Institute of Actuaries of India

ACET December 2021 Solutions

Mathematics

1. D. Let $y = \sin^{-1} x^2$ so that $\sin y = x^2$ and therefore $x^2 \le 1 \Rightarrow |x| \le 1$.

2. B. Let $P = 3^{\frac{1}{2}}.9^{\frac{1}{4}}.27^{\frac{1}{8}} \dots \infty = 3^{\frac{1}{2}}.3^{\frac{2}{4}}.3^{\frac{3}{8}} \dots \infty = 3^{\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\cdots\infty} = 3^S$ (say),
where $S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \cdots \infty \Rightarrow \frac{1}{2}S = \frac{1}{4} + \frac{2}{8} + \cdots \infty$.
Thus $S - \frac{1}{2}S = \frac{1}{2}S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \infty = \frac{1/2}{1-1/2} = 1 \Rightarrow S = 2$.
Hence the required value $P = 3^2 = 9$.

3. A. $adj(3A) = 3^{4-1} adj\ A = 27 adj\ A$, as order of A is 4.

4. C. Both 1-1 and onto, since $h(2m) = 2m - 1$ and $h(2m-1) = 2m$, i.e., consecutive odd and even numbers are interchanged.

5. A. $f'(x) = -\frac{1}{x^{\frac{1}{3}}}\left(1 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \to \infty$, as $x \to 0$. Thus $f'(0)$ does not exist.
But $f(0) = 1 > f(x), \forall\, x$ in the neighbourhood of $x = 0$.
Thus, $f(x)$ has a maximum at $x = 0$.

6. D. The given equation reduces to $r\sin(\theta + \varphi) = c$, where $3 = r\cos\varphi$, $4 = r\sin\varphi$, so that $r = 5$.
Hence the equation takes the form $\sin(\theta + \varphi) = \frac{c}{5}$, it is solvable only when $\left|\frac{c}{5}\right| \le 1$, i.e., $-5 \le c \le 5$.

7. B. $7^{103} = 7.(7^2)^{51} = 7.(50-1)^{51} = 7.(50^{51} - C_1^{51}50^{50} + C_2^{51}50^{49} - \cdots - 1) = 7.(25k - 1) = 25.(7k-1) + 18$, where k is an integer.

8. A. By solving the first two equations one gets $x = 2$, $y = -1$, which satisfies the third equation too. Therefore, this is the unique solution.

9. D. $\mu = \int_3^5 \frac{2\log x}{2\log x + \log(8-x)^2} dx = \int_3^5 \frac{\log x}{\log x + \log(8-x)} dx = \int_3^5 \frac{\log(8-x)}{\log(8-x) + \log x} dx = \mu$.
Hence $2\mu = \int_3^5 dx = 2$, i.e., $\mu = 1$.

10. D. If the coefficient matrix is non-singular, then it is invertible and therefore the only solution is $x = y = z = 0$.
For there to be more than one solution, the coefficient matrix has to be singular (i.e., has to have determinant equal to 0), which happens only when $a = 4$.

11. B. The required equation in Cartesian form is $\left(x+\frac{7}{2}\right)^2+y^2=\left(\frac{7}{2}\right)^2$.

The corresponding polar form becomes $\left(r\cos\theta+\frac{7}{2}\right)^2+(r\sin\theta)^2=\frac{49}{4}$, which reduces to $r=-7\cos\theta$.

The range may be ascertained by ensuring that the graph of the circle should lie only on the second and third quadrants.

The correct range ensures that r is always positive.

12. D. Here $x^2=t-3$ and $y^2=4-t$, so that $x^2+y^2=1$, representing the equation of a circle.

13. D. $\lim_{x\to 0-}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}=\frac{0-1}{0+1}=-1$, but $\lim_{x\to 0+}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}=1$. Hence the limit does not exist.

14. A. At $x=0$ $\log|x|$ is not defined. Therefore, $f(x)$ is undefined, and hence discontinuous at that point.

At $x=-1,1,$ $\log|x|=0$ and therefore $f(x)$ is undefined and discontinuous.

For all other values of x the function $\log|x|$ has a non-zero value and is continuous. Therefore its inverse is also continuous at those points.

Thus the number of points of discontinuities is 3.

15. D. The range of $\operatorname{cosec}^{-1}\theta$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$.

16. C. Since the function $f(y)=e^y-e^{-y}$ for $-1\le y\le 1$ has the derivative $f'(y)=e^y+e^{-y}>0$, it is an increasing function, with maximum at $y=1$ and the maximum value of the function is $f(1)=e-\frac{1}{e}=2.72-0.37<2.5$.

Therefore, the left hand side $e^{\sin x}-e^{-\sin x}$ of the given equation can never attain the value 2.5 on the right hand side.

17. B.

18. A. Since x^2-1 is an even function, the given integral simplifies as

$$\int_{-3}^{3}|x^2-1|dx=2\int_0^3|x^2-1|dx=2\int_0^1(1-x^2)dx+2\int_1^3(x^2-1)dx$$

$$=2-\frac{2}{3}+\frac{2}{3}(27-1)-2(3-1)=2-\frac{2}{3}+18-\frac{2}{3}-4=\frac{44}{3}.$$

19. B. Note that $\Delta^4 y_0=(E-1)^4 y_0=0$, i.e., $y_4-4y_3+6y_2-4y_1+y_0=0$.

Hence, $81-4y_3+54-12+1=0$.

It follows that $y_3=31$.

20. C. The projection of $\vec{i}+\vec{j}+\vec{k}$ on the plane of the unit vectors $\vec{i}$ and $\vec{j}$ is $\vec{i}+\vec{j}$.

Therefore the cosine of the angle between these two vectors is the dot product between the corresponding unit vectors, which is $\frac{2}{\sqrt{2\times 3}}=\sqrt{2/3}$.

Statistics

21. D. Since the arithmetic mean of the four consecutive integers starting with x is y, we have $y = x + 6/4$.

    Therefore, the arithmetic mean of the eight consecutive integers starting with $x + 2$ is $(x + 2) + 28/8 = x + 11/2 = x + 3/2 + 4 = y + 4$.

22. C. Among the three medians in the three groups, the largest could be 13.

    The next largest median could be 10, provided two smaller integers are put in the first group (along with 13, 14 and 15).

    Likewise, the largest median of the third group is 7, which may be achieved by putting two smaller numbers in the second group (along with 10, 11 and 12).

    Thus, the largest possible average is 10.

    One of the possible choices of the three groups that achieves the three largest medians is as follows.

    1 2 13 14 15
    3 4 10 11 12
    5 6 7 8 9

23. B. Given all the conditions the only way to arrange the revenues of the five companies is

    $$100 \quad 300 \quad 600 \quad 1000 \quad 1000$$

    Hence the difference between the highest and the lowest of the revenues among the five companies is $1000 - 100 = 900$.

24. A. Standard deviation is the square root of the difference between the mean of squares of the numbers and the square of mean of the numbers.

    Therefore $SD = \sqrt{232/4 - (28/4)^2} = \sqrt{58 - 49} = 3$.

25. A. $\lambda = 1/60$. The probability of not completing within 90 minutes is $\exp(-\lambda \times 90) = \exp(-3/2)$.

26. D. Here $\lambda = 5/60$ and $t = 3$. Therefore, $P(0) = \exp(-\lambda \times 3) = \exp(-1/4)$.

27. C. The number of errors in eight pages is the sum of eight iid Poisson random variables, which has the Poisson distribution.

    Mean number of errors in 8 pages is $8 \times 0.2 = 1.6$ with SD $\sqrt{1.6}$.

    So, CV is $1/\sqrt{1.6}$.

28. B. Probability of a perfect recording is 0.8.

    Number of perfect recordings out of 10 has a Binomial distribution

    with $p = 0.8$ and $1 - p = 0.2$,

    so that the probability of at least one perfect recording is

    $1 - P(\text{No perfect recording}) = 1 - 0.2^{10}$.

29. C. If x is the 87th percentile, $P[Z \leq (x-65)/15] = 0.87 = 1 - 0.13$, that is, $(x-65)/15 = 1.13$, or $x = 65 + 15 \times 1.13 = 82$ (Approximately).

30. D. $P(Z < 2.58) = 0.995 = P(Z > -2.58)$.
$P(X > 100) = 0.995$.
$(Z > (100 - \text{mean})/1) = 0.995$.
$\text{Mean} = 100 + 2.58 = 102.58$.

31. B. $Corr\ (X, Y) = 1/4$.
$Cov(X, Y) = Corr(X, Y)\ \times \sqrt{Var\ X}\sqrt{Var Y}\ = 3/4$.
$Cov(\ Y, X + Y) = Cov(\ X, Y) + Var(Y) = 3/4 + 9\ = 39/4$.

32. D. $E(X) = 4/2 = 2$.
$E(Y) = E[E(Y|X)] = E[1/X] = \int_1^3 (1/x)(1/2)\ dx = (1/2)\log 3$.
$E(XY) = E[E(XY|X)] = E[XE(Y|X)] = E[X/X] = 1$.
$Cov(X, Y) = E(XY) - E(Y)E(X) = 1 - 2(1/2)\log 3. = 1 - \log 3$.

33. C. $Cov(X, Y) = Corr(X, Y) \times SD(X) \times SD(Y) = 7.5$.
$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y) = 25 + 9 + 15 = 49$.
$SD(X + Y) = \sqrt{Var(X + Y)} = \sqrt{49} = 7$.

34. D. If the regression line of X on Y is $4X = 5Y - 33$, and that of Y on X is $9Y = 20X - 107$, then $b_{xy} = 5/4$ and $b_{yx} = 20/9$, so that the correlation coefficient r satisfies $r^2 = b_{xy}b_{yx} = 100/36 > 1$, which is not possible.
If the regression line of Y on X is $5Y = 4X + 33$ and that of X on Y is $20X = 9Y + 107$, then $b_{yx} = 4/5$ and $b_{xy} = 9/20$, so that $r^2 = b_{xy}b_{yx} = 36/100$.
Since the regression coefficients are positive, $r > 0$. So $r = 6/10 = 3/5$.

35. C. Since (EX, EY) is the point of intersection of the two regression lines, solving the two equations we get $EX = 13/14$ and $EY = -33/14$.

36. A. The regression of P on S is
$P - E(P) = Corr(P, S)[SD(P)/SD(S)]\big(S - E(S)\big)$, i.e.,
$P - 50 = 0.5[4/2](S - 60)$, which simplifies to $P = \text{S} - 10$.

37. C. Let n (> 0) be the number of sides.
Then the number of diagonals is $\binom{n}{2} - n = 170$.
Therefore, $n^2 - 3n - 340 = 0$, i.e., $n = 20$.

38. A. Let the three particular objects be considered as a single entity. This group has to occur in every permutation of 5 objects out of 10.

There are ${}^7P_2$ ways that the other 2 objects can be permuted from the remaining set of 7 to choose from.

The group of 3 can be put between 2 selected objects in 3 ways.

The group itself can be permuted among themselves in 3! ways.

Thus, the required number is ${}^7P_2 \times 3 \times 3! = 42 \times 3 \times 6 = 756$.

39. D. The concerned set is equivalent to $(B - A) \cap (A \cap B) = \phi$.

40. A. $X \cap Z, Y \cap Z$ and $X \cap Y \cap Z$ are subsets of, Z so that
$P(X \cap Z) \leq P(Z) = 0, P(Y \cap Z) \leq P(Z) = 0, P(X \cap Y \cap Z) \leq P(Z) = 0$.
Therefore, $P(X \cap Z) = 0 = P(X)P(Z)$, $P(Y \cap Z) = 0 = P(Y)P(Z)$,
$P(X \cap Y \cap Z) = 0 = P(X)P(Y)P(Z)$. Hence X, Y, Z are independent.

41. C. The following is the contribution from unit X in each year.

Year	X
2016	$150/(150+188+173+139) = 23.08\%$
2017	$241/(241+138+207+309) = 26.93\%$
2018	$205/(205+290+296+236) = 19.96\%$
2019	$275/(275+130+365+127) = 30.66\%$
2020	$140/(140+99+232+158) = 22.26\%$

42. D. X never had the maximum production level;
Z never had the minimum production level.

43. B. The table below shows the average of sales in two cities against each model variant. Model 2 has the same average across the two cities.

Model 1	160	179
Model 2	177	177
Model 3	178	165
Model 4	170	165
Model 5	182	176
Model 6	165	184
Model 7	166	192

44. A. The table below shows, against each model variant, the excess in number of units sold (taking together all brands) in Jaipur over that in Ahmedabad. Model 1 has the largest excess.

	Ahmedabad	Jaipur	Excess of Jaipur over Ahmedabad
Model 1	641	714	73
Model 2	707	707	0
Model 3	713	659	−54
Model 4	679	660	−19
Model 5	729	703	−26
Model 6	660	734	74
Model 7	664	766	102

45. D. <u>Cost of one gram of vitamin 1 in 2017-18</u>
Apple: $0.140/0.04\% = 350$ rupees; Guava: $0.160/0.03\% = 533$ rupees;
Mango: $0.135/0.025\% = 540$ rupees; Grapes: $0.150/0.025\% = 600$ rupees.

46. A. <u>Cost of one gram of vitamin 2 in 2017-18</u>
Apple: $0.140/0.025\% = 560$ rupees; Guava: $0.160/0.03\% = 533$ rupees;
Mango: $0.135/0.025\% = 540$ rupees; Grapes: $0.150/0.045\% = 333$ rupees.
The ratio of largest to smallest is $560/333 = 1.68$.

47. C. The table below gives the price of 0.25 gram of Vitamin 1 in each fruit in 2019-20.

Apple	$(0.150/0.04\%)/4 = 375/4 = 94$ rupees
Guava	$(0.155/0.03\%)/4 = 517/4 = 129$ rupees
Mango	$(0.130/0.025\%)/4 = 520/4 = 130$ rupees
Grapes	$(0.150/0.025\%)/4 = 600/4 = 150$ rupees
Total	503 rupees

48. A. If 1 kg of apple, 2 kg of guava, 3 kg of mango and 4 kg of grapes are mixed, then, out of the total weight of 10000 grams,

weight of Vitamin 1 and 2 in apples is $0.4 + 0.25 = 0.65$ grams;

weight of Vitamin 1 and 2 in apples is $(0.3 + 0.3) \times 2 = 1.2$ grams;

weight of Vitamin 1 and 2 in apples is $(0.25 + 0.25) \times 3 = 1.5$ grams;

weight of Vitamin 1 and 2 in apples is $(0.25 + 0.45) \times 4 = 2.8$ grams.

A total of 6.15 grams or 0.0615% will be constituted by Vitamin 1 and Vitamin 2.

49. C. Company C had grown from 20 Crores in2019 to 30 Crores in 2020 with the sales growth rate of 50% only, lowest among all the companies.

	Sales Growth Rate
A	75.00%
B	150.00%
C	50.00%
D	166.67%
Others	100.00%

50. D. The current sales growth rate for Others category is 100%, leading to 10 crores business in 2020.
If the sales growth rate becomes 200%, then the business is of 15 crores in 2020. Therefore it is a 205 crores market instead of a 200 crores market in 2020.

51. A. Sales of other companies will become 20 crores instead of 10 crores in 2020. Hence the ratio of sales in 2020 to 2019 is $20/5 = 4$.

52. A.
53. C.
54. B.
55. A.
56. B.
57. A.
58. C.
59. D.
60. A.
61. B.
62. B.

Logical reasoning

63. C.

64. D. One cannot determine the last day of the next month which has 31 days because one does not know the number of days in the present month which may be 30, 31, 28 or 29 (in a leap year).

65. B. The order is S, Q, P, T, R.

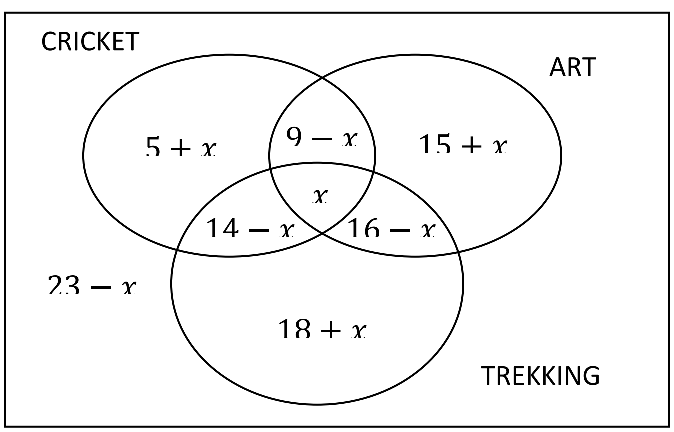
66. A.

67. B.

68. B. We know that angle traced by hour hand in 12 hours is $360°$
From 7 to 1, there are 6 hours.
Angle traced by the hour hand in 6 hours is $6 \times (360/12) = 180°$.

69. C. If the cardboard box is flattened then the shortest crawling path is the line joining the source and destination points, which is a diagonal of a rectangle of sides 1 meter and 2 meters. The length of the diagonal is $\sqrt{5}$ meters.

70. C. If x is the number of students in all three activities,
then the numbers of students in the various segments are as given below.
All three: x.
Art and Trekking only: $16 - x$.
Trekking and Cricket only: $14 - x$.
Cricket and Art only: $9 - x$.
Art only: $40 - (16 - x) - (9 - x) - x = 15 + x$.
Cricket only: $28 - (14 - x) - (9 - x) - x = 5 + x$.
Trekking only: $48 - (16 - x) - (14 - x) - x = 18 + x$.
None: $100 - (18 + x) - (5 + x) - (15 + x) - (9 - x) - (14 - x) - (16 - x) - x = 23 - x$.



Since none of these numbers can be negative, x can be at most 9.

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