

# Institute of Actuaries of India

ACET October 2020

## Mathematics

1. The approximate value of  $(1.02)^8$ , rounded up to four decimal places, is
- A. 1.1712.
  - B. 1.1716.
  - C. 1.1717.
  - D. 1.1715.
- 2 marks
2. The abscissa of the point on the curve  $f(x) = \sqrt{x}(7x - 6)$ , where the tangent is parallel to  $x$ -axis is
- A.  $\frac{7}{2}$ .
  - B.  $\frac{7}{6}$ .
  - C.  $\frac{6}{7}$ .
  - D.  $\frac{2}{7}$ .
- 1 mark
3. If  $g(x) = \sqrt{x + \sqrt{x + \sqrt{x} + \dots \infty}}$ , the value of  $g'(x)$  is equal to
- A.  $2g(x) - 1$ .
  - B.  $2g(x) + 1$ .
  - C.  $(2g(x) + 1)^{-1}$ .
  - D.  $(2g(x) - 1)^{-1}$ .
- 1 mark
4. If  $\cos^{-1} x + \cos^{-1} y = \pi$ , then  $\sin^{-1} x + \sin^{-1} y$  is equal to
- A.  $\pi$ .
  - B. 0.
  - C.  $2\pi$ .
  - D.  $\frac{\pi}{2}$ .
- 1 mark
5. If  $X = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ ,  $10Y = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & c \\ 1 & -2 & 3 \end{bmatrix}$  and  $Y = X^{-1}$ , then the value of  $c$  is
- A. 5.
  - B. -1.
  - C. -2.
  - D. 2.
- 1 mark

6. The value of  $\sqrt[3]{9} \cdot \sqrt[9]{9} \cdot \sqrt[27]{9} \dots$  is equivalent to
- 1.
  - 3.
  - 9.
  - 27.
- 1 mark
7. If  $[x]$  is the greatest integer less than or equal to  $x$  and  $-1 \leq a < 0, 0 \leq b < 1, 1 \leq c < 2$ , the value of  $\begin{vmatrix} [a] + 1 & [b] & [c] \\ [a] & [b] + 1 & [c] \\ [a] & [b] & [c] + 1 \end{vmatrix}$  is
- 1.
  - 0.
  - 1.
  - 2.
- 1 mark
8. The value of  $1 + i^2 + i^4 + i^6 + \dots + i^{2n}$ , where  $i$  is the usual square-root of  $-1$  and  $n$  is a natural number,
- is 0.
  - is positive.
  - is negative.
  - cannot be specified.
- 1 mark
9. Two finite sets have  $m$  and  $n$  number of elements. The number of subsets of the first set is 112 more than that of the second set. Then  $m - n$  is equal to
- 1.
  - 9.
  - 3.
  - 5.
- 1 mark
10. The value of  $\int_0^2 e^{-x+[x]} dx$ , where  $[x]$  is the greatest integer less than or equal to  $x$ , is equal to
- $e^2 - 1$ .
  - $e^2 + 1$ .
  - $(e^2 + 1)(e - 1)$ .
  - $(e^2 - 1)(e + 1)$ .
- 1 mark

11. If the vectors  $a\vec{i} + \vec{j} + \vec{k}$ ,  $\vec{i} + b\vec{j} + \vec{k}$ ,  $\vec{i} + \vec{j} + c\vec{k}$  are coplanar and  $a$  and  $b$  are distinct, then the rank of the matrix  $\begin{pmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{pmatrix}$  is

- A. 0.
- B. 1.
- C. 2.
- D. 3.

1 mark

12. The numbers  $\tan\left(\frac{10\pi}{3}\right)$ ,  $\tan\left(\frac{9\pi}{4}\right)$  and  $\tan\left(\frac{7\pi}{6}\right)$

- A. are in AP.
- B. are in GP.
- C. are in HP.
- D. have sum equal to 1.

1 mark

13. The least positive integer  $m$  for which  $\left(\frac{1-i}{1+i}\right)^m + 1 = 0$  is

- A. 2.
- B. 3.
- C. 1.
- D. 4.

1 mark

14. The set of real values of  $x$  satisfying  $\log_{\frac{1}{2}}\left(\frac{x+2}{x}\right) \leq 1$  is

- A.  $(-\infty, -4] \cup (0, \infty)$ .
- B.  $[4, \infty)$ .
- C.  $(-\infty, -4]$ .
- D.  $(0, \infty)$ .

2 marks

15. The value of  $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots$  equals to

- A. 3.
- B.  $\frac{3}{2}$ .
- C.  $\frac{5}{2}$ .
- D. 2.

2 marks

16. Let  $f(x)$  be a continuous function such that  $f(a - x) + f(x) = 0, \forall x$  in  $[0, a]$ , then the value of the integral  $\int_0^a \frac{dx}{1+e^{f(x)}}$  equals to

- A.  $\frac{a}{2}$ .
- B.  $f(a)$ .
- C.  $a$ .
- D.  $\frac{f(a)}{2}$ .

2 marks

17. The unit vector perpendicular to both the vectors  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$  and making an acute angle with the vector  $\hat{k}$  is

- A.  $\frac{1}{\sqrt{26}}(-4\hat{i} + \hat{j} + 3\hat{k})$ .
- B.  $\frac{1}{\sqrt{26}}(4\hat{i} - \hat{j} + 3\hat{k})$ .
- C.  $\frac{1}{\sqrt{26}}(4\hat{i} - \hat{j} - 3\hat{k})$ .
- D.  $\frac{1}{\sqrt{26}}(-4\hat{i} - \hat{j} + 3\hat{k})$ .

3 marks

18. The digit in the unit place in the sum  $1! + 2! + 3! + 4! + 5! + \dots + 99!$  is

- A. 5.
- B. 4.
- C. 3.
- D. 2.

2 marks

19. The value of  $I = \int_{\alpha}^{\beta} \frac{|x|}{x} dx$  ( $\beta > \alpha$ ) is

- A.  $\beta - \alpha$ .
- B.  $\alpha - \beta$ .
- C.  $|\alpha| - |\beta|$ .
- D.  $|\beta| - |\alpha|$ .

2 marks

20. If  $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} X \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = I_2$ , where  $X$  is a matrix and  $I_2$  is the  $2 \times 2$  identity matrix, then  $X$  is equal to

- A.  $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ .
- B.  $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ .
- C.  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ .
- D.  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ .

3 marks

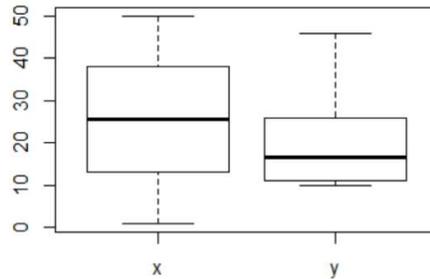
## Statistics

21. Suppose  $x_1, x_2, x_3, \dots, x_n$  are the  $n$  observations so that  $\sum x_i = 80, \sum x_i^2 = 400$ . One possible value of  $n$  is

- A. 5.
- B. 17.
- C. 12.
- D. 15.

1 mark

22. Box plots of two data sets, labelled as  $x$  and  $y$ , are given below.



The summary statistic of  $y$  that exceeds the corresponding statistic of  $x$  is

- A. the third quartile.
- B. the inter-quartile range.
- C. the first quartile.
- D. the minimum.

1 mark

23. Let  $X = \{2, 3, 4, \dots, 21, 22, 23\}$ . A number is chosen at random from the set  $X$  and it is found to be prime. The probability that it is more than 13 is

- A.  $9/22$ .
- B.  $3/22$ .
- C.  $1/3$ .
- D.  $1/9$ .

1 mark

24. In a random arrangement of the letters of the word UNIVERSITY, the probability that the two I's do not come together is

- A.  $1/10$ .
- B.  $9/10$ .
- C.  $1/5$ .
- D.  $4/5$ .

1 mark

25. If  $P(X) = \frac{2}{5}, P(Y) = \frac{3}{10}, P(X \cap Y) = \frac{1}{5}$ , then the value of  $P(X^c | Y^c)$  is

- A.  $5/7$ .
- B.  $1/2$ .
- C.  $3/7$ .
- D.  $6/7$ .

1 mark

26. For the data set

6, 9, 10, 10, 10, 15, 15, 25, 35,

- A. the mean is larger than the mode but smaller than the median.
- B. the median is larger than the mode but smaller than the mean.
- C. the mode is larger than the mean but smaller than the median.
- D. the median is equal to the mode, which is smaller than the mean. 1 mark

27. A card is drawn from a pack of cards. The card is replaced and the pack is shuffled again. If this process is executed six times, the probability that 2 spades, 2 clubs and two red cards are drawn is

- A.  $\frac{45}{2} \times \left(\frac{3}{4}\right)^4$ .
- B.  $\frac{90}{2^{10}}$ .
- C.  $90 \times \left(\frac{1}{4}\right)^6$ .
- D.  $\frac{\binom{13}{2} \times \binom{13}{2} \times \binom{26}{2}}{\binom{52}{6}}$ . 2 marks

28. Urn A contains 2 white balls and 1 black ball, whereas urn B contains 1 white ball and 5 black balls. A ball is drawn at random from urn A and placed in urn B. A ball is then drawn from urn B. It happens to be white. The probability that the ball transferred was white is

- A. 4/5.
- B. 1/5.
- C. 2/7.
- D. 5/7. 1 mark

29. If  $X \sim \text{Bin}\left(8, \frac{1}{2}\right)$ , then  $P(|X - 4| \leq 2)$  is

- A. 9/128.
- B. 119/128.
- C. 91/128.
- D. 109/128. 2 marks

30. A bag contains 5 red and 4 blue balls. One ball is drawn at random and the ball is not replaced. Its colour is also not noted. Next another ball is drawn at random. The probability of drawing a red ball in the second draw is

- A. 1/2.
- B. 4/9.
- C. 5/9.
- D. 5/8. 1 mark

31. The first of two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with sum 3900 and sum of squares 64200, the standard deviation of the second group is
- A. 4.
  - B. 3.67.
  - C. 16.
  - D. 4.5.
- 2 marks
32. Replacement times for CD players are normally distributed with a mean of 7 years 7 months and a standard deviation of 1 year 6 months. If you are the manufacturer and want to provide a warranty such that about 98% of the CD players need replacement *after* the warranty expires, how long should the warranty period approximately be? (You can use the fact that for a standard normal random variable  $Z$ ,  $P(Z \leq 2.05375) = 0.98$ .)
- A. 2 years 11 months.
  - B. 4 years 8 months.
  - C. 4 years 6 months.
  - D. 10 years 8 months.
- 2 marks
33. Security cameras in a railway reservation office indicate that the number of customers arriving before the opening of counters on any given day has the Poisson distribution with mean 3. The minimum number of counters it should open so that there is at least 90% chance of all the waiting customers being immediately served is
- A. 3.
  - B. 4.
  - C. 5.
  - D. 6.
- 3 marks
34. It is known that 15% of all items of a particular kind produced in a factory are defective. If defects occur independently in all items, then the expected number of items to be sampled serially in order to get the first non-defective item is
- A. 0.176.
  - B. 1.176
  - C. 5.667.
  - D. 6.667.
- 1 mark
35. If the two linear regression equations involving the variables  $Y$  and  $X$  are  $4X - 5Y + 33 = 0$  and  $20X - 9Y - 107 = 0$ , and the variance of  $X$  is 9, the standard deviation of  $Y$  is
- A. 1.44.
  - B. 4.
  - C. 3.
  - D. 16.
- 3 marks

36. Two unbiased die are thrown. The expected value of the sum of the numbers on the faces that turn up is

- A. 6.
- B. 6.5.
- C. 7.
- D. 7.5.

1 mark

37. Suppose the random variable  $X$  has the probability density function

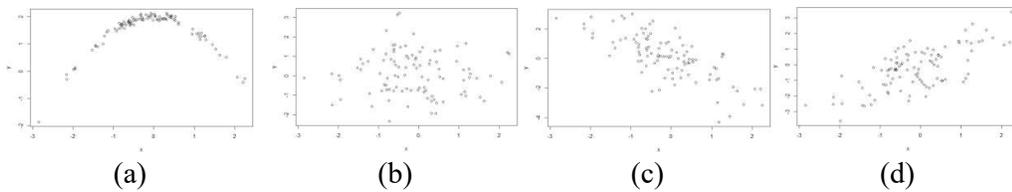
$$f(x) = \begin{cases} ce^{x/3}, & x \leq 0, \\ ce^{-x/3}, & x > 0, \end{cases}$$

for some positive constant  $c$ . The value of  $P(X > 6|X > 0)$  is

- A.  $e^{-2}$ .
- B.  $ce^{-2}$ .
- C. 0.
- D.  $1 - e^{-2}$ .

2 marks

38. Scatter plots of four paired data sets are shown below.



The data set having largest correlation between the variables is

- A. (a).
- B. (b).
- C. (c).
- D. (d).

1 mark

39. A thousand candidates appear for an examination. Their scores are independent and have the continuous uniform distribution over the range 0 to 100. The variance of the number of candidates having score above 30 is

- A. 210.
- B. 700.
- C.  $25/3$ .
- D. 300.

2 marks

40. The random variables  $X$  and  $Y$  are independent, and have variances 3 and 4, respectively. Then the covariance of  $Z = X - Y$  and  $X$  is

- A.  $-3$ .
- B. 3.
- C. 4.
- D.  $-4$ .

1 mark

## Data Interpretation and Data Visualization

Answer the questions 41-43 based on the following information.

A team in a five-a-side cricket tournament consists of Rahul, Virat, Rohit, Laxman and Vijay. The table given below represents the runs scored by each player in four matches of the tournament.

	Match 1	Match 2	Match 3	Match 4
Rahul		90		45
Virat	78	65		50
Rohit			100	
Laxman	67	68	15	48
Vijay	50		70	
Total	240	270	210	170

The two missing values in each column are the two lowest individual scores for the team, which are strictly smaller than the three reported scores. No missing value is more than 10% of the total runs made by the team in that match, given in the last row of that column.

41. Based on the partial information given above, the maximum possible contribution made by Rahul to the aggregate team score in the four matches (taken together) is

- A. 15.2%.
- B. 19.4%.
- C. 20.2%.
- D. 24.2%.

2 marks

42. Based on the partial information given above, the player who has the lowest value of the minimum possible aggregate runs (in the four matches taken together) is

- A. Rahul.
- B. Rohit.
- C. Laxman.
- D. Vijay.

2 marks

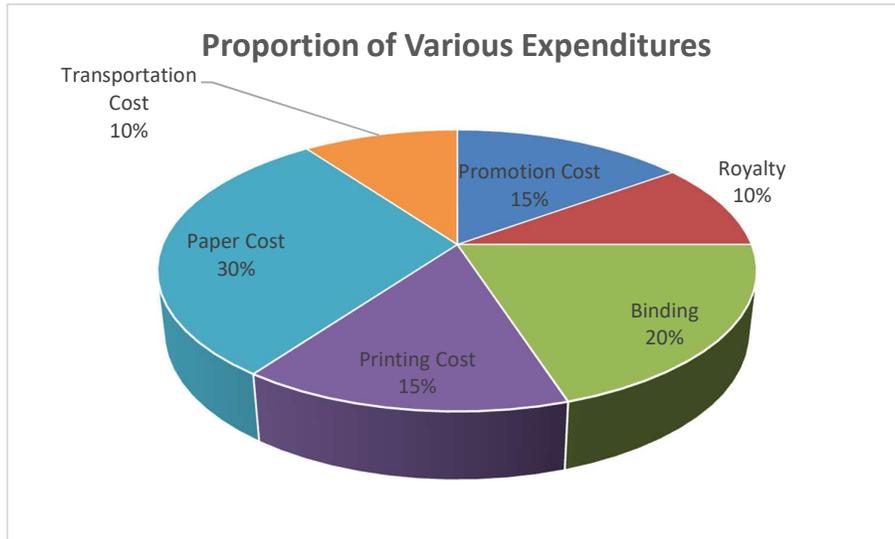
43. Suppose the players are ranked 1 to 5 based on the total runs scored by them in the four matches, with the highest scorer getting rank 1. If it is known that no two players scored exactly the same number of total runs, then the number of players whose ranks can be exactly determined from the incomplete information is

- A. 0.
- B. 1.
- C. 2.
- D. 3.

2 marks

Answer questions 44-46 based on the following pie chart.

Pie chart showing the break-up of expenditure incurred in publishing a book



44. If for a certain quantity of books the publisher pays Rs. 25000 as printing cost, then the amount of royalty to be paid for these books is approximately

- A. 16667.
- B. 10000.
- C. 25000.
- D. 37500.

1 mark

45. The price of the book is marked 15% above the cost of production (including all components given in the chart). If that price is Rs. 200, the cost of paper used in a single copy of the book is approximately

- A. Rs. 100.
- B. Rs. 69.
- C. Rs. 60.
- D. Rs. 52.2.

1 mark

46. Royalty on the book is less than the printing cost by approximately

- A. 33.3%.
- B. 66.7%.
- C. 50%.
- D. 40%.

1 mark

Answer questions 47-49 based on the data given in the following table.

Expenditure of a company (in Rs. Lakh) per annum over various heads in five years

Year	Item of expenditure				
	Salary	Fuel and transport	Bonus	Interest on loans	Taxes
2015	188	88	3.00	13.4	73
2016	242	102	2.52	22.5	98
2017	224	91	3.84	31.6	64
2018	236	123	3.68	26.4	78
2019	320	132	3.96	39.4	88

47. The total amount of bonus paid by the company during the given period, as a percentage of the total amount of salary paid during the same period, is approximately

- A. 0.7%.
- B. 1.2%.
- C. 1.4%.
- D. 0.9%.

1 mark

48. The ratio between the total expenditure on taxes for all the years and the total expenditure on fuel and transport for all the years respectively is approximately

- A. 6 : 7.
- B. 5 : 6.
- C. 4 : 5.
- D. 3 : 4.

1 mark

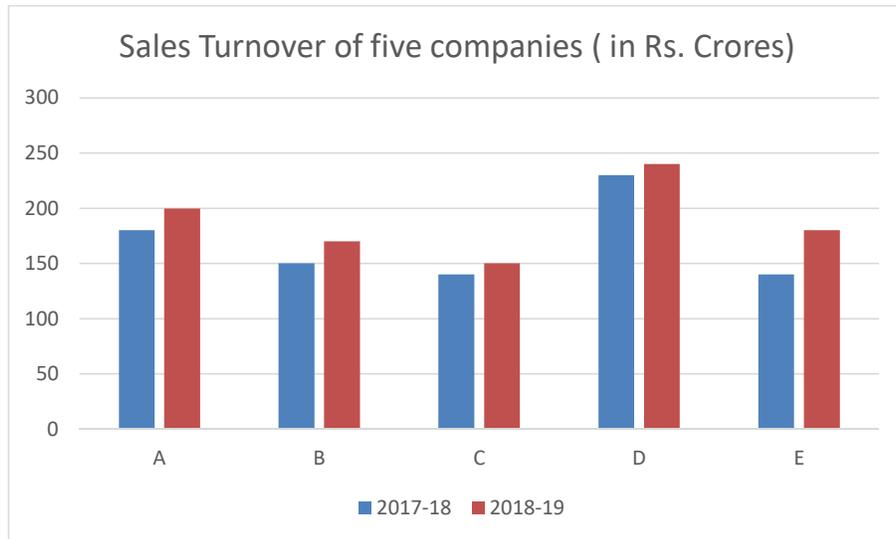
49. The largest percentage increase in any particular year (over the previous year) happened under the head

- A. Salary.
- B. Fuel and transport.
- C. Bonus.
- D. Interest on loans.

2 marks

Answer questions 50-51 based on the information given in the following bar chart.

Sales turnover of five companies (in Rs. Crores) in two successive years



50. The excess of sales turnover in 2018-2019 over 2017-2018 would have been the same for companies B and C if the 2018-19 turnover of Company C had been

- A. Rs. 155 crores.
- B. Rs. 160 crores.
- C. Rs. 165 crores.
- D. Rs. 170 crores.

1 mark

51. The average sales turnover of all the five companies put together differed between the years 2018-2019 and 2017-2018 by

- A. Rs. 10 crores.
- B. Rs. 15 crores.
- C. Rs. 20 crores.
- D. Rs. 30 crores.

1 mark

## English

52. Select the word which is not a synonym of "Prosecute":
- A. Litigate.
  - B. Arraign.
  - C. Sue.
  - D. Halt.
- 1 mark
53. Select the word which is not a synonym of "Debacle":
- A. Eruption.
  - B. Fiasco.
  - C. Collapse.
  - D. Disaster.
- 1 mark
54. Select the word which is not an antonym of "Plausible":
- A. Unlikely.
  - B. Farfetched.
  - C. Preferred.
  - D. Impossible.
- 1 mark
55. Select the word which is not an antonym of "Voracious":
- A. Satisfied.
  - B. Hungry.
  - C. Content.
  - D. Glutted.
- 1 mark
56. Select the most appropriate word(s) to fill the blank in " 'The criminal escaped \_\_\_\_\_ police custody while he was being taken to the court '":
- A. through
  - B. from
  - C. out of
  - D. in spite of
- 1 mark
57. Select the correct sentence:
- A. Beware from that man, he can stoop at any level.
  - B. Beware from that man, he can stoop to any level.
  - C. Beware of that man, he can stoop to any level.
  - D. Beware of that man, he can stoop at any level.
- 1 mark

58. One who pays too much attention to his clothes and appearance is called

- A. Amateur.
- B. Dandy.
- C. Pauper.
- D. Stoic.

1 mark

59. One who is not very sophisticated or socially savvy is called

- A. Bumpkin.
- B. Dumb.
- C. Immature.
- D. Hedonist.

1 mark

60. Given below are five parts of a single sentence marked as P, Q, R, S and T. Rearrange them so that they constitute a meaningful sentence.

The students

- P: to complain about their marks but
- Q: were surprised to be called in
- R: were satisfactorily addressed after that
- S: were hesitant to enter the principal's office
- T: by the principal himself and the issues

- A. RSQPT.
- B. SPQTR.
- C. QTRPS.
- D. STRPQ.

2 marks

61. Select the correct sentence:

- A. Since my tea was cold, I heated it in a microwave oven.
- B. My tea was cold, I heated it in a microwave oven.
- C. Because my tea was cold, so I heated it in a microwave oven.
- D. As my tea was cold, so I heated it in a microwave oven.

2 marks

Read the passage below and answer Question No. 62.

In India, President's rule is the suspension of State Government and imposition of direct Central Government rule in a state. Under Article 356 of the Constitution of India, in the event that a State Government is unable to function according to constitutional provisions, the Central Government can take direct control of the state machinery. Subsequently, executive authority is exercised through the centrally appointed Governor, who has the authority to appoint other administrators to assist him. Under normal circumstances, a state Government is run by an elected council of ministers responsible to the state's legislative assembly (Vidhan Sabha). The council is led by the Chief Minister, who is the de facto Chief Executive of the state; the Governor is only a de jure Constitutional Head. However, during President's rule, the council of ministers is dissolved, vacating the office of Chief Minister. Furthermore, the Vidhan Sabha is either prorogued or dissolved, necessitating a new election. Following its landmark judgment in the 1994 Bommai case, the Supreme Court of India has restricted arbitrary impositions of President's rule. Chhattisgarh and Telangana are the only states where the President's rule has not been imposed so far.

- I. Under President's rule, the state machinery is directly controlled by
  - (i) the President of India.
  - (ii) the Governor of the state.
  - (iii) the Chief Minister of the state.
- II. After President's rule is imposed in a state,
  - (i) its Vidhan Sabha is dissolved.
  - (ii) its Vidhan Sabha is not dissolved.
  - (iii) its Vidhan Sabha may or may not be dissolved.
- III. In the state of Andhra Pradesh, President's rule
  - (i) was imposed at least once.
  - (ii) was never imposed after 1994.
  - (iii) was never imposed before 1994.

62. The correct answers to I, II and III are:

- A. i, iii, i, respectively.
- B. ii, iii, i, respectively.
- C. ii, i, i, respectively.
- D. ii, iii, ii, respectively.

3 marks

## Logical Reasoning

63. A class in a high school has 10 members in the school football team and 14 members in its science club. Among the students of this class, 5 belong to both the football and science teams. The number of students belonging to only one of the two groups is

- A. 9.
- B. 14.
- C. 19.
- D. 24.

1 mark

64. Eight men  $A, B, C, D, E, F, G$  and  $H$  are sitting around a circular table facing the centre for having their lunch. It is known that

- $G$  is not an immediate neighbour of  $C$ .
- $A$  is third to the right of  $C$ .
- $E$  is second to the left of  $C$ .
- $C$  is second to the left of  $B$ .
- $F$  is second to the left of  $D$ .
- $A$  is second to the left of  $F$ .

The man sitting in the position to the immediate right of  $C$  is

- A.  $G$ .
- B.  $F$ .
- C.  $H$ .
- D.  $D$ .

2 marks

65. Pointing to a man, a woman said, "His mother is the only daughter of my mother." The man is the woman's

- A. son.
- B. father.
- C. nephew.
- D. grandson.

1 mark

66. An accurate clock shows 7 o'clock in the morning. The number of degrees through which the hour hand will rotate till the clock shows 3 o'clock in the afternoon is

- A.  $120^\circ$ .
- B.  $180^\circ$ .
- C.  $240^\circ$ .
- D.  $150^\circ$ .

1 mark

67. If 8<sup>th</sup> February, 2005 was Tuesday, then 8<sup>th</sup> February, 2004 was

- A. Monday.
- B. Wednesday.
- C. Thursday.
- D. Sunday.

1 mark

68. There are two student unions in a college –  $A$  and  $B$ . Students of union  $A$  always speak the truth and the students of union  $B$  always lie. A student named Gopal says: “I always lie”. Then it may be concluded that

- A. Gopal must belong to union  $A$ .
- B. Gopal must belong to union  $B$ .
- C. Gopal belongs to one of the two unions.
- D. Gopal cannot belong to either of the two unions.

2 marks

69. If Kapil attends an event, then Sunil, Mohinder and Chandrashekhar also attend it. Mark the correct inference:

- A. If it is found that Kapil has not attended the event, it may be concluded that Sunil has not attended it either.
- B. If it is found that Sunil has not attended the event, it may be concluded that Kapil has not attended it either.
- C. If it is found that Mohinder has not attended the event, it may be concluded that Chandrashekhar has not attended it either.
- D. If Sunil, Mohinder and Chandrashekhar are found to have attended the event, it may be concluded that Kapil has attended it too.

1 mark

70. In the following, two statements numbered as 1 and 2 are given, followed by two conclusions, numbered as I and II. You have to take the two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows the given two statements, disregarding the known facts.

*Statements*

- 1. All Roses are flowers.
- 2. No Lily is a Rose.

*Conclusions*

- I. No Lily is a flower.
- II. Some Lilies are flowers.

- A. Only conclusion I follows.
- B. Only Conclusion II follows.
- C. Both conclusions I and II follow.
- D. Neither conclusion I nor II follows.

1 mark