

Institute of Actuaries of India

ACET 2021

Mathematics

1. Let $f: R \rightarrow R$ be a function defined by $f(x) = x^2 - 3x + 4$ for all $x \in R$. Then the pre-image of $\{2\}$ is
- A. $\{5\}$.
 - B. $[1,2]$.
 - C. $\{1,2\}$.
 - D. $\{1\}$.

1 mark

2. Suppose $f(n) = \frac{2f(n-1)+1}{2}$ for $n = 1, 2, \dots$ and $f(1) = 2$. Then $f(51)$ is
- A. 52.
 - B. 26.
 - C. 54.
 - D. 27.

1 mark

3. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 2 \sin^{-1}\left(\frac{1}{2}\right)$ is
- A. $\frac{\pi}{2}$.
 - B. $\frac{\pi}{3}$.
 - C. $\frac{2\pi}{3}$.
 - D. π .

1 mark

4. The set of values satisfying the inequality $4x^2 + x - 3 > 0$ is
- A. $-1 < x < \frac{3}{4}$.
 - B. $x > \frac{3}{4}$ or $x < -1$.
 - C. $-1 \leq x \leq \frac{3}{4}$.

D. $x \geq \frac{3}{4}$ or $x \leq -1$.

2 marks

5. If $\log_{15} 5 = a$, then $\log_{15} 81$ in terms of a is

A. $4(a - 1)$.

B. $4(1 - a)$.

C. $3(a - 1)$.

D. $3(1 - a)$.

1 mark

6. The limit $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k(k+2)}$ is equal to

A. 1.

B. $\frac{3}{2}$.

C. $\frac{1}{2}$.

D. $\frac{3}{4}$.

1 mark

7. If α and β are the roots of the equation $3x^2 - 6x + 1 = 0$, then the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ is

A. $x^2 - 30x + 9 = 0$.

B. $x^2 + 30x + 9 = 0$.

C. $x^2 - 9x + 30 = 0$.

D. $x^2 + 9x + 30 = 0$.

2 marks

8. In the expansion of $(1 + x)^n$, the fifth term is 4 times the fourth term and the fourth term is 6 times the third term. Then n is

A. 9.

B. 10.

C. 11.

D. 12.

3 marks

9. The third order divided difference of the function $f(x) = x^3 - 2x$ with arguments 2,4,9 and 10 is
- A. 1.
 - B. 2.
 - C. 3.
 - D. 4.

2 marks

10. The approximate value of the $I = \int_0^1 \frac{1}{1+x} dx$, using Simpson's one-third rule with interval width $h = \frac{1}{2}$, is
- A. $\frac{17}{36}$.
 - B. $\frac{17}{24}$.
 - C. $\frac{25}{24}$.
 - D. $\frac{25}{36}$.

2 marks

11. The maximum value of the function $f(x) = 4 \sin x + 3 \cos x$ is
- A. 1.
 - B. -3.
 - C. 5.
 - D. ∞ .

1 mark

12. If the curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at $x = 1$, then
- A. $a + 3b = 1$.
 - B. $a + 3b = 0$.
 - C. $3a + b = 1$.
 - D. $3a + b = 0$.

1 mark

13. The derivative of $\log(x + \sin x)$ with respect to $x + \cos x$ is
- A. $\frac{1+\cos x}{(x+\sin x)(1-\sin x)}$.

- B. $\frac{1-\cos x}{(x+\sin x)(1-\sin x)}$.
- C. $\frac{1-\cos x}{(x-\sin x)(1+\cos x)}$.
- D. $\frac{1+\cos x}{(x-\sin x)(1-\cos x)}$.

2 marks

14. The value of the integral $\int x \log 2x \, dx$ is

- A. $\frac{x^2}{2} \log 2x + \frac{x^2}{4} + \text{constant}$.
- B. $\frac{x^2}{2} \log 2x - \frac{x^2}{4} + \text{constant}$.
- C. $\frac{x}{2} \log 2x + \frac{x^2}{4} + \text{constant}$.
- D. $\frac{x}{2} \log 2x - \frac{x^2}{4} + \text{constant}$.

2 marks

15. The value of the integral $\int_{-1}^1 \log \left(\frac{5-x}{5+x} \right) dx$ is

- A. 0.
- B. 1.
- C. 2.
- D. $\frac{4}{5}$.

1 mark

16. The value of the integral $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$ is

- A. $\frac{\pi^2}{16}$.
- B. $\frac{\pi}{32}$.
- C. $\frac{\pi^2}{32}$.
- D. $\frac{\pi}{16}$.

1 mark

17. If $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and if $\vec{b} = 3\vec{i} + 2\vec{j} - \mu\vec{k}$ are perpendicular to each other, then the value of μ is

- A. 0.

- B. 1.
- C. 2.
- D. 3.

1 mark

18. The vectors $a\vec{i} + \vec{j} + \vec{k}$, $\vec{i} + b\vec{j} + \vec{k}$ and $\vec{i} + \vec{j} + c\vec{k}$, where none of the coefficients a , b and c is equal to 1, are coplanar. Then a vector that is perpendicular to all the three vectors is

- A. $a\vec{i} + b\vec{j} + c\vec{k}$.
- B. $(1 - a)\vec{i} + (1 - b)\vec{j} + (1 - c)\vec{k}$.
- C. $\frac{1}{a}\vec{i} + \frac{1}{b}\vec{j} + \frac{1}{c}\vec{k}$.
- D. $\frac{1}{1-a}\vec{i} + \frac{1}{1-b}\vec{j} + \frac{1}{1-c}\vec{k}$.

3 marks

19. The inverse of the matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ (a, b, c are non-zero real numbers) is

- A. $\begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$.
- B. $\frac{1}{abc} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.
- C. $\frac{1}{abc} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
- D. $\frac{1}{abc} \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$.

1 mark

20. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

- A. is one.
- B. is two.
- C. is three.
- D. does not exist.

1 mark

Statistics

21. A five-digit number divisible by 3 is to be formed using the numbers 0, 2, 3, 4, 6, 8 without repetitions. The total number of ways this can be done is
- A. 192.
 - B. 240.
 - C. 288.
 - D. 312.

1 mark

22. A student is to answer 6 out of 8 questions in an examination. How many choices of questions are there if she must answer at least 3 of the first 4 questions?
- A. 14.
 - B. 16.
 - C. 22.
 - D. 28.

1 mark

23. Let A and B be events with $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Then $P(A^c|B^c)$ equals
- A. $\frac{5}{36}$.
 - B. $\frac{5}{24}$.
 - C. $\frac{3}{8}$.
 - D. $\frac{5}{8}$.

1 mark

24. It is known from the past experience that the probability that a new worker who has attended the company's training program will meet the production quota is 0.8 and the corresponding probability is 0.5 for a new worker who has not attended the company's training program. If two-thirds of all new workers attended the training program, what is the probability that a new worker will meet the production quota?
- A. 0.8.
 - B. 0.7.
 - C. 0.75.
 - D. 0.5.

2 marks

25. The mean and the second central moment of two numbers are 8 and 4, respectively. The two numbers are
- A. 4, 12.
 - B. 6, 10.
 - C. 3, 13.
 - D. 5, 11.

1 mark

26. Which of the following statements is true?
- A. If two samples A and B have the same standard deviation, but the mean of B is greater than that of A , then the coefficient of variation of B is greater than that of A .
 - B. If in a certain symmetric distribution, the third quartile is 130 and the median is 112, then the interquartile range is 34.
 - C. If $y = a + bx$, where a and b are constants, then $sd(y) = b.sd(x)$.
 - D. Suppose the highest value 68.2 of a set of 50 observations is changed to 168.2, then the mean of the observations increases by 2.

2 marks

27. Consider the numbers 0, 1, 2, ..., n included in a list with frequencies 1, $\binom{n}{1}$, $\binom{n}{2}$, ..., $\binom{n}{n}$. Then the mean of the numbers in the list is
- A. $\frac{n-1}{2}$.
 - B. $\frac{n}{2}$.
 - C. $\frac{n}{2} + 1$.
 - D. $\frac{n+1}{2}$.

2 marks

28. Consider a set of n numbers, of which two are a and b and the remaining $(n - 2)$ numbers are all equal to $\frac{a+b}{2}$. The mean deviation about mean is
- A. $\frac{|a-b|}{n}$.
 - B. $\frac{|a+b|}{2n}$.
 - C. $|a - b|$.
 - D. $\frac{|a-b|}{2}$.

1 mark

29. Consider a biased coin so that $P(\text{head}) = 2P(\text{tail})$. Let X denote the total number of heads obtained in three independent tosses of the coin. Then $P(1 < X \leq 3)$ equals

- A. $\frac{8}{27}$.
- B. $\frac{12}{27}$.
- C. $\frac{26}{27}$.
- D. $\frac{20}{27}$.

1 mark

30. Let X be a discrete random variable which takes the values $-1, 0$ and 1 . If $P(X = 0) = 1/4$ and $E(X) = 1/4$, then the variance of X is

- A. $\frac{10}{16}$.
- B. $\frac{11}{16}$.
- C. $\frac{3}{4}$.
- D. $\frac{5}{8}$.

2 marks

31. A certain kind of sheet metal has, on the average 2 defects per 5 square-foot. It is assumed that the number of defects follows the Poisson distribution. Then the probability that a 10 square-foot sheet of the metal will have at most two defects is

- A. $5e^{-2}$.
- B. $2e^{-2}$.
- C. $6e^{-4}$.
- D. $13e^{-4}$.

1 mark

32. Which of the following statements is true?

- A. The expected number of points to be obtained from a throw of a fair die is 3.
- B. For a random variable X , $(E(X))^2$ is always larger than $E(X^2)$.
- C. Let X be a random variable such that $E(X - a)^2$ exists for all real a . Then $E(X - a)^2$ has a maximum when $a = E(X)$.
- D. If a discrete random variable X assumes only the values 2, 3, 5, 7, 9, 10 with equal probability, then $P[|X - 8| < 4] = \frac{2}{3}$.

33. The cumulative distribution function of a random variable X is given by

$$F(x) = 1 - e^{-x} - xe^{-x}, \quad 0 \leq x < \infty.$$

The mode of the distribution is

- A. 0.5.
- B. 1.
- C. 2.
- D. e .

2 marks

34. Suppose X follows a normal distribution with mean μ and variance σ^2 . Then which of the following statements is true?

- A. $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ depends only on the value of μ .
- B. $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ depends on the values of both μ and σ .
- C. $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$ does not depend on the values of μ and σ .
- D. $P(\mu \leq X \leq \mu + 2\sigma)$ depends on the value of σ .

1 mark

35. Consider a random rectangle whose sides are X and $(1 - X)$. Let X have the probability density function $f(x) = 3x^2$, $0 < x < 1$, zero elsewhere. The expected value of the area of the rectangle is

- A. 0.15.
- B. 0.12.
- C. 0.10.
- D. 0.1875.

1 mark

36. Let X be a continuous random variable with probability density function

$$f(x) = \frac{3}{7} \exp\left(-\frac{3}{7}x\right), \quad x > 0.$$

Then $P\left[X \geq \frac{17}{5} \mid X \geq 2\right]$ equals

- A. $\exp\left(-\frac{51}{35}\right)$.
- B. $\exp\left(-\frac{6}{7}\right)$.
- C. $\exp\left(-\frac{3}{5}\right)$.

D. $\exp(-1)$.

1 mark

37. If X is a standard normal variable and $Y = X^2$, then the correlation coefficient between X and Y is
- A. 0.
 - B. 0.8.
 - C. 0.5.
 - D. 1.

2 marks

38. The value of k for which the function given by

$$f(x, y) = kxy, \text{ for } x = 1, 2, 3; \ y = 1, 2, 3$$

can serve as a joint probability distribution is

- A. $\frac{1}{36}$.
- B. $\frac{1}{14}$.
- C. $\frac{1}{12}$.
- D. $\frac{1}{18}$.

1 mark

39. Which of the following statements is true?

- A. If correlation coefficient between X and Y is positive then the correlation between $-X$ and $-Y$ is negative.
- B. If the random variables X and Y are independent, then $\text{Var}(X - Y) = \text{Var}(X) - \text{Var}(Y)$.
- C. Let X and Y be independent random variables with standard deviations σ_x and σ_y , respectively, then the correlation coefficient between X and $X - Y$ is $\sigma_y / \sqrt{\sigma_x^2 + \sigma_y^2}$.
- D. If X and Y have the same standard deviations, then $X + Y$ and $X - Y$ are uncorrelated.

3 marks

40. Two variables x and y have the least squares regression lines $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$. The mean values of the variables are
- A. $(\bar{x}, \bar{y}) = (-2, 1)$.
 - B. $(\bar{x}, \bar{y}) = (1, -2)$.

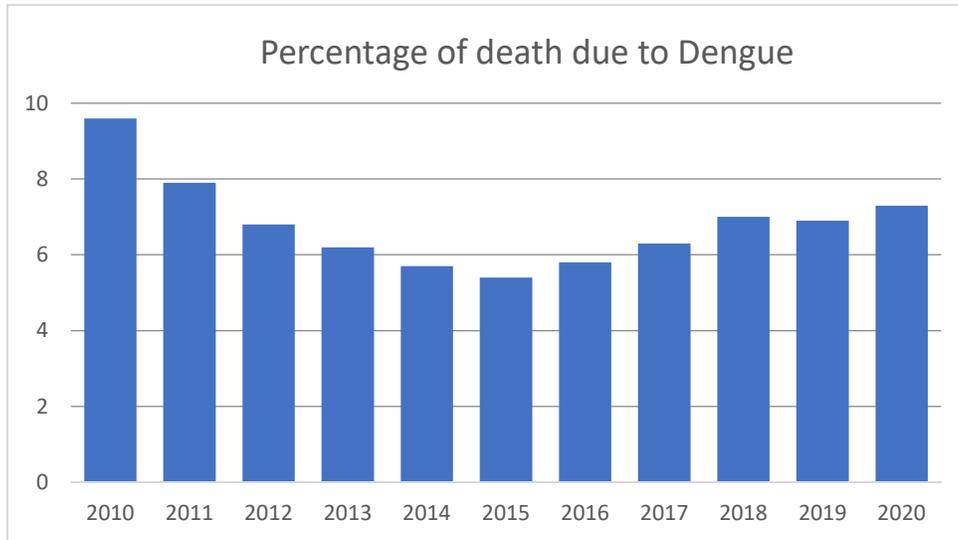
C. $(\bar{x}, \bar{y}) = (1, -1)$.

D. $(\bar{x}, \bar{y}) = (-1, 1)$.

1 mark

Data Interpretation

Answer Questions 41 and 42 based on the following graph, which represents the percentage of deaths due to dengue in a certain region in the years from 2010 to 2020.



41. In which year is the percentage of death due to dengue the second smallest?

- A. 2014.
- B. 2015.
- C. 2016.
- D. 2017.

1 mark

42. In how many years is the percentage of death below 6%?

- A. 3.
- B. 4.
- C. 5.
- D. 7.

1 mark

Answer Questions 43 and 44 based on the data given the following table. The table gives the numbers of Telephone calls received in 250 one-minute intervals.

No. of calls	0	1	2	3	4	5	6	7	Total
Frequency	10	25	40	55	50	40	25	5	250

43. The percentage of time intervals with more than 3 calls is
- A. more than 52.
 - B. 48.
 - C. below 40.
 - D. 46.

1 mark

44. The mode of the distribution of number of telephone calls is
- A. 55.
 - B. 40.
 - C. 2.
 - D. 3.

1 mark

Answer Questions 45-48 based on the data given the following table, which gives the number of permanent workers of an organization and the percentage of permanent and casual workers over the period 2008 -2016.

Year	Number of Permanent workers	Percentage of total workers	
		Permanent	Casual
2008	320	80	20
2009	336	80	20
2010	351	78	22
2011	345	75	25
2012	336	70	30
2013	340	68	32
2014	338	65	35
2015	377	65	35
2016	372	62	38

45. The percentage increase in total workers of the organization from 2008 to 2016 is
- A. 16.25%.
 - B. 40%.
 - C. 50%.
 - D. 55%.

2 marks

46. The largest percentage of increase of casual workers over the previous year occurred in

- A. 2010.
- B. 2011.
- C. 2012.
- D. 2016.

2 marks

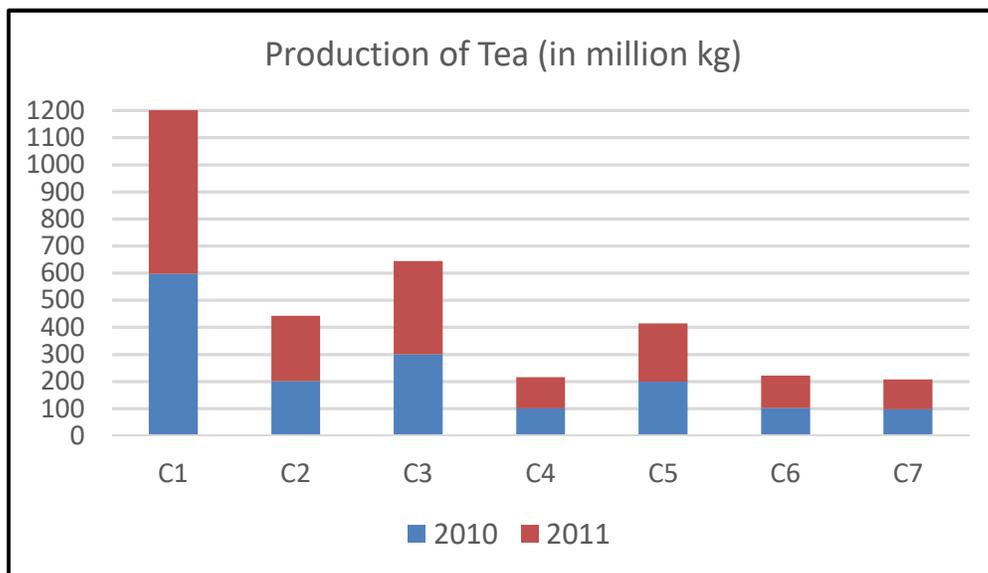
47. The difference between the numbers of permanent and casual workers is minimum in the year
- A. 2008.
 - B. 2012.
 - C. 2014.
 - D. 2016.

2 marks

48. The percentage increase of the number of casual workers of the organization from 2008 to 2016 is
- A. 90%.
 - B. 120%.
 - C. 185%.
 - D. 200%.

2 marks

Answer Questions 49 to 51 based on the chart given below, which shows production of tea (in million kg) of seven countries C1, C2, C3, C4, C5, C6 and C7 in the years 2010 and 2011.



49. The country having lowest percentage of increase of tea production from 2010 to 2011 is

- A. C1.
- B. C2.
- C. C5.
- D. C6.

1 mark

50. Which country has the least tea production in 2010?

- A. C4.
- B. C5.
- C. C6.
- D. C7.

1 mark

51. The total production of tea of seven countries in 2010

- A. is less than 1400 million kg.
- B. lies between 1400 and 1550 million kg.
- C. lies between 1550 and 1650 million kg.
- D. more than 1650 million kg.

1 mark

English

52. Choose the word closest in meaning to the word 'Belligerent'.

- A. Restless.
- B. Noisy.
- C. Aggressive.
- D. Pretty.

1 mark

53. Choose the word(s) closest in meaning to the word 'Fusillade'.

- A. Official files.
- B. Fussy footballers.
- C. A discharge of firearms.
- D. Barricade.

1 mark

54. Select the word that is opposite in meaning to the word "Jittery".

- A. Confident.
- B. Winner.
- C. Warrior.
- D. Shaken.

1 mark

55. Select the word that correctly completes the following sentence: I am expecting a compensation _____ with my experience.

- A. Commensal.
- B. Commensurate.
- C. Equal.
- D. Proportionate.

1 mark

56. Select the word that is closest in meaning to the word 'somnolence'.

- A. Loudness.
- B. Boredom.
- C. Sleepiness.

D. Silence.

1 mark

57. Select the word that is opposite in meaning to the word “Hostile”.

- A. Friendly.
- B. Tolerant.
- C. Kind.
- D. Gentle.

1 mark

58. If you took an extra sheet and then found you didn’t need it, you would

- A. Keep it back.
- B. Put it up.
- C. Put it back.
- D. Keep it up.

1 mark

59. Which of these pairs has the same relationship as: ‘Colourful, colourless’

- A. Frigid, gelid.
- B. Torrid, horrid.
- C. Smelly, fragrant.
- D. Torrid, frigid.

1 mark

60. Select the incorrect sentence.

- A. He said me the tickets had arrived.
- B. What are you here for?
- C. I don't believe you, but I'm listening.
- D. You're wilfully misunderstanding me.

2 marks

61. Select the incorrect sequence.

- A. Small, medium, big, large.
- B. Tropical, topical, optical, clear.
- C. Graceful, deft, uncoordinated, clumsy.

D. Venal, venial, proper, praiseworthy.

2 marks

Read the passage below and answer Question No. 62.

India's role on the international stage was moulded by history rather than by the pomp and circumstances of war. As a separate entity of the British empire, India became a founder member of the League of Nations in the same way as Australia and Canada, and thus acquired a somewhat unique international status long before independence.

During the days of the League when India's interests were affected, such as in matters of trade or the position of Indians overseas, the Indian delegation did not hesitate to make known its independent position, for what it was worth. Some Indian delegates were in demand for chairing meetings riddled with contention and controversy, because of their reputation for impartiality and talent for mediation. These were the small beginnings of India's later entry into the United Nations in 1945, again as a founder member before independence.

The drafting of Chapter IX and X of the UN Charter, which deal with international social and economic cooperation, was entrusted to a group that was presided over by an Indian. It is important, however, to bear in mind the fact that the Government of independent India had no part in the drafting of the UN Charter, although it accepted the obligations contained therein. The UN was primarily the creation of three powers - USA, UK and Russia.

The size, population, resources and potential of India lent it the weight of a medium power in the functioning of the United Nations. The fact that India was among the first nations to liberate itself from the imperialist domination through non-violent means endowed it with a moral obligation to work in and outside the UN for the independence of nations still under colonial rule. India has served on the Security Council for 10 years or 5 terms, on the Trusteeship Council for 12 years or 4 terms and on the Economic and Social Council for 21 years or 7 terms.

- I. Indian delegates were in demand for chairing controversial sessions because
 - i. They were well respected.
 - ii. They had a talent for mediation.
 - iii. They were experienced in international matters.
- II. Among the following, the most suitable title for the passage is
 - i. India's International Relations.
 - ii. Early History of India at UN.
 - iii. India and Her Neighbours.
- III. India acquired a unique status in the UN even before independence because
 - i. India was a founder member of the League of Nations.
 - ii. An Indian drafted Chapters IX and X of the UN Charter.
 - iii. All of the above.

62. The correct answers to I, II and III are

- A. i, ii, iii, respectively.
- B. ii, ii, iii, respectively.
- C. ii, i, iii, respectively.
- D. ii, i, i, respectively.

3 marks

Logical Reasoning

63. In a mall 200 people are interviewed. 140 have come to the mall for shopping, 120 have come for eating and 80 have come to do both. The number of people who have come to the mall for at least one of the two activities mentioned above and the number of people who have come for neither of the two are _____ and _____ respectively.
- A. 100 and 40.
 - B. 180 and 20.
 - C. 60 and 0.
 - D. 140 and 60.

1 mark

64. Arrange the following words in a logical sequence:
1. Doctor 2. Sickness 3. Medicine 4. Pharmacy Shop 5. Diagnosis 6. Infection
- A. 6, 2, 1, 5, 3, 4.
 - B. 6, 1, 2, 5, 1, 3.
 - C. 6, 2, 1, 5, 4, 3.
 - D. 4, 3, 5, 6, 1, 2.

1 mark

65. At 8.30 am, what is the angle in degrees between the minute hand and the hour hand in a clock?
- A. 80.
 - B. 75.
 - C. 60.
 - D. 105.

1 mark

66. In the questions below are given few statements followed by two conclusions. You have to take the given statements to be true even if they seem to be at variance from commonly known facts. Read all the conclusions and then decide which of the given conclusions logically follow from the given statements disregarding commonly known facts.

Statements:

All cashew nuts are peanuts.

No peanut is a cactus.

Some cactuses are almonds.

Conclusions:

- I. All almonds can never be peanuts
- II. Some cactuses are cashew nuts

- A. Only conclusion I follows.
- B. Only conclusion II follows.
- C. Neither conclusion I not conclusion II follows.
- D. Both conclusion I and conclusion II follow.

1 mark

67. Consider the group of words:

Cub, Calf, Foal, Cat

Which of these is the odd one out?

- A. Cub.
- B. Calf.
- C. Foal.
- D. Cat.

1 mark

68. The year 2016 had the same calendar as

- A. 1980.
- B. 1984.
- C. 1986.
- D. 1988.

2 marks

69. A cube is painted on all the six sides and then cut into small identical cubes. Amongst the smaller cubes there were eight cubes which had none of the sides painted. The number of small cubes which had exactly one sides painted was

- A. 12.
- B. 24.
- C. 16.
- D. 18.

1 mark

70. The following facts are known about a group of persons.

A is the only son of C.

G is the father of J.

A is married to M.
A and J are siblings.
P is the brother of M.
C has only one son.
R is the son of A.
P is the father of X.
Z is the daughter of J.
P has no daughters.

How many female members are there among the persons mentioned above?

- A. 1.
- B. 2.
- C. 3.
- D. 4.

2 marks
