match our liability of 517.83 will be as follows:
Risk Free Asset (a) = N (-d2) * K * e^{-rt}
= 0.0730502329817 * 100000 * $e^{(-0.06*20)}$
= 2200.23073529259
Risky Asset (b) = - N(-d1) = - 0.0168240129374

We can also reconcile this Portfolio Value with the Guarantee Liability at t=0.

The Portfolio Value is: = 2200.23073529259 + (- 0.0168240129374) * 100000
= 517.8294416 = **517.83** which is our Guarantee Liability at t=0.

<u>b) Self-Financing Portfolio:</u>

Taking this discussion a bit further, let us say, at time t=1, the Contract Value is changed to 95000. Then, the Guarantee Liability will change as follows:

d1 = 2.048 d2=1.377 N(-d1) = 0.020289748722 N(-d2)= 0.084259114171
Guarantee Liability at t=1 = 610.31. (We use formulae as explained in 3.3.2 of Part-2 to get this)

Portfolio at t=1 will be:

a = N (-d2) * K * $e^{(-rt)}$
= 0.084259114171 * 100000 * exp $^{(-0.06 * 20)}$
= 2537.83574891546

b= -N (-d1) = - 0.020289748722

At t=1, the portfolio value is = a + bS_1
= 2537.83574891546 + (- 0.020289748722) * 95000
= 610.31 which is our Guarantee Liability too.

We can see the change in 'a' and 'b' at time t=1, and we shall now establish that there was no additional money necessary in this rebalancing of Hedge Portfolio.

Change in (a) = 2537.83574891546 - 2200.23073529259 = 337.6050136
Change in stock in (b) = 0.020289748722 - 0.0168240129374 = 0.003465736
Amount realized due to change in stock = 0.003465736 * 95000 (i.e. .current stock price) = 329.2448995.

We can see that the change in stock (b) almost finances the increase in (a). Therefore, the portfolio we developed using Black Scholes is called 'self financing' portfolio.

4.6.3 Hedging Using Greeks

Here we shall illustrate how to construct the Hedge Portfolio using Greeks as key parameters. We shall demonstrate Gamma and Vega Hedging. Also we use Delta Hedging to address Residual Sensitivity.

Let us assume the following Greeks for Liability portfolio.

Greek	Delta	Gamma	Vega
Value	-54	0.3	127

Now, let us assume the availability of following Options with given Delta, Gamma and Vega:

Instrument	Delta	Gamma	Vega
A - Put Option	-0.3	0.0012	1.641
B - Put Option	-0.6	0.0036	2.392

To Hedge Gamma and Vega, we need to find how many of A (say, a) and how many of B (say, b) we need to purchase/sell. We can find this as follows:

Gamma of (A) * a + Gamma of (B) * b = Gamma of Liability
= 0.0012 * a + 0.0036 * b = 0.3
Vega of (A) * a + Vega of (B) * b = Vega of Liability
= 1.641 * a + 2.392 * b = 127

Solving for (a) and (b), we get **a = -85.73 = -86; and b = 111.91 = 112.**

This solution means - we need to short 86 A-Put Options and long 112 B-Put Options to perfectly hedge our Guarantee Liability.

Residual Sensitivity – Delta Hedging:

If we have a = -86 and b= 112 in our Hedge Portfolio, the residual sensitivity of Delta, Gamma and Vega can be calculated as follows:

For Delta = Delta of (Liability) – [Delta of (A) * a + Delta of (B) * b]
= -54 - [-0.3 * -86 + -0.6 * 112] = -12.6
For Gamma = Gamma of (Liability) – [Gamma of (A) * a + Gamma of (B) * b]
= 0.3 – [0.0012 * -86 + 0.0036 * 112] = 0
For Vega = Vega of (Liability) – [Vega of (A) * a + Vega of (B) * b]
= 127 – [1.641 * -86 + 2.392 * 112] = 0.222

Greek	Delta	Gamma	Vega
Residual Sensitivity	-12.6	0	0.222

Now to hedge residual Delta we have to short 12 future contracts of Delta of 1, Gamma and Vega of zero so that the residual sensitivity of Delta becomes -0.6.

4.7 Risk Management Options – Advantages & Disadvantages

Table-9 briefly presents Advantages and Disadvantages of the Risk Management Options we discussed.

Table-9: Advantages & Disadvantages of Risk Management Options

Risk Management Option	Advantages	Disadvantages
Running Risk Naked	Easy to implement, No upfront cost associated with reinsurance premiums or hedging costs, Highest level of profits on an expected basis	High capital requirements, Significant earnings volatility, Exposure to large and potentially catastrophic losses.
Reinsurance	Customizable, familiar, easy to implement, Insurance and financial risks are covered and provides certainty to pricing, Credit for reserves	Expensive, Limited coverage especially for tail exposure, Counter party credit risk exposure, irreversible, illiquid
Static Hedging	There's little or no ongoing rebalancing, Less Trading Costs, Limited internal controls because this is a buy-and-hold strategy	Potentially expensive, Long Term - No liquidity, Exposure to the Credit risk of the counterparty. Limited exposure to actual volatility, there are no established secondary markets for these types of contracts, Can not address variances in expected persistency.
Dynamic Hedging	Costs not known at the time when it is implemented, but it may prove to be cheaper than static hedging since it covers the actual volatility rather than the estimated volatility. Offers more liquidity, Easily addresses variances in lapses and market conditions, and Uses the most liquid hedging instruments that are generally exchange traded, Limited counterparty credit exposure.	More complex to manage than static hedging. Carries the risks that the instruments being used are not always available or not available at desirable prices or at the exact times needed There are a lot of internal approvals required to implement a Dynamic Hedging program. There can be some operational risks of trading execution if proper controls are not in place. There will be some residual risks associated with Dynamic Hedging. Extent of exposure to basis risk needs to be considered too. Sophisticated systems and expertise are needed to execute a Dynamic Hedging program

4.8 Relevance to Indian Context:

In the developed markets, it has been proved that Variable Annuities with Guarantees are good instruments for attracting investors' attention and insurance companies have enjoyed fulfillment of financial objectives through this area of business.

Considering the Indian Scenario, the economic indicators and the stock market indices seem to be making complementary contributions for the growth of Indian economy as a whole. In the present day context, the Indian insurers experience tremendous growth in business from Unit Linked Product family which is similar to Variable Policies. These products, if offered with guarantees, will have better market appeal and can attract the segment of 'Mutual Funds' investors too.

Maintaining a balance between marketability of annuity products and viability of holding huge funds for longer periods has been a rope walk because of unpredictability of investment returns. This would call for a robust Risk Management framework appropriate for managing the Unit Linked Annuity Guarantees in Indian scenario. Hedging, in itself, is a necessary but not sufficient strategy in the total Risk Management Process, since, in reality, it is a simultaneous interplay of several risks that we have discussed. Actuarial elite can throw further light on Risk Management techniques suitable for Indian scenario and develop suitable models, so that the Indian Insurance company is well equipped to offer the Unit Liked Annuity Guarantees.

4.9 Conclusion:

The VA guarantees change the very nature of the product offered. It leads the product into a mix of various finance aspects from a pure Insurance one. We wish to conclude that an analysis of these guarantees lead us to believe that there is a huge potential for growth in the Insurance industry in this direction and challenge the actuarial world to constantly fine tune the Risk Management Framework.

Part-5 Appendices/Glossary/References

5.1.1 Appendix-A Features of Guaranteed Minimum Accumulation Benefit (GMAB)

Feature	Description
Election of the Guarantee	At issue or in administration – some companies offer only at issue. Maximum issue age is typically specified
Guarantee Benefit Base	Original Purchase payments (plus purchase payments made within a specified period of 4 to 6 months) less adjustment for withdrawals. Withdrawals are adjusted proportionately to Contract Value. Instead of adding 100%, only a specified percentage of purchase payments can be used in calculating base. This percentage may depend on age and/or duration at which premiums are paid. At the end of the benefit period - contract value is increased to GMAB Base if contract value is less than GMAB Base. The maximum GMAB Base may be specified as 2 times the premiums paid or even in absolute terms like $5 million.
Reset	Reset can be exercised after a specified number of years (subjected to a maximum age) from the rider start date. Benefit period will be extended on Reset. Some companies allow Reset only if contract value is greater than GMAB base.
Withdrawals/Annuity Payments	The withdrawals affect Base proportionately as against $ to $. Reduction from base is calculated as = Base * (withdrawal amount/Contract value)
Death of Owner	Spousal continuance allowed.
Charges	Specified percentage of Benefit Base or as a specified percentage of daily value of assets invested in each fund after fund expense is deducted. Typically, the charges are deducted on the contract anniversary.
Investment Choices	Model allocation can be selected out of the models offered. Change of models allowed but transfers other than change of model will terminate GMAB. Once GMAB is terminated in this way, it can not be re-elected.
Termination/ Cancellation	GMAB terminates on termination of contract, annuitization, annuity start date, full surrender, or death of owner. Typically cancellation is not allowed.
Relation to other riders	Typically, this benefit is not offered along with GMWB, GMIB, DCA, Interest Sweeps, and Automatic Rebalancing.
Others	If the contract value is zero for any reason other than full withdrawal or annuitization but contract has positive base, the contract and GMAB remains in force – and there will not be any charges up to rider maturity.

5.1.2 Appendix-B Features of Guaranteed Minimum Income Benefit (GMIB)

Feature	GMIB Description
Election of the Guarantee	Typically available only at issue. Maximum issue ages are specified based on the age of the annuitant. The maximum issue age also varies based on re-set Options available under the rider.
Exercising Option	Typically, this Option should be exercised after the 10th or subsequent anniversary but not later than the anniversary immediately following the attainment of specified age of the annuitant.
Guarantee Benefit Base	Purchase payments received within a specified period after contract issue date is used for calculation of Guaranteed Income Base (GIB). The specified period is typically 3 months, while it may even extend up to 2 years. At times, the actual contract value is used as the starting GIB if the rider is added after the issue of the contract
Changes to the Base	Reset, Roll-up, Ratchet/Step-up are few Options to lock in gains and increase GIB, as we see with other Guarantees discussed.
No lapse provision	If the contract value is less than zero during the initial term of the rider, the contract will be annuitized immediately – GIB as on that date and age of the annuitant as on that date are used to calculate annuity value.
Withdrawals/ Annuity Payments	Withdrawals reduce GIB on $-$ basis or proportionate to contract value. Surrender charges may be waived for withdrawals up to specified percentage of GMIB and if the withdrawals in a year exceed that percentage, surrender charges are applicable. Annuity Payments – Greater of • GIB * Guaranteed Purchase Rates • Current contract value * Current Annuity Rates as on the date of annuitization
Death of contract owner	Spousal Continuation allowed
Charges	Specified percentage of GIB or as a specified percentage of daily value of assets invested in each fund after fund expense is deducted. Typically, the charges are deducted on the contract anniversary.
Investment choices	Model allocation can be selected out of the models offered. Change of models allowed but transfers other than change of model will terminate GMIB. Once GMIB is terminated in this way, can not be re-elected. If payments are applied to a contract in a different allocation than model, they are allowed but model allocation adjustments will be made periodically as decided in the contract. Sometime, fixed funds may not be allowed.
Termination/ Cancellation	This rider terminates in the following cases – Date of termination of basic contract, Annuitization, On the anniversary following a specified birth day of annuitant, Full withdrawal, Death of the annuitant. Rider typically can not be dropped once selected.
Relation to other riders	Can not go with GMAB, GMWB, and Dollar Cost Average. Interest Sweep or Automatic Rebalancing provisions.

5.1.3 Appendix-C Binomial Approach for Liability Valuation

The Binomial Model values the Option's key underlying variables using a Binomial tree, for a given number of time intervals between the valuation date and the Option expiry date. Each node in the tree represents a possible price (or value) of the underlying, at a particular point in time. This price evolution forms the basis for the Option valuation. The valuation process involves the following three steps.

a) Price Tree Generation
b) Decide liability at the expiration
c) Calculation of value of the Option at each node, working backwards from the final node.

First, the Price Tree is produced by working forward from the first node to the expiration. At each step, it is assumed that the underlying instrument will move up or down by a specific factor (*u* or *d*) per step of the tree. If *S* is the current price, then in next period the price will either be S * u on the upside, or S * d on the downside. The up and down factors are calculated using the underlying volatility, σ and the time duration of a step't'.

$$u = e^{\sigma\sqrt{t}}$$
$$d = e^{-\sigma\sqrt{t}} = 1/u$$

Then, the liability at the expiration is decided. At the expiry, the Option value is simply its exercise value which is the liability from Insurer's perspective. For the Put Options like VA Guarantees, this liability is Max [(K – S), 0], where K is the Strike Price (Guarantee) and S is the Stock Price (Contract Value).

Finally, we move back wards from the final node to calculate value of the Option at each node using the following formula.

Binomial Value of the Guarantee = [p * Option up + (1-p) * Option down] × e^{-rt}

Where $p = [e^{(r-q)\,t} - d]/\,[u - d]$ and **q** is the dividend yield of the underlying corresponding to the life of the Option

<u>Illustration of Binomial Approach:</u>

Now, let us illustrate this three-step process taking the following assumptions under a VA Guarantee.

Contract Value at time t=0 (S) = 100000
Guarantee Amount – (K) = 100000
Risk Free Rate (r) = 6%
Volatility (σ) = 15%
Time period (T) =20 Years
Number of Binomial Steps=5
Time Period per Binomial Step (t) = T/5 = 20/5 = 4
Dividend Yield (q) =0

Step-1 Price Tree Generation:

a) Up Step Size:

$$u = e^{\sigma\sqrt{t}} = \exp(0.15 * 2) = 1.349858807576$$

b) Down Step Size:

$$d = e^{-\sigma\sqrt{t}} = 1/u = 1/1.3499 = 0.74081822068$$

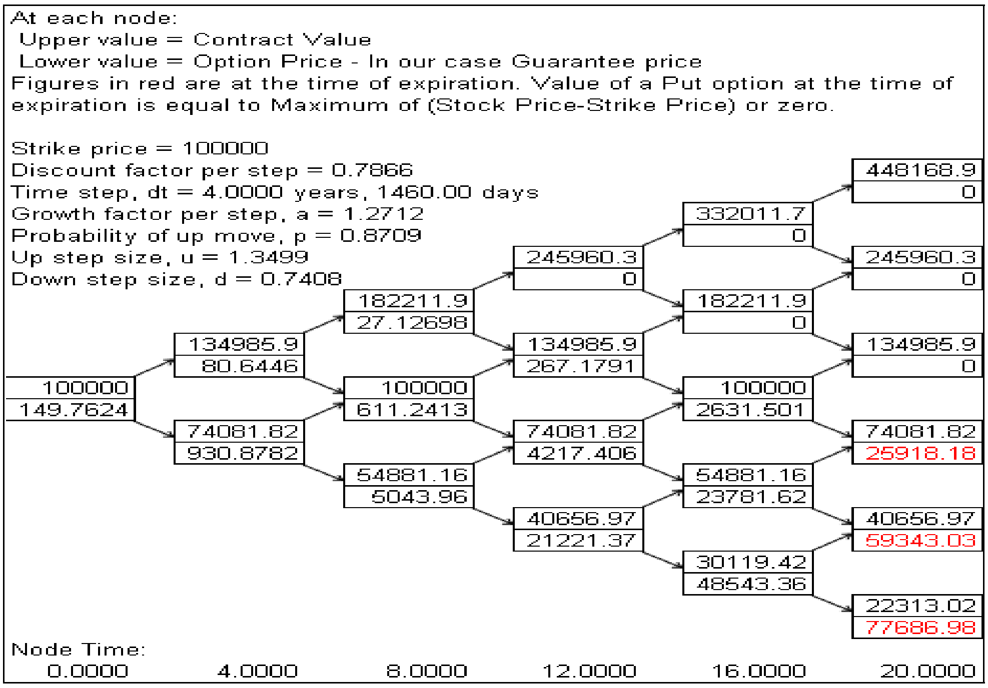
At the first step –

Contract Value if it goes up = S*u = 100000.00 * 1.349858807576=134985.8807576 = 134989.88
Contract Value if it goes down = S*d = 100000*0.740818220682=74081.8220682=74081.82

Similarly, we can calculate the Up and down values at each node of the tree to complete Price Tree.

Step 2: Liability at the expiration:

At the final node (bottom most node), the Option value is
= Maximum of [(Guarantee value –Contract Value) or Zero].
= Maximum of [(100000 - 22313.016), Zero)] = 77686.984
This value is marked in red in the Binomial Tree given below.



(Tool Used: DerivaGem Software - http://www.rotman.utoronto.ca/~hull/software/)

Step 3: Value of Guarantee (Option):

Now, we calculate the value of the Guarantee working back wards:

The Guarantee value at t=16 is when the contract value is 30,119.42 is calculated as below:

Probability of Up move = p = $[e^{(r-q)\,t} - d]/(u\text{-}d)$
= {exp [(0.06-0)4] - 0.740818220682}/ (1.349858807576-0.740818220682)
= 0.870928705006 or 0.8709

Probability of Down move = (1-p) = 1-0.870928705006 =0.129071294994

Binomial Value of the Guarantee = [p * Option up + (1-p) * Option down] × e^{-rt}
= {(0.870928705006 *59343.0340259401) + (0.129071294994*77686.983985157} exp (-0.06*4)
= 48543.364915435100 or 48543.36

Therefore, the value of Guarantee is **48543.36**

5.1.4 Appendix-D Black Scholes Formulae to Calculate Greeks

Greek	Call Option	Put Option
Delta	N(d1)	N(d1) - 1
Gamma	N(d1)/S$\sigma\sqrt{t}$	N(d1)/S$\sigma\sqrt{t}$
Vega	S * N(d1) * $\sqrt{t}$	S * N(d1) * $\sqrt{t}$
Theta	[-S * N(d1) * σ)/ 2$\sqrt{t}$] – [r*K*e^{-rt} * N(d2)]	[-S * N(d1) * σ)/ 2$\sqrt{t}$] + [r*K*e^{-rt} * N(-d2)]
Rho	K*t *e^{-rt} * N(d2)	-K*t *e^{-rt} * N(-d2)

5.2 Glossary

Aggregate claim limits	The maximum sum of recoveries payable under those reinsurance agreements that provide an overall maximum loss limitation.
Arbitrage	The process in which professional traders simultaneously buy and sell the same or equivalent securities for a riskless profit.
At-the-money	A situation in an Option contract where the strike price equals the price of underlying asset
Call Option	A contract giving holder the right, but not the obligation, to buy an underlying asset (a stock or index) at a specific price on or before a certain date.
Counterparty	The opposite side in a financial transaction.
Derivative	A financial instrument whose price depends on or derived from the price of another asset
Dynamic Hedging	Process of hedging an Option in which the portfolio composition is periodically changed based on changes to the underlying variables.
Future	A standardized contract calling for the delivery of a specified quantity of a commodity at a specified date in the future.
Hedging	A conservative strategy used to limit investment loss by effecting a transaction which offsets an existing position.
Historic Volatility	The realized volatility of a financial instrument over a given time period.
Implied Volatility	Volatility implied from Option price.
Interest Sweeps	The transfer of Interest earned on the fixed account to Variable funds at periodic intervals under an insurance or annuity contract
In-the-money	A situation in a Put Option where the strike price is greater than the price of underlying asset. A call Option will be in-the-money if strike price is less than the price of the underlying. Options are also said to be 'under-the-water' in this situation.
Long Position	A position involving purchase of an asset. Buyer is said to be in the Long Position. 'Going Long' refers to buying.

Marking to Market	Practice of revaluing an instrument to reflect the current values of the relevant market variables.
Modified Coinsurance	Indemnity life reinsurance where the reserves are transferred back to the ceding company while the risk remains with the reinsurer – also called Modco.
Natural Hedging	Process of identification of liabilities within insurer's portfolio that moves in opposite directions for a given change in underlying assets.
No arbitrage Principle	The assumption that there are no arbitrage opportunities in the market prices. Further, this implies that two identical cash flows must have the same value at any time.
Non-proportional Reinsurance	A form of reinsurance where the reinsurer's liability is not fixed in advance, but is dependent on the number or amount of claims incurred in a given period.
Option	An Option is a contract giving the buyer the right, but not the obligation, to buy or sell an underlying asset (a stock or index) at a specific price on or before a certain date.
Out-of-the-money	A situation in a Put Option where the strike price is less than the price of underlying asset. A call Option will be Out-of-the-money if strike price is greater than the price of the underlying.
Put Option	A contract giving holder the right, but not the obligation, to sell an underlying asset (a stock or index) at a specific price on or before a certain date.
Replicating portfolio	A portfolio that exactly replicates the Option payoff. The process of finding a portfolio that exactly replicates the Option payoff is called 'Replication'.
Risk neutral Assumption	An assumption where investors are assumed to require no extra return for bearing risks. A valuation of an Option with Risk neutral assumption is called Risk Neutral Valuation.
Risk Premium Reinsurance	A form of life reinsurance under which the risks are transferred to the reinsurer for a premium that varies each year with the amount at risk and the ages of the insureds. Also known as Yearly Renewable Term (YRT) reinsurance
Self Financing Portfolio	A Hedge portfolio in which change in the value of each asset is precisely sufficient to finance the changes in the value of another asset

Short Position	A position involving selling of an asset. Seller is said to be in the Short Position. ‘Going short’ refers to selling.
Static Hedging	Strategy of not changing the hedge portfolio once it is constructed.
Stop-loss Reinsurance	A form of reinsurance which indemnifies the reinsured against the amount by which the losses incurred in a specific period exceed the agreed amount under the reinsurance treaty.
Volatility	A measure of uncertainty of changes in the price of an asset.

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About the Author:

Mr. Rajasekhar Mallela

Qualifications:

Professional:

- Fellow of Insurance Institute of India, Mumbai
- Fellow, Life Management Institute (FLMI), LOMA, USA
- Associate, Reinsurance Administration (ARA), LOMA
- Certified Software Quality Analyst (CSQA), QAI, USA
- Student Member of ASI (Credit in Two subjects)

Educational:

Bachelors in Computer Science
PG Diploma in Planning & Project Management
- Central University, Hyderabad

Career Profile:

Life Insurance Corporation of India (LIC)

- 10 years of experience - important positions held:
  - Asst. Branch Manager
    - Life Branch
    - Pension & Group Schemes
  - Asst. Administrative Officer
    - New Business & Actuarial
    - Claims

Computer Sciences Corporation (CSC)

- Working with Life Insurance Domain Team as
  - Deputy Manager
- 3 years' experience in
  - US Life & Annuity Practices
  - Providing Business Logic Consultation
  - Requirements Gathering process
  - Conducting Gap Analysis
  - Preparation of Business Specifications
  - Analysis of Product/features and provide business support to design the software application

Special Interests: Pursue Actuarial Studies and involve in Actuarial work

Conferences Attended: First GCA at New Delhi in 1999Actuarial Students Conference at Hyderabad when Paul Thornton visited

Address:

CSC India Pvt Ltd,
Electronics Complex
Indore – 452010 (MP).
Email: rsmallela@gmail.com

Mr. Purnananda Kumar Divakaruni

Qualifications:

Professional:

- Fellow of Insurance Institute of India, Mumbai
- Fellow, Life Management Institute (FLMI), LOMA, USA
- Certified Software Quality Analyst (CSQA), QAI, USA
- Student Member of ASI

Educational:

Bachelors in Commerce

Career Profile:

Life Insurance Corporation of India (LIC)

- 12.5 years of experience in
  - Underwriting
  - Policy Servicing
  - Claims
  - Accounts

Computer Sciences Corporation (CSC)

- Working with Life Insurance Domain Team as
  - Deputy Manager – Life Insurance
- 3 years' experience in
  - Life Insurance Products and Insurance Company processes in US
  - Providing Domain consultancy support to software engineering team.
  - Analysis of Requirements and preparation of Business specifications
  - Quality Assurance of Software

Special Interests:

Research and Analysis in Life Insurance.

Address:

CSC India Pvt Ltd,
Electronics Complex
Indore – 452010 (MP).
Email: ananddivakaruni@rediffmail.com