Institute of Actuaries of India

ACET January 2021 Solutions

Mathematics

1. A. $\sum_{i=1}^{10}\sum_{j=1}^{10} ij = (\sum_{i=1}^{10} i)^2 = \left(\frac{10\times 11}{2}\right)^2 = 55^2 = 3025.$

2. D. $A = \{1,2,3,4,5,6\}$ and the relation $R = \{(x,y): y = x+1\}$.
R is not relexive: $(1,1) \notin R$.
R is not symmetric: $(2,3) \in R$ but $(3,2) \notin$ R.
R is not transitive: $(3,4) \in R$ and $(4,5) \in R$ but $(3,5) \notin$ R..

3. C. $f^\circ g\left(-\frac{7}{2}\right) = f(g\left(-\frac{7}{2}\right) = f\left(\left|-\frac{7}{2}-1\right|\right) = f\left(\left|-\frac{9}{2}\right|\right) = f(|-4.5|) = f(4.5) = 4.$

4. D. For some x, y ; $x \neq y; f(x) = f(y), f$ is not one to one. The range of is $\{-1,0,1\}$ which is a subset of real line, it is not onto. Hence *f* is neither one to one nor onto.

5. C. Let $f(x) = x^3 - 2x + 5$. Then $f'(x) = 3x^2 - 2$, which is 0 only at $\sqrt{2/3}$ and $-\sqrt{2/3}$, negative only in between these roots and positive outside the interval $\left[-\sqrt{2/3}, \sqrt{2/3}\right]$. Therefore, it is decreasing in between these two roots and increasing outside this interval. Further, there is a local minimum at $\sqrt{2/3}$, where the value of the function is positive. This means $f(x)$ is positive for $x > -\sqrt{2/3}$, and can only be zero at some unique $x < -\sqrt{2/3}$. Checking the values at the boundaries of the intervals, we have $f(0) = 5\ (+ve)$, $f(-1) = 6(+ve)$, $f(-2) = 1\ (+ve)$, $f(-3) = -16(-ve)$. Thus, the root lies in $(-3,-2)$.

6. D. $|1-2x| - 2x \geq 0$ implies $1 - 2x \geq 2x\ or\ 1 - 2x \leq -2x$.
Hence, $x \leq \frac{1}{4}$. The other choice is inadmissible and $S = \left(-\infty, \frac{1}{4}\right]$.

7. B. The forward difference table is:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	2	3	-1	a-8	56-4a
2	5	2	a-9	48-3a	
3	7	a-7	39-2a		
4	a	32-a			
5	32				

Since $f(x)$ is a cubic polynomial, $\Delta^4 y = 56 - 4a = 0$. Hence $a = 14$.

8. A. Let α and β be the zeroes of the polynomial expression $p(x) = x^2 + bx - 2$.
$\alpha^2 + \beta^2 = 5$ (given). Further, $\alpha + \beta = -b, \alpha\beta = -2$.
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 5$; Hence $b = \pm 1$.

9. B. $T_{r+1} = \binom{9}{r}\left(\frac{4x^2}{3}\right)^r\left(-\frac{3}{2x}\right)^{9-r} = \binom{9}{r}\left(\frac{4}{3}\right)^r\left(-\frac{3}{2}\right)^{9-r} x^{3r-9}$.
When $3r - 9 = 0$, or $r = 3$ in T_{r+1}, we get the constant term,
which is $\binom{9}{3}\left(\frac{4}{3}\right)^3\left(-\frac{3}{2}\right)^6$.

10. D. Let $y = 16^{\sin^2 x}$. Then $y + \frac{16}{y} = 10$, i.e., $y^2 - 10y + 16 = 0$, i.e., $y = 2$ or 8.
Therefore, $\log_2 y = 4\sin^2 x = 1$ or 3. Since $\sin x$ cannot be negative in the range $0 < x < \pi$, it follows that $\sin x = \frac{1}{2}$ or $\frac{\sqrt{3}}{2}$. The solutions in the given range are $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$. Hence $\tan x = \frac{1}{\sqrt{3}}$ or $\sqrt{3}$.

11. B. Put $= \frac{1}{x}$, then as $x \to \infty, y \to 0$.
Hence, $\lim\limits_{y\to 0}\frac{y^2 - 2\tan^{-1} y}{y}$, which has the $\frac{0}{0}$ form.
By L'Hopital's rule, $\lim\limits_{y\to 0}\frac{2y - 2\frac{1}{1+y^2}}{1} = -2$.

12. A. $y = (\log_e x)^{\cos x}$ implies $\log_e y = \cos x \log_e(\log_e x)$.
$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{1}{\log_e x}\frac{1}{x} + \log_e(\log_e x)(-\sin x)$.
Hence, $\frac{dy}{dx} = (\log_e x)^{\cos x}\left[\cos x \frac{1}{x\log_e x} - \sin x \log_e(\log_e x)\right]$.

13. C. $f'(x) = 2x - 1$. Hence, $f'(x) = 0 \Rightarrow x = \frac{1}{2}$.
It is seen that for $x \in \left(-1, \frac{1}{2}\right)$, $f'(x) < 0$ and $x \in \left(\frac{1}{2}, 1\right)$, $f'(x) > 0$.
Hence, $f(x)$ is neither increasing nor decreasing in $(-1,1)$.

14. B. $\frac{x}{(x+1)(x+2)} = \frac{M}{x+1} + \frac{N}{x+2} \Rightarrow x = M(x+2) + N(x+1)$.
$\Rightarrow M = -1$ and $N = 2$.
$\int \frac{x}{(x+1)(x+2)}dx = \int \frac{(-1)}{x+1}dx + \int \frac{2}{x+2}dx$
$= -\log_e|x+1| + 2\log_e|x+2| + C = \log_e\left[\frac{(x+2)^2}{|x+1|}\right] + C$.

15. A. $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx = \int_{e^{-1}}^{1}\left|\frac{\log_e x}{x}\right|dx + \int_{1}^{e^2}\left|\frac{\log_e x}{x}\right|dx$

$= \int_{e^{-1}}^{1}\frac{-\log_e x}{x}dx + \int_{1}^{e^2}\frac{\log_e x}{x}dx = \int_{-1}^{0} -y dy + \int_{0}^{2} y dy = \frac{y^2}{2}\Big|_0^{-1} + \frac{y^2}{2}\Big|_0^{2} = \frac{5}{2}$.

16. D. $\left|\vec{a}+\vec{b}\right|^2 - \left|\vec{a}-\vec{b}\right|^2 = 4\vec{a}.\,\vec{b}$ implies $29 - \left|\vec{a}-\vec{b}\right|^2 = 20$.
Thus $\left|\vec{a}-\vec{b}\right|^2 = 9$, giving $\left|\vec{a}-\vec{b}\right| = 3$.

17. A. The area of the parallelogram is the magnitude of the vector

$$\vec{a}\times\vec{b} = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix} = (-1+21)\vec{i} - (1-6)\vec{j} + (-7+2)\vec{k} = 20\vec{i} + 5\vec{j} - 5\vec{k}.$$

$\left|\vec{a}\times\vec{b}\right| = \sqrt{20^2+5^2+(-5)^2} = \sqrt{450} = 15\sqrt{2}$ sq. units.

18. C. When P is multiplied by k, every summand in the expression of the determinant gets multiplied by k^n.

19. B. Since $|M| \neq 0$, M^{-1} exits, Hence $M^2 - M + I = 0$ implies
$M^2 - M + MM^{-1} = 0$.
Thus, $M(M - I + M^{-1}) = 0$ giving $M^{-1} = I - M$.

20. C. $P(x)P(y) = \begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$= \begin{bmatrix}\cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0\\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$= \begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} = P(x+y).$

Statistics

21. B. Number of students opted Mathematics only = $28 - 22 = 6$.
Number of students opted Biology only = $30 - 22 = 8$.
Number of students opted neither Mathematics nor Biology
$= 56 - (6 + 8 + 22) = 20$.
The required probability = $20/56 = 5/14$.

22. A. The number of ways, the man can visit the cities is $4! = 24$.
The possible cases that the man visits C_1 before C_2 and C_2 before C_3 are $C_1C_2C_3C_4$, $C_1C_2C_4C_3$, $C_1C_4C_2C_3$, $C_4C_1C_2C_3$.
The number of cases favourable to the event is 4.
The required probability = $4/24 = 1/6$.

23. C. $P(A \cup B^c) = 1 - P(A^c \cap B) = 1 - \left(P(B) - P(A \cap B)\right) = 1 - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{11}{12}$.

24. D. $P(E|F) > P(E)$
$\Rightarrow \frac{P(E\cap F)}{P(F)} > P(E) \Rightarrow P(E \cap F) > P(E)P(F) \Rightarrow P(F|E) > P(F)$.

25. B. $P(A_1 \cup A_2 \cup A_3) = 1 - P(A_1^c \cap A_2^c \cap A_3^c)$
Since A_1, A_2, A_3 are independent, A_1^c, A_2^c, A_3^c are also independent.
$P(A_1^c \cap A_2^c \cap A_3^c) = P(A_1^c)P(A_2^c)P(A_3^c) = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right) = \frac{9}{32}$.
$P(A_1 \cup A_2 \cup A_3) = 1 - \frac{9}{32} = \frac{23}{32}$.

26. C. The probability that the man speaks truth is 0.80.
Let E be the event that the man reports that the number occur is more than 4 in the throwing of the die.
E_1 = the event that the number occurs is more than 4.
E_2 = number occurs is less than or equal to 4.
$P(E_1) = \frac{2}{6} = \frac{1}{3}$. $P(E_2) = \frac{4}{6} = \frac{2}{3}$.
$P(E|E_1) = 0.80$, $P(E|E_2) = 0.20$.
$P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E_2)P(E|E_2)}$.
$P(E|E_1)P(E_!) + P(E|E_2)P(E_2) = 0.80 \times \frac{1}{3} + 0.20 \times \frac{2}{3} = 0.4$.
So $P(E_!|E) = \frac{0.80 \times \frac{1}{3}}{0.4} = \frac{2}{3}$.

27. D. Median = (20th observation + 21st observation)/2 = (50+53)/2 = 51.5.

28. C. Since x takes only the values 0 and 1, $y = x^2 = x$. Therefore,

$Var(y) = Var(x) = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left[\frac{1}{n}\sum_{i=1}^{n} x_i\right]^2 = \frac{1}{n}\sum_{i=1}^{n} x_i - \left[\frac{1}{n}\sum_{i=1}^{n} x_i\right]^2 = \frac{n_1}{n} - \frac{n_1^2}{n^2}$.

29. B. The mean earning per worker of the whole organization is

$\frac{50\times15000+50\times13000+100\times18500}{50+50+100} = \frac{750000+650000+1850000}{200} = 16250.$

(Alternatively, average salary of 100 employees in D_1 and D_2 is $\frac{15000+13000}{2} = 14000$. Therefore, average salary of 200 employees in the whole organization is $\frac{14000+18500}{2} = 16250$.)

30. D.

Weights in Kg	Number of persons	Cumulative frequency
35.5 – 40.5	8	8
40.5 – 45.5	12	20
45.5 – 50.5	25	45
50.5 – 55.5	40	85
55.5 – 60.5	45	130
60.5 – 65.5	60	190
65.5 – 70.5	46	236
70.5 – 75.5	12	248
75.5 – 80.5	2	250

From the frequency distribution, it is clear that $\frac{n}{2} = 125$, and the median class is 55.5 – 60.5.

Modal class is 60.5 – 65.5. The mode is not 60.5.

The values on the left side are more spread out than those on the right side, and hence the distribution is negatively skewed. So the coefficient of skewness would be negative.

31. A. Range is independent of change of origin, but it depends on the scale of measurement.
If each observation is increased by 5, range remains same, that is 20.
If each observation is multiplied by -3, then the new maximum value is -3 times the earlier minimum value and the new minimum value is -3 times the earlier maximum value. Therefore, range becomes 3 × 20 = 60.
If each observation is multiplied by 3 and then 5 added to it, then range becomes $3 \times 20 = 60$.
If each observation is multiplied by 5 and then 3 subtracted from it, then range becomes $5\times 20 = 100$.

32. A. The probability mass function of X is $P(X = x) = \frac{1}{7}$, $x = 7, 12, 15, 20, 23, 25, 30$.
$P(X > 18 | X < 26) = \frac{P(18<x<26)}{P(X<26)} = \frac{P(X=20)+P(X=23)+P(X=25)}{P(X=7)+P(X=12)+P(X=15)+P(X=20)+P(X=23)+P(X=25)} = \frac{3}{6} = \frac{1}{2}$.

33. D. $p =$ probability that an item is defective = 5/25 = 1/5 = 0.2
Let X be the number of defective items in a sample of 4 items selected from a lot.
$X \sim \text{Binomial}(4, 0.2)$.
Probability that he accepts a lot is $P(X = 0) = 0.8^4 = 0.4096$.

34. C. $X \sim \text{Poisson}(\lambda)$. $P(X = 1) = P(X = 2) \Rightarrow \lambda e^{-\lambda} = \frac{\lambda^2}{2} e^{-\lambda} \Rightarrow \lambda = 2$.
$p =$ Probability that a page contains at most one misprint is
$P(X \le 1) = P(X = 0) + P(X = 1) = e^{-2} + 2e^{-2} = 3e^{-2}$.
Let Y be the number of pages containing at most one misprint. $Y \sim \text{Binomial}(500, p)$.
Expected number of pages with at most one misprint is $500 \times 3e^{-2} = 1500e^{-2}$.

35. B. $f(x) = \begin{cases} \lambda e^{-\lambda(x-\alpha)}, & x \ge \alpha \\ 0, & x < \alpha \end{cases}$ Therefore $E(X) = \int_{\alpha}^{\infty} x\, \lambda e^{-\lambda(x-\alpha)} dx = \int_0^{\infty} \lambda(u + \alpha) e^{-\lambda u}\, du = \int_0^{\infty} \lambda u e^{-\lambda u}\, du + \alpha \int_0^{\infty} \lambda e^{-\lambda u}\, du = \frac{1}{\lambda} + \alpha = \frac{\alpha\lambda+1}{\lambda}$.

36. B. $X \sim N(20, 2^2)$. 90th percentile of X is $x_{0.90}$.
$P(X \le x_{0.90}) = 0.90 \Rightarrow P\left(\frac{X-20}{2} \le \frac{x_{0.90}-20}{2}\right) = 0.90 \Rightarrow P\left(Z \le \frac{x_{0.90}-20}{2}\right) = 0.90$,
$Z \sim N(0, 1)$. Given 90th percentile of Z is 1.65. So $\frac{x_{0.90}-20}{2} = 1.65$. $\Rightarrow x_{0.90} = 23.3$. Thus, 90th percentile of X is 23.3.

37. D. Skewness = $\frac{\mu_3}{(\sqrt{\mu_2})^3}$, $\mu_2 = E\big(X - E(X)\big)^2$, $\mu_3 = E\big(X - E(X)\big)^3$.

$E(X) = \int_{-1}^{1} \frac{x}{2}\, dx = 0$. $\mu_3 = E\big(X - E(X)\big)^3 = E(X^3) = \int_{-1}^{1} \frac{x^3}{2} dx = 0$

Skewness = 0.

(Alternatively, since the density is symmetric about 0, the skewness is 0.)

38. A.

y \ x	1	2	Total
0	$0.4 + p$	$0.3 - p$	0.7
1	$0.3 - p$	p	0.3
Total	0.7	0.3	1

$E(X) = 1 \times 0.7 + 2 \times 0.3 = 1.3$.

$E(Y) = 0 \times 0.7 + 1 \times 0.3 = 0.3$.

$E(XY) = 1 \times 0 \times (0.4 + p) + 2 \times 0 \times (0.3 - p) + 1 \times 1 \times (0.3 - p) + 2 \times 1 \times p = 0.3 + p$.

Cov(X, Y) $= 0.3 + p - 1.3 \times 0.3 = p - 0.09$.

$Var(X) = Var(X - 1) = 0.7 \times 0.3 = 0.21$.

$Var(Y) = 0.7 \times 0.3 = 0.21$.

Correlation Corr$(X, Y) = \frac{p - 0.09}{\sqrt{0.21 \times 0.21}} = \frac{p - 0.09}{0.21}$.

Corr(X, Y) $= 0.1 \Rightarrow p = 0.111$.

39. A. $U = aX + b$ and $V = cY + d$. Correlation coefficient between X and V is $\rho_{XV} = \frac{Cov(X,V)}{\sqrt{Var(X)Var(V)}}$.

$Cov(X, V) = Cov(X, cY + d) = c\, Cov(X, Y)$, $Var(V) = c^2 Var(Y)$

So $\rho_{XV} = \frac{c\, Cov(X,Y)}{\sqrt{Var(X) c^2 Var(Y)}} = \frac{c}{|c|} \rho_{XY}$ If $c > 0, \rho_{XV} = \rho_{XY}$.

40. C. Two regression lines are $4x - 5y + c = 0$ and $20x - 9y - d = 0$.

First identify the regression of y on x and the regression of x on y.

The product of the slopes of the regression lines is r^2, which should be less than 1. Hence, the identification of regression lines with both the slopes greater than 1 must be incorrect, and the one with both the slopes less than 1 must be correct. Therefore, first equation is regression of y on x. $y = \frac{4}{5}x + \frac{c}{5}$;

second equation is regression of x on y. $x = \frac{9}{20}\, y + \frac{d}{20}$.

$b_{yx} = \frac{4}{5}$. $b_{xy} = \frac{9}{20}$.

$r^2 = \frac{4}{5} \times \frac{9}{20} = \frac{36}{100}$. So $r = \frac{3}{5}$

Now $b_{yx} = r \times \frac{\sqrt{var(y)}}{\sqrt{Var(x)}} \Rightarrow \frac{4}{5} = \frac{3}{5} \times \left(\frac{\sqrt{var(y)}}{\sqrt{Var(x)}}\right) \Rightarrow \frac{Var(y)}{Var(x)} = \frac{16}{9}$.

Data Interpretation

41. C. Number of candidates obtained marks 60 and above is 120.
Total number of candidates is 250
So the number of candidates who have scored less than 60 is $250 - 120 = 130$.

42. D. The number of candidates passed the examination is 200.
Percentage of candidates passed of the examination is $(200/250) \times 100 = 80$.

43. D. The annual percentage increase is the largest when the ratio of numbers of candidates in consecutive years is the largest. These ratios for the successive years are $\frac{6.7}{6.6} = 1.015$, $\frac{6.85}{6.7} = 1.022$, $\frac{7.05}{6.85} = 1.029$, $\frac{7.2}{7.05} = 1.021$, $\frac{7.45}{7.2} = 1.035$, $\frac{7.7}{7.45} = 1.034$, $\frac{8.05}{7.7} = 1.045$, $\frac{8.2}{8.05} = 1.019$.
The largest of the ratios is $\frac{8.05}{7.47} = 1.045$, which occurs in 2017-18.

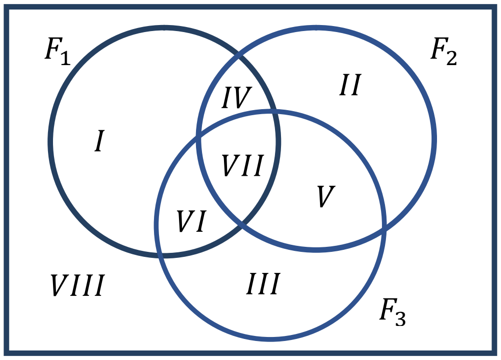
44. C.

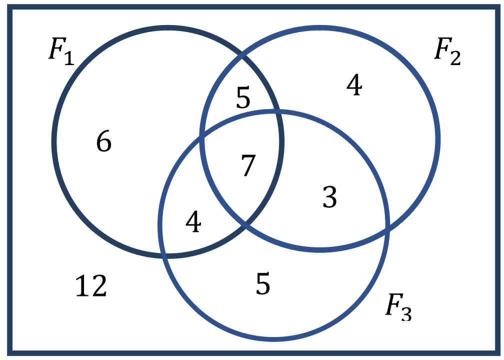
Year	Male (in lakhs)	Female (in lakhs)	Total (lakhs)	Percentage of Female candidates
2011	3.40	3.20	6.60	(3.20/6.60)*100 = 48.5
2012	3.38	3.32	6.70	(3.32/6.70)*100 = 49.6
2013	3.45	3.40	6.85	(3.40/6.85)*100 = 49.6
2014	3.53	3.52	7.05	(3.52/7.05)*100 = 49.9
2015	3.55	3.65	7.20	(3.65/7.20)*100 = 50.7
2016	3.60	3.85	7.45	(3.85/7.45)*100 = 51.7
2017	3.70	4.00	7.70	(4/7.7)*100 = 51.9
2018	3.85	4.20	8.05	(4.20/8.05)*100 = 52.2
2019	3.95	4.25	8.20	(4.25/8.20)*100 = 51.8

No need to calculate percentages for 2011-2014, where the number of male candidates is more than that of female candidates. Percentages to be calculated from 2015 to 2019.

45. A. From 2011 -2014: number of male candidates is more than that of female candidates. Annual difference of female and male candidates for the years 2015 - 2019 are as follows.
2015: 10000
2016: 25000
2017: 30000
2018: 35000
2019: 30000

Diagrams for 46-48:





46. D. The number of students who liked exactly one type of food is $I + II + III = 6 + 4 + 5 = 15$.

47. A. The number of students who liked F_1 and F_2 but not F_3 is the region $IV = 5$.

48. B. The number of students who liked F_1 or F_3 is $I + III + IV + V + VI + VIII = 6 + 5 + 5 + 3 + 4 + 7 = 30$.

49. B. (All calculations are in crores of rupees.)
Total export in 2010-11 is 5000.
Total export in 2011-12 is $5000 \times 1.07 = 5350$.
Export of P3 and P4 in 2010-11 is $5000(0.15 + 0.10) = 1250$.
Export of P3 and P4 in 2011-12 is $5350(0.16 + 0.12) = 1498$.
Total export of P3 and P4 in two years is $1250 + 1498 = 2748$.

50. A. (All calculations are in crores of rupees.)
Total export of P1 and P2 in 2010-11 is $5000 - 1250 = 3750$.
Total export of P1 and P2 in 2011-12 is $5350 - 1498 = 3852$.
From 2010-11 to 2011-12, total export of the commodities P1 and P2 is increased by $(3852 - 3750) \times 100/3750 = 102 \times 100/3750 = 2.72\%$.

51. D. Sales decreased:
From 2007 to 2008,
From 2008 to 2009,
From 2010 to 2011,
From 2014 to 2015,
From 2017 to 2018.

English

52. B.
53. A.
54. B.
55. D.
56. D.
57. A.
58. C.
59. C.
60. A.
61. C.
62. B.

Logical reasoning

63. C.

64. B. The substitution code is:
R - 7
U - 5
B - 8
B - 8
E - 9
R - 7
Thus, RUBBER would be coded as 758897.

65. B.

66. D.

67. C. The cubes cut out from the corners will have three of their faces brown. Since there are 8 corners for a cube, there will be 8 such cubes.

68. B. 1895 is not a leap year. So it will have 1 odd day.
Since 1896 is a leap year, it will add 2 odd days.
Similarly 1987, 1898, 1899, 1900 will add 1,1,1,1 odd days.
These odd days add up to 7.
So the following year (1901) will have the same calendar as 1895.

69. D. The original circular arrangement in clockwise manner is SDARPY. After the exchanges, the arrangement is SYAPRD, in which S is to the left of D.

70. A. Suppose Aaquib is a *Jhoota*. Then the first statement means that Wasim is either a *Sachha* or a *Badla*, and the second statement means that Imran is either a *Badla* or a *Sachha* (but not the same as Wasim). There is no contradiction.

Suppose Wasim is a *Jhoota*. The first statement means that Aaquib is a *Badla*, which means Imran must be a *Sachha*, and this contradicts the second statement.

Suppose Imran is a *Jhoota*. Then the second statement means that Wasim must be a *Sachha*. Therefore Aaquib has to be a *Badla*, which contradicts the first statement.

Since only Aaqib can logically be a *Jhoota*, the correct option is A.