1st Capacity Building Seminar on
Key aspects of Risk Management
in Life Insurance Companies

Economic Scenario Generator
and Stochastic Modelling

Jonathan Lau, FIA

Moody’s Analytics

9 August 2014, Mumbai
Stochastic Modelling for Insurance

Economic Scenario Generator

Jonathan Lau, FIA, Solutions Specialist
Jonathan.Lau@Moodys.com

9 August 2014
Moody’s Analytics Overview – beyond credit ratings

Research-Led Risk Management Solutions for Financial Institutions
Strong & Growing Presence in the Global Insurance Market

- 200 Insurance Relationships
- 70% of Insurers in Global Fortune 500 clients
- Combine B&H & Moody’s expertise to extend what we offer to the insurance sector
- Focus on supporting the Capital modeling & ERM activities of insurers
- Leveraging both the research expertise and enterprise infrastructure.
Agenda – Stochastic Modelling for Insurance Companies

» Stochastic Modelling for Insurance and Asset Management
  • ESG (Economic Scenario Generator) Overview
  • Different Uses of ESGs

» ESG Model Selection and Calibration

» Stochastic Modelling for Indian Insurers and Key Challenges

» ESG Models
Objectives

» Explain the use of ESG by insurance companies
  – Market Consistent ESG and Real World ESG

» Explain the approach to validating ESGs for insurance companies
  – Choosing the appropriate asset model
    • ESG is NOT a black-box
    • Validation and documentation
  – The challenges for calibrating models to Indian markets
  – Answering the challenges for Indian Insurers

» Example of ESG models (Interest Rates, Equity and Credit)
Overview – Stochastic Modelling
What are Stochastic Simulations?

» Future is *unknown*

» We may have *expectations* about the future but we are never *certain* about it

» Simulate *many* future scenarios based on mathematical stochastic models

» Use scenarios in *Monte Carlo* simulations by ALM systems

» *Average* of the *Monte Carlo* simulations converge to our expectation
Stochastic Economic Scenario Generator

The ESG uses Monte Carlo Simulation to generate thousands of simulations of risk factors across multiple time periods.

Example: 10-year Spot Rate Projected over 5 years
Stochastic Economic Scenario Generator

The ESG uses Monte Carlo Simulation to generate thousands of simulations of risk factors across multiple time periods

Example: 10-year Spot Rate Projected over 5 years
Stochastic Economic Scenario Generator

The ESG uses Monte Carlo Simulation to generate thousands of simulations of risk factors across multiple time periods

Example: 10-year Spot Rate Projected over 5 years

Simulation 9
Risk Factors generated by the ESG

» The ESG generates Monte Carlo simulations for the joint behaviour of multiple risk factors:
  – Nominal Interest Rates
  – Real Interest Rates
  – Inflations Indices
  – Equity and dividend returns
  – Property and rental returns
  – Credit Spreads, rating transitions, risky bonds returns
  – Alternative asset returns
  – Interest rate implied volatility and equity implied volatility
  – Exchange rates
  – Macroeconomic indicators such as GDP, wage indices
  – Non market risk such as mortality and lapse rates

» Coherent modelling in Real World and Market Consistent environment
Joint distribution

» Correlation relationships between shocks driving each model

» Economically rational structure
ESG Global Multi Economy Model Structure

INTER-ECONOMY CORRELATIONS

- Equity Returns
- Property Returns
- Alternative Asset Returns
- Corporate Bond Returns
- Credit risk

- Initial swap and government nominal bonds
- Index linked government bonds

- Nominal short rate
- Real short rate

- Real-economy
- Nominal minus real is inflation expectations
- Realised Inflation and “alternative”
- Exchange rate
- Foreign nominal short rate and inflation
Use of the ESG in the insurance sector

- Calculation of cost of options and guarantees (EV, Fair Value, Best Estimate Reserves)
- Economic Capital calculation
- ALM, Asset Allocation, Business Planning
- Hedging
- Pricing and product development
- Retail advisory

- Technical Provision (Time Value)
- Internal models, ORSA
- Advanced uses of stochastic models
Stochastic Economic Scenario Generator

Historic Analysis & Expert Judgement

Establish economic targets for factors of interest:
- Interest rates
- Equity
- Credit
- Correlations
- Alternatives

Stochastic Models

Choose models that will best represent the risk factors and the specific modelling problem.

Calibrate – Establish model parameters to meet targets

Multiple Economies

Correlations

Multiple Time Steps

ESG

Visualise Output

Validation

Communication
Market Consistent ESG – Example
Market Consistent ESG

- Mathematical models used to value complex cashflows
  - Can be asset or liability cashflow
  - No arbitrage theory
- Model prices replicate market prices
  - Models calibrated to market prices to achieve this
- Model simulates scenarios that can be used to value cashflows where a market price does not exist
Valuation of Path Dependent Insurance Liability
Deterministic Market-Consistent Roll Forward Using Risk-Free Rates

Intrinsic Value = 0
Valuation of Path Dependent Insurance Liability
Run ALM Many Times Using Stochastic Market-Consistent Scenarios

» Average value represents stochastic value

» The difference between the stochastic value and the intrinsic value is the time value
Real World ESG – Example
### Use of the ESG in the insurance sector

<table>
<thead>
<tr>
<th>Activity</th>
<th>Use Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation of cost of options and guarantees (EV, Fair Value, Best Estimate Reserves)</td>
<td>Technical Provision (Time Value)</td>
</tr>
<tr>
<td>Economic Capital calculation</td>
<td>Internal models, ORSA</td>
</tr>
<tr>
<td>ALM, Asset Allocation, Business Planning</td>
<td></td>
</tr>
<tr>
<td>Hedging</td>
<td></td>
</tr>
<tr>
<td>Pricing and product development</td>
<td></td>
</tr>
<tr>
<td>Retail advisory</td>
<td></td>
</tr>
</tbody>
</table>

Use Test
Example Use – Determine the tail for SCR

» Real World ESG models are calibrated to realistic distributional targets

» Probability distribution of risk factors (equity, interest rates, etc) translated into probability distribution of the Net Asset Value

» Holistic approach captures dependency between risk factors

» Internal model approach also contains Use Test information such as risk exposure decomposition and reverse stress test material.
Approach (1): Stress and Correlate

- Interest Rate
  - Shift
  - Twist
  - Curvature
  - Volatility
- Equity/Property
  - Level
  - Volatility
- Credit
  - Spread Level
  - Transitions
- Other Markets
  - FX
- Non ESG Risks
  - Catastrophe
  - Longevity
  - Mortality
  - Lapse
  - Morbidity
  - Expense

Problems:
- Does not capture dependency effects that are firm specific
- Capital aggregation matrix requires subjective input and does not reflect actual correlations between risk factors

*Capital Aggregation Matrix does not reflect actual correlations between risk factors

Risk Capital

V@R

Capital Aggregation Correlation Matrix*
## Approach (2): Holistic Balance Sheet

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>Equity/Property</th>
<th>Credit</th>
<th>Other Markets</th>
<th>Non ESG Risks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift</td>
<td>Level</td>
<td>Spread Level</td>
<td>FX</td>
<td>Catastrophe</td>
</tr>
<tr>
<td>Twist</td>
<td>Volatility</td>
<td>Transitions</td>
<td></td>
<td>Longevity</td>
</tr>
<tr>
<td>Curvature</td>
<td></td>
<td></td>
<td></td>
<td>Lapse</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
<td>Mortality</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Morbidity</td>
</tr>
</tbody>
</table>

**ESG dependency**

**RSG dependency**

### Cashflow engine

- Risk Capital reflect company specific risk profile
- Contains useful metrics beyond Stress and Correlate approach
  - Probability of insolvency
  - Upside potential statistics
  - Conditional tail expectation
Solvency Capital / Economic Capital

» Capital allocation
  • By risk factors
  • By line of businesses/products

» Capital efficiency through optimising
  • Investment strategy
  • Management action
  • New business strategy
  • M&A strategy

» Risk framework that are specific to the insurance company
  • Specific to risk profile and cashflow of the company
  • Provide financial confidence internally and externally
Other uses of Real World ESG
Experience from B&H

Strategic Asset Allocation and Portfolio Optimisation

» Maximises investment returns
  • Minimises volatility
  • Minimises VaR
  • Minimises risk capital

» Used by insurance companies (life and non-life), pensions funds and asset managers

ALM Hedging

» Matching investment strategies to liability profile

Retail Advisory

» Spectrum charts instead of simplistic “high-medium-low” numbers

» Welcomed by regulators and policyholders for increased transparency
Choosing Stochastic Models
Stylised Facts & Data

Goal is to produce realistic and justifiable projections of financial and macroeconomic variables.

Use all credible historical data, market expectations via options and expert judgement.

Our approach involves 3 main activities:
1) Developing and documenting a set of stylized facts and beliefs.
2) Use these to select/build/structure, calibrate and validate models.
3) Look at real world markets to validate and review the stylized facts and models.

These are all ongoing activities:
» Frequent calibration
» Regular Real World Target updates and methodology reviews
Weighting Schemes & Data

Calibration is an art

» Subjectivity in: data sources, data policies, weighting, judgement

Goal is to produce **realistic** and **justifiable** projections of financial and macroeconomic variables.

Use all credible data available:

» Combine with market data of expectations: e.g. option implied volatility, consensus data

» Filter and clean data: liquidity of instruments, depth of market

» Exponentially-weighted moving average ensures more weight is placed on recent observations

» Consistency across asset classes
Models & Calibrations

Interest Rates
- Vasicek
- Black-Karasinski
- Cox-Ingersoll-Ross
- Libor Market Model

Equity Indices
- Time varying deterministic volatility
- Stochastic Volatility Jump Diffusion
- Constant Volatility

Multi Factor, Stochastic Volatility

And others for credit, inflation, exchange rates, MBS, derivatives etc.

All models documented in academic literature and MA research papers
B&H Economic Scenario Generator (ESG)

Mathematical stochastic models simulates returns of financial assets

Correlation ensures plausible economic relationship between asset classes and economies
Correlations and Dynamic Behaviours

Inter-economy Correlations

- Equity Returns
- Property Returns
- Alternative Asset Returns (e.g., commodities)
- Corporate Bond Returns
- Initial swap and government nominal bonds
- Index linked government bonds

Nominal short rate

Real short rate

Realised inflation and “alternative” inflation rates (i.e., Medical)

Nominal minus real is inflation expectations

Real-economy; GDP and real wages

Exchange rate (PPP or Interest rate parity)

Foreign nominal short rate and inflation

Intra-economy Correlations
Communicating Stochastic Models
Knowledge transfer

» MA/B&H ESG is NOT a black box.
  – Transparency is a core value to the B&H services

» Knowledge transfer is provided through
  – ESG trainings
  – Bespoke trainings/workshops
  – Detailed model documentations
  – Calibration reports (economic analysis + validation reports)
  – ESG Users group meetings (current topics and presentation of new models)
  – Access to online research library
  – Access to technical support
Knowledge Database

» Models methodologies, Economic research,

» Calibration documentation and Technical Advisory Panel
2.2 Calibration method

Using the fact that spread is an increasing function of \( \pi \) value (with all other parameters fixed), we can write the cumulative distribution function of spread in terms of the cumulative distribution function of \( \pi \) as follows:

\[
\operatorname{Pr}[S < s] = \operatorname{Pr}[(\pi > s)] = \operatorname{Pr}[(\pi > \pi^{-1}(s))] = F(\pi^{-1}(s))
\]

where \( F \) is the cumulative distribution function of the non-central chi-squared distribution and \( \pi^{-1}(s) \) is the \( \pi \) value corresponding to the spread level \( s \), obtained by numerical inversion of the relationships above (spread = default probability = \( \pi \)).

Similarly, any desired percentile of the spread distribution can be found by computing the corresponding percentile of the \( \pi \) distribution and converting that \( \pi \) value to a spread.

Since our calibration targets are expressed in terms of moments of the spread distribution (i.e., average and standard deviation), we use the following expression for the \( p^{th} \) moment:

\[
E[S^p] = \int_0^\infty s^{p-1} Pr[S > s] \, ds
\]
The ESG proposition of B&H

» **Software**
  - Professional software, Intuitive User interface
  - Compatible with many operating systems and ALM solutions
  - Includes an API
  - Grid computing

» **ESG modelling**
  - Joint stochastic modelling of multiple assets, multiple economies, multiple use
  - Bond portfolios and composite portfolios
  - MBS and derivatives (FRNs, swaps, swaptions, options...)

» **Calibration Services**
  - Standard calibrations for a variety of economies and variety of assets
  - Bespoke calibrations services
  - Access to calibrations tools
  - Economic research
  - Automation platform

» **Support, maintenance, training**
  - Support
  - Training
  - Documentation
  - Maintenance services
Key Challenges for Indian Insurers
Challenges in Indian Capital Markets

Mathematical assets need to be calibrated to market data (bond yields, equity prices, etc)

» Lack of good quality data
  • Data coverage is not consistent
  • Market data does not have long enough history
  • Lack of liquidity in certain parts of asset market
    o Affects frequency of data
    o Bid-Offer spread/transaction costs mask the underlying market values

» High volatility challenges the stability of results

Answering the challenge:

» Consistent choice of index across all economies for consistent and comparable data
» Adjust weighting scheme to reflect the shorter data history
» Set global targets to make economic sense of the stochastic scenarios instead of blindly calibrating to poor quality data. B&H provides model calibrations to 28+ economies.
Beyond Market Risks

Insurance capital should also cover non-market risks/insurance risks

» Non-market risks often only affects the Liability side of the balance sheet

» Quite often insurance companies model non-market risks and market risks independently
  • But need to bear in mind potential dependencies. E.g. equity risks and lapse risks
B&H Economic Scenario Generator (ESG)

Mathematical stochastic models simulates returns of financial assets

Correlation ensures plausible economic relationship between asset classes and economies
Nominal Rates
Extended 2Factor Black Karasinski (2FBK)

» Log-Normal model

» Simulate the short rate

» Model dynamics:

• Mean-Reverting processes

\[
d \ln r(t) = \alpha_1 (\ln m(t) - \ln r(t))dt + \sigma_1 dZ_1 \\
d \ln m(t) = \alpha_2 (\mu'(t) - \ln m(t))dt + \sigma_2 dZ_2
\]
Example Market Yield Curve vs Realised Yield Curve
Additional Parameter: Market Price of Risk

» In the risk-neutral world the expected return on all assets (e.g. bonds) is the risk-free rate.

» In the reality investors demand a premium for holding bonds (e.g. Interest rate risk)

» The Market Price of Risk ($\gamma$) adjusts the Brownian motions

\[ dZ_1 = dW_1 + \gamma_1(t)dt \]
\[ dZ_2 = dW_2 + \gamma_2(t)dt \]

» $\gamma$ value set to adjust interest rate paths

  o We set target short rate paths and calibrate $\gamma$
B

Equity
B&H Equity Six-Factor Model

Parent Equity Assets: shared exposure to factors controls correlations and tail-dependency

Child Equity Assets: exposure to Parent via CAPM
B&H Equity Model Choices

Excess Return Model

» Constant Volatility
  • Black-and-Scholes type Geometric Brownian Motion

» Time Varying Deterministic Volatility

» Stochastic Volatility Jump Diffusion
Constant Volatility
Time Varying Deterministic Volatility
B&H Equity - Stochastic Volatility Jump Diffusion

Simulation of the SVJD model

- Index No Jumps
- Index Jumps Only
- Index
- Stochastic Volatility

Month

Total return index

Stochastic volatility

0% 10% 20% 30% 40% 50% 60% 70% 80%
Lognormal Model: Dynamics

- The process for equity (excess return) index:
  \[ dS = \mu S dt + \sigma S dZ \]

- From Ito’s lemma
  \[ d\ln S = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dZ \]

- So that changes in the index follow
  \[ \ln \left( \frac{S(t + \Delta t)}{S(t)} \right) = \left( \mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \sqrt{\Delta t} Z \]
Time-Varying Deterministic Volatility (TVDV) model

- Volatility varies (deterministically) with time

\[ d\ln S = \left( \mu - \frac{\sigma(t)^2}{2} \right) dt + \sigma(t) dZ \]

- Fit market data term structure

- Implied Volatility:

\[ \sigma_{imp}(T) = \sqrt{\frac{1}{T} \int_0^T \sigma(t)^2 dt} \]
**Equity – Stochastic Volatility Jump Diffusion**

**Stochastic Volatility Jump Diffusion**

» Stochastic volatility part, Heston model *(red)*

» Jump diffusion part, Merton model *(blue)*

\[
\begin{align*}
\frac{dS_t}{S_t} &= (\mu - \lambda \bar{\mu})dt + \sqrt{v_t}dW_t^{(1)} + (\eta_t - 1)dN_t \\
fv_t &= \alpha(\theta - v_t)dt + \xi \sqrt{v_t}dW_t^{(2)} \\
\text{corr} \left( W_t^{(1)}, W_t^{(2)} \right) &= \rho \\
dN_t &\sim \{0,1\} \\
\eta_t &\sim e^{N(\mu, \sigma^2)}
\end{align*}
\]
Tail Dependence: Definition

- Factor exposure implies tail dependency
- What happened to other indices given on the conditional another index is in its tail?
- Tail dependence is not targeted (limited amount of data) but is measured

Example: Equity tail-dependency – GBP Equity against other developing countries
Properties and Alternatives

Equity type assets:

• Properties
• Infrastructure
• Commodities (Generic, Energy, Precious Metal, etc)
• Private Equity
• Hedge Fund
Setting Targets - Equity
MA B&H Real-World Equity Volatility targets

» **Short Term**: 30-day at-the-money option implied volatilities
  - Adjusted by a scalar of 0.98
  - Scalar determined through regression on long term historical data

» **Long Term**: Exponentially weighted moving average of up to 120 years of historical data
  - Average age of data for developed markets = 25 years
  - Average age of data for developing markets = 12.5 years

» **Medium to Long Term**: Produce “volatility term structure” to bridge short and long term
  - Volatility decay by regressing 21-day ahead volatilities against realised volatilities
  - Negative correlation between volatility and returns
  - “Volatility of volatility”
Setting Targets - Credit
MA B&H Real-World Credit targets

The Credit model is made up of a number of elements:

» Transition Matrix
» Credit Spread Level
» Credit Spread Distribution
» Default Recovery Assumption
» Correlation and Tail Dependency with Equity Asset

B&H ESG simulate stochastically:

- Spreads
- Transitions/Defaults
- Recovery upon Default
Annual Transition Matrix

Ratings at end of period

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>94.04%</td>
<td>5.69%</td>
<td>0.23%</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>AA</td>
<td>2.13%</td>
<td>89.53%</td>
<td>7.28%</td>
<td>0.36%</td>
<td>0.30%</td>
<td>0.23%</td>
<td>0.03%</td>
<td>0.14%</td>
</tr>
<tr>
<td>A</td>
<td>1.63%</td>
<td>3.44%</td>
<td>89.52%</td>
<td>4.40%</td>
<td>0.39%</td>
<td>0.39%</td>
<td>0.03%</td>
<td>0.20%</td>
</tr>
<tr>
<td>BBB</td>
<td>1.54%</td>
<td>1.56%</td>
<td>5.20%</td>
<td>87.95%</td>
<td>2.06%</td>
<td>0.63%</td>
<td>0.63%</td>
<td>0.43%</td>
</tr>
<tr>
<td>BB</td>
<td>0.07%</td>
<td>0.56%</td>
<td>1.28%</td>
<td>6.68%</td>
<td>82.26%</td>
<td>6.70%</td>
<td>0.70%</td>
<td>1.75%</td>
</tr>
<tr>
<td>B</td>
<td>0.02%</td>
<td>0.05%</td>
<td>1.28%</td>
<td>1.58%</td>
<td>6.26%</td>
<td>80.56%</td>
<td>5.71%</td>
<td>4.53%</td>
</tr>
<tr>
<td>CCC</td>
<td>0.02%</td>
<td>0.04%</td>
<td>0.95%</td>
<td>1.46%</td>
<td>2.66%</td>
<td>8.74%</td>
<td>73.01%</td>
<td>13.13%</td>
</tr>
<tr>
<td>Default</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

- **Markovian:** \( p_2(A \rightarrow D) = p_1(A \rightarrow D)p_1(D \rightarrow D) + p_1(A \rightarrow A)p_1(A \rightarrow D) \)
- Multiply matrices at different periods to calculate default probabilities
- Two type of transition matrices: real world (RW) vs risk neutral (RN)
**Spreads**

Generally, corporate bonds are cheaper (i.e. offer a higher yield) than an equivalent government bond.

The spread is the difference in yield between a corporate bond and a government bond:

\[ y_{corp} = r_f + s \]

Spreads vary with time (our model must allow \( q \) to vary with time)
Break-even spreads

The spread $s$ accounts for the risk-neutral probabilities of default. The break-even spread $s_{BE}$ accounts for the real-world probabilities of default.
Pricing Credit Risk
RW vs RN pricing of credit risky ZCB

Real World pricing:

» probability $p$ of bond defaulting before maturity
» recovery rate $R$
» expected cashflow at maturity $= R \times p + (1 - p)$

$\text{Price} = \text{Expected present-value of cashflow}$

$P_{\text{corp}}(T) = e^{-dT} [R \times p + (1 - p)]$

where $d = r_f + \text{RiskPremium}$

» Requires a $\text{RiskPremium}$ to compensate for uncertain return
Pricing Credit Risk

Risk Neutral pricing of credit risky ZCB

Risk Neutral pricing:

» Risk Neutral probability \( q \) of bond defaulting before maturity

» recovery rate \( R \)

» expected cashflow at maturity = \( R \times q + (1 - q) \)

» Price = Expected present-value of cashflow

\[
P_{\text{corp}}(T) = e^{-r_f T} [R \times q + (1 - q)]
\]

where \( r_f \) is the risk-free rate

» Does not require any risk premium to compensate for uncertain return. Earns risk-free rate on average.

NOTES:

» Real World and Risk Neutral agree on today's price.

» Positive Risk Premium implies \( q > p \), i.e. RN is more pessimistic.

» RN pricing means default probabilities need to be unrealistically pessimistic
Transition and Spreads

How to simulate both in the same credit model?
Risk Neutral Transition Matrix
The $\pi$ process

» RN matrix is used to calculate credit-risky bond prices and spreads

» RN transition matrix is calculated by scaling the RW generator matrix $\Lambda_p$ by the credit stochastic driver $\pi_t$:

$$\Lambda_Q = \pi_t \Lambda_p$$

» $\pi_t$ is calibrated to market spreads and follows a CIR process:

$$d\pi_t = \alpha (\mu - \pi_t) dt + \sigma \sqrt{\pi_t} dZ_t$$

- $\pi_t > 0 \Rightarrow$ positive spreads
- Model displays mean reversion and has an analytical solution
Simulation of Credit Transitions

» Credit transitions are generated by slicing up the standard normal distribution according to the transition probabilities.

» The issuer’s rating at the end of a period will depend on the value of a generated standard normal variable $Z_i$.

» For a CCC-rated bond, we have:

![Diagram showing the relationship between number of standard deviations and credit ratings.](CCC bond)
Intra-Sector Correlation

» We must incorporate correlation between different issuers’ credit experience

» This can be done by generating a correlated $Z_i$:

$$Z_i = \sqrt{\rho}Z_M + \sqrt{1-\rho}Z_{S_i}$$

where

$Z_M$ is the associated equity shock

$Z_{S_i}$ is the issuer specific shock

$\rho$ is the correlation between $Z_i$ of different issuers (Intra-sector Correlation)
Stochastic Spread in Real-World

Introduce scaling factors:

\[ \Lambda_Q^{BASE} = K \Lambda_P^{BASE} \]

where \( K \) is a diagonal matrix

» one scaling factor for each credit rating

Build a more general credit model from Base generator matrices:

\[ \Lambda_P = (A_P + B_P \pi_t) \Lambda_P^{BASE} \]
\[ \Lambda_Q = (A_Q + B_Q \pi_t) \Lambda_P^{BASE} \]

» Real-World transition matrix can vary stochastically.

» Stochastic process drives variation in Credit Risk Premium and Default Probability

» More freedom when calibrating: better fit to market data.
Split between default and spread

Real-world transitions: $A = 1; B = 0$
Credit spread changes purely a result of changes in credit risk-premium

Real-world transitions: $A = 0.75; B = 0.25$
~25% of credit spread changes are due to changes in RW expected default losses
Example market spreads

» Calibrate to a credit spread (e.g. A7)
Inflation
Setting Targets - Inflation
MA B&H Real-World Inflation Calibration

» Index-Linked Bonds

» Inflation Targets
  • Global Targets
  • Specific Targets
    o By Country
    o By Sector

Inflation Modelling: Affected by Nominal Rates and Real Rates
B&H Inflation models

» Derived inflation (Basic model)
  • Difference between Nominal Rates and Real Rates

» InflationPlus
  • Adding unexpected inflation
  • Additional uncertainty

» Inflation Wedge
  • Specific inflation, e.g. Medical, Wage
  • Evolve around base inflation (CPI/RPI)
B&H Inflation models (Mathematics)

» Derived inflation (Basic model)
  • $\text{inflation } \pi = \text{nominal rate } - \text{real rate}$

» InflationPlus
  • $\pi_{\text{realised}}(t) = \pi_{\text{expected}}(t) + MUI + \sigma_{\text{inflation}}Z_{\text{inflation}}(t)$
  • $MUI = \text{mean unexpected inflation}$
  • $\pi_{\text{expected}}(t) = r_{\text{nominal}}(t - 1) - r_{\text{real}}(t - 1)$

» Inflation Wedge
  • $\pi_{\text{specific}} = x_t + \pi_{\text{core}}$
  • $dx_t = \alpha(\mu_x - x_t)dt + \sigma_{\text{specific}}dW_t$
FX, Correlations and Tail Dependency
Real World FX Modelling

Goal:

» Capture economically coherent outcomes for the purposes of projection.

Desirable features:

» Respect Purchasing Power Parity in the long run
» Use interest rate differentials in the short term

B&H RW Implementation

» Model real exchange rate
» As a mean reverting process
» Subject to random shocks in the short term but is pulled towards some mean level over the long term
Example PPP Model Simulation

![Graph of exchange rate over years showing spot rate and PPP trend](image-url)

- **Exchange Rate (domestic units/foreign units)**
- **Year**

The graph illustrates the exchange rate over years, comparing the spot rate and PPP trend.
Real World Correlation targets

Correlation targets are unconditional and global

» Currently investigating rationale for economy-specific targets

» Weak evidence to suggest statistically credible alternatives.

EndJune2012
Tail Dependency

- Tail dependency in bond defaults
- Tail dependency in equity markets
- Tail dependency is not targeted (limited amount of data) but is measured

Example: Equity tail-dependency
Asset Allocation, Views and Biases
Real-World Scenarios and Asset Allocation

» Another typical use of Real-World scenarios is for **Strategic Asset Allocation**

» Investment portfolio optimised using techniques such as Mean-Variance Optimisation
  • Maximise Return
  • Given a Volatility target

» Real-World scenarios as input
  • To analyse risk-return performances
  • Given different asset-mixes

» But Real-World embeds views such that
  • Asset allocation biased towards certain assets
  • Because of views taken on risk-premiums being different to market Average
Dynamic Equilibrium Calibration

» Assumes market-equilibrium in the long-term based on global capitalisation

» Possible application of Black-Litterman weighting to blend between views and equilibrium.

Bias towards corporate bonds because of views on credit risk-premia  
Dynamic Equilibrium scenarios based on global capitalisation
Dynamic Equilibrium Calibration

- Black-Litterman weighting between views and equilibrium.
- Shift between importance of best-estimate views (subjective) and market equilibrium (based on global capitalisation)

Weighting on Views

Weighting on Equilibrium
Examples

&

Question and Answer