Institute of Actuaries of India

ACET March 2023

Mathematics

1. The range of the function $f(x) = ||x| - x|$, for x real, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x, is

   A. $(0,1)$.
   B. $(0,1]$.
   C. $[0,1)$.
   D. $(0, \infty)$.

   1 mark

2. Let f and g be real functions such that $f(x) = x^2 + 4$ and $g(x) = \frac{x}{x-2}; x \neq 2$. The value of $f°g(3)$ is

   A. 13.
   B. 12.
   C. 4.
   D. 5.

   1 mark

3. The value of $\sqrt{7^2 + \sqrt{7^2 + \sqrt{7^2 + \cdots}}}$ is

   A. $1 - \sqrt{197}$.
   B. $1 + \sqrt{197}$.
   C. $\frac{1-\sqrt{197}}{2}$.
   D. $\frac{1+\sqrt{197}}{2}$.

   2 marks

4. The coefficient of $a^5 b^7$ in $(a - 3b)^{12}$ is

   A. $\binom{12}{5}3^7$.
   B. $\binom{12}{5}(-3)^7$.
   C. $\binom{12}{5}(-3)^5$.
   D. $\binom{12}{5}(3)^5$.

   1 mark

5. If $\log_m 7 - 3 \log_m 2 = 2$, then m is equal to

   A. $\frac{7}{8}$.
   B. $-\frac{7}{8}$.

C. $-\sqrt{\dfrac{7}{8}}$.

D. $\sqrt{\dfrac{7}{8}}$.

<div align="right">1 mark</div>

6. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + 1 = 0$, then a relationship between a and b is

A. $b^2 - a^2 = 2a$.
B. $b^2 + a^2 = 2a$.
C. $b^2 - a^2 = 2b$.
D. $b^2 + a^2 = 2b$.

<div align="right">2 marks</div>

7. Let $a_1, a_2, \dots, a_n, \dots$ be defined by $a_n = 3 + 4n$. The sum of the first 15 terms is

A. 625.
B. 635.
C. 525.
D. 555.

<div align="right">1 mark</div>

8. The value of

$$\sum_{i=1}^{n}\sum_{j=1}^{i} j$$

is equal to

A. $\binom{n}{3}$.

B. $\binom{n+2}{3}$.

C. $\binom{n+1}{3}$.

D. $\binom{n-2}{3}$.

<div align="right">2 marks</div>

9. If $z = \dfrac{3+i}{2-i}$, then z^{16} is equal to

A. $256i$.

B. $128i$.

C. 256.

D. 128.

<div align="right">2 mark</div>

10. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be mutually perpendicular unit vectors. The value of $|\vec{a} + \vec{b} + \vec{c}|$ is

A. 3.
B. $\sqrt{3}$.
C. 1.
D. 9.

11. The value of x for which the vectors $\vec{i} - x\vec{j} + 3\vec{k}$, $-2\vec{i} + 3\vec{j} - 4\vec{k}$ and $-\vec{j} + 2\vec{k}$ are coplanar is

   A. -2.
   B. 2.
   C. 1.
   D. -1.

<div align="right">1 mark</div>

12. In the approximation of $y = f(x)$ through a second degree polynomial, obtain the missing values of (y_1, y_3) from the following table.

x	$x_0 = 21$	$x_1 = 22$	$x_2 = 23$	$x_3 = 24$	$x_4 = 25$
y	3.0	y_1	2.0	y_3	2.4

The missing values (y_1, y_3) are

   A. $(2.925, 2.225)$.
   B. $(1.925, 1.025)$.
   C. $(3.925, 1.225)$.
   D. $(2.325, 2.025)$.

<div align="right">3 marks</div>

13. The approximate value of the integral $\int_0^2 \log_e \sqrt{1+x}\ dx$, by Simpson's $\frac{1}{3}$ rule using $h = 1$ (given that $\log_e \sqrt{2} = 0.3466$ and $\log_e \sqrt{3} = 0.5493$), is

   A. 0.6452.
   B. 0.5342.
   C. 1.2345.
   D. 0.3932.

<div align="right">1 mark</div>

14. $\lim\limits_{x \to 2} \dfrac{x^6 - 2}{x^4 - 2}$ is equal to

   A. 6.
   B. $\dfrac{31}{7}$.
   C. $\dfrac{195}{3}$.
   D. 4.

<div align="right">1 mark</div>

15. If $y = \tan^{-1}\left(\dfrac{\sin x}{1 + \cos x}\right)$, then $\dfrac{dy}{dx}$ is equal to

   A. 1.
   B. 2.
   C. $\dfrac{1}{2}$.
   D. $\dfrac{1}{3}$.

<div align="right">2 marks</div>

16. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$, then

$$\frac{d^2y}{dx^2} + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$$

is

A. y^2.
B. e^{x-2}.
C. e^x.
D. y.

2 marks

17. The value of the integral

$$\int \frac{\sqrt{5 + \log x}}{x} \, dx$$

is equal to

A. $\frac{2}{3}(5 + \log x)^{\frac{3}{2}} + c$.
B. $(5 + \log x)^{\frac{3}{2}} + c$.
C. $\frac{2}{3}(5 + \log x)^{\frac{1}{2}} + c$.
D. $\frac{3}{2}(5 + \log x)^{\frac{3}{2}} + c$.

3 marks

18. The value of the integral

$$\int_{-1}^{1} x^{15} \cos^6 x \, dx$$

is equal to

A. 1.
B. 0.
C. 3.
D. 2.

1 mark

19. Let

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$

The characteristic roots of $A + B$ are

A. $(1,6)$.
B. $(2,5)$.
C. $(1,-6)$.
D. $(-2,5)$.

1 mark

20. The inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

is

A. $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -7 \end{bmatrix}$.

B. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

C. $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$.

D. $\begin{bmatrix} 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{5} & 0 \\ \frac{1}{7} & 0 & 0 \end{bmatrix}$.

1 mark

Statistics

21. Twenty books are placed at random in a shelf. The probability that a particular pair of books shall never be together is

    A. $\frac{1}{20}$.

    B. $\frac{1}{10}$.

    C. $\frac{19}{20}$.

    D. $\frac{9}{10}$.

    1 mark

22. A box contains 24 balls out of which x are red balls. If one ball is drawn at random from the box, the probability that it will be red is p. If 12 more red balls are put in the box, the probability of drawing a red ball is now $2p$. The value of x is

    A. 6.
    B. 8.
    C. 9.
    D. 12.

    1 mark

23. The probability that both E and F occur, the probability that E occurs and F does not occur and the probability that E does not occur and F occurs are equal to p. Then, the probability that either E or F occurs is

    A. p.
    B. $2p$.
    C. $3p$.
    D. $3p^2$.

    1 mark

24. If E and F are independent events having probability strictly less than 1, which of the following statements is incorrect?

    A. E and $\bar{F}$ are independent.
    B. $\bar{E}$ and $\bar{F}$ are independent.
    C. $P(E \cup F) = P(E) + P(\bar{E})P(F)$.
    D. $P(E|F)P(F|E) = 1$.

    2 marks

25. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the probability that the number 5 has appeared at least once?

    A. $\frac{1}{5}$.

    B. $\frac{2}{5}$.

    C. $\frac{1}{2}$.

    D. $\frac{3}{4}$.

    1 mark

26. The following table gives the frequency distribution of car accidents for 100 days in a stretch of national highway.

Number of car accidents	0	1	2	3	4	5
Frequency	20	30	25	12	11	2

The mean, median and mode of the distribution of car accidents are, respectively,

A. $1.7, 1, 1$.
B. $1.5, 1.5, 1$.
C. $1.7, 1, 5$.
D. $1.7, 1.5, 1$.

1 mark

27. A set of n numbers consist of $(n-2)$ zeros and the numbers -1 and 1. The mean absolute deviation of these numbers about the mean is

A. $\frac{1}{n}$.
B. $\frac{2}{n}$.
C. $\frac{1}{2n}$.
D. 0.

1 mark

28. The standard deviation and coefficient of variation of a set of observations are 3.8 and 7.6%, respectively. If each observation is multiplied by 2, then the coefficient of variation of new observations is

A. 3.8%.
B. 7.6%.
C. 15.2%.
D. 38%.

1 mark

29. A discrete random variable X takes the values 21, 22, 23, 25, 27, 29, 30, 32, 35, 40 with equal probability. Then $P(X > 25 | X \geq 23)$ equals

A. $\frac{3}{4}$.
B. $\frac{3}{8}$.
C. $\frac{6}{7}$.
D. $\frac{7}{8}$.

1 mark

30. Let X be a random variable with probability density function $f(x) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right), x > 0, \theta > 0$. If the median of X is 25, the mean of X is

A. $25 \log_e 2$.

B. $\dfrac{25}{\log_e 2}$.

C. $25 \log_e 0.5$.

D. $\dfrac{25}{\log_e 0.5}$.

1 mark

31. Suppose X follows binomial distribution with mean and variance $\dfrac{4}{3}$ and $\dfrac{8}{9}$, respectively. Then $P(X \geq 2)$ equals

A. $\dfrac{4}{9}$.

B. $\dfrac{5}{9}$.

C. $\dfrac{7}{81}$.

D. $\dfrac{33}{81}$.

1 mark

32. Let X and Y be two correlated random variables with respective variances σ_1^2 and σ_2^2, and correlation coefficient ρ. Then ρ equals

A. $\dfrac{Var(X)+Var(Y)+Var(X-Y)}{2}$.

B. $\dfrac{Var(X-Y)-Var(X)-Var(Y)}{4}$.

C. $\dfrac{Var(X)-Var(Y)+Var(X-Y)}{2\sigma_1^2\sigma_2^2}$.

D. $\dfrac{Var(X)+Var(Y)-Var(X-Y)}{2\sigma_1\sigma_2}$.

1 mark

33. Which of the following statement is incorrect?

A. Suppose X follows the Poisson distribution with mean 3. Then $P(X \leq 1) = 4e^{-3}$.

B. Probability that a Poisson variable X take values at least 1 is $1 - e^{-3}$. Then the variance of X is 3.

C. Suppose X follows a Poisson distribution with $P(X = 2) = 2P(X = 1)$. Then standard deviation of the distribution is 4.

D. Suppose X follows the Poisson distribution with mean 1. Then $E\left(\dfrac{1}{1+X}\right) = 1 - e^{-1}$.

2 marks

34. Suppose Z follows normal distribution with mean 0 and variance 1. If $X = 3 + Z$, then $E(X^3)$ equals

A. 27.
B. 0.
C. 36.
D. 54.

2 marks

35. Consider the joint probability mass function of X and Y
$$p(x, y) = \frac{x + y}{n^2(n + 1)}, \qquad x, y = 1, 2, \ldots, n$$
Then $P(X = Y)$ equals

A. $\frac{1}{n}$.

B. $\frac{2}{n}$.

C. $\frac{1}{n+1}$.

D. $\frac{2}{n+1}$.

1 mark

36. In a school 40% of the students have 100% attendance and 60% are irregular. Previous years' results indicate that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in the annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance?

A. $\frac{2}{5}$.

B. $\frac{14}{17}$.

C. $\frac{12}{17}$.

D. $\frac{6}{7}$.

2 marks

37. From 20 pairs of observations on (x, y), the following information is obtained. $\bar{x} = 1.2$, $\bar{y} = 12$, the regression coefficients of y on x is 3.2 and x on y is 0.2. Then which of the following statements is true?

A. The correlation coefficient between x and y is -0.8.
B. The standard deviations of x and y are related by $s_x = 4\, s_y$.
C. The regression of y on x is $y - 3.2x - 8.16 = 0$.
D. The regression x on y is $x + 0.2y - 1.2 = 0$.

2 marks

38. If X and Y have joint probability density function
$$f(x, y) = 4x(1 - y), 0 < x < 1, 0 < y < 1,$$
then $P\left(0 < X < \frac{2}{3} \middle| 0 < Y < \frac{1}{3}\right)$ equals

A. $\frac{4}{9}$.

B. $\frac{5}{9}$.

C. $\frac{20}{81}$.

D. $\frac{5}{81}$.

2 marks

39. Let *X* be a random variable with probability mass function
$$P(X = x) = \frac{\lambda^x}{x!\,(e^\lambda - 1)}, \qquad x = 1, 2, \dots, \infty$$

The variance of *X* is

A. λ^2.

B. $\frac{\lambda e^\lambda}{e^\lambda - 1}\left(e^\lambda - 1\right)$.

C. $\frac{\lambda e^\lambda}{e^\lambda - 1}\left(e^\lambda - \lambda\right)$.

D. $\frac{\lambda e^\lambda}{\left(e^\lambda - 1\right)^2}\left(e^\lambda - \lambda - 1\right)$.

3 marks

40. The joint distribution of the random variables *X* and *Y* is provided in the following table.

		*X*		
		0	1	2
Y	0	$\frac{1}{12}$	0	0
	1	$\frac{2}{12}$	$\frac{3}{12}$	0
	2	$\frac{1}{12}$	$\frac{4}{12}$	$\frac{1}{12}$

The correlation coefficient between *X* and *Y* is

A. $\frac{27}{\sqrt{3009}}$.

B. $-\frac{\sqrt{6}}{4}$.

C. $\frac{\sqrt{6}}{4}$.

D. $-\frac{27}{\sqrt{3009}}$.

3 marks

Data Interpretation

An organization selected 1800 families in a town at random and surveyed income level and number of vehicles in a family. The information obtained is listed in the following frequency table. Answer questions 41-43 based on your interpretation of the table.

Monthly income (in Rupees)	Number of vehicles per family			
	0	1	2	Above 2
Less than 50000	53	22	0	0
50000 - 75000	32	90	3	0
75000 - 100000	28	165	7	0
100000 - 125000	16	290	14	0
125000 - 150000	10	258	30	2
150000 - 175000	6	307	62	5
175000 - 200000	5	159	75	11
200000 or more	3	65	62	20

41. The highest number of families owning at least two vehicles belongs to the monthly income group

    A. 125000 - 150000.
    B. 150000 - 175000.
    C. 175000 - 200000.
    D. 200000 or more.

    2 marks

42. Number of families owning at least one vehicle is

    A. 1356.
    B. 1509.
    C. 1609.
    D. 1647.

    2 marks

43. Number of families whose earning is Rs. 100000 or more per month owning exactly 1 vehicle

    A. 1079.
    B. 1244.
    C. 1322.
    D. 1360.

    1 mark

The following diagram shows share (%) of households with tap water connections of 10 states S1 – S10 in 2019 and 2022. Answer questions 44-47 based on your interpretation of the diagram.

Share (%) of households with tap water connections
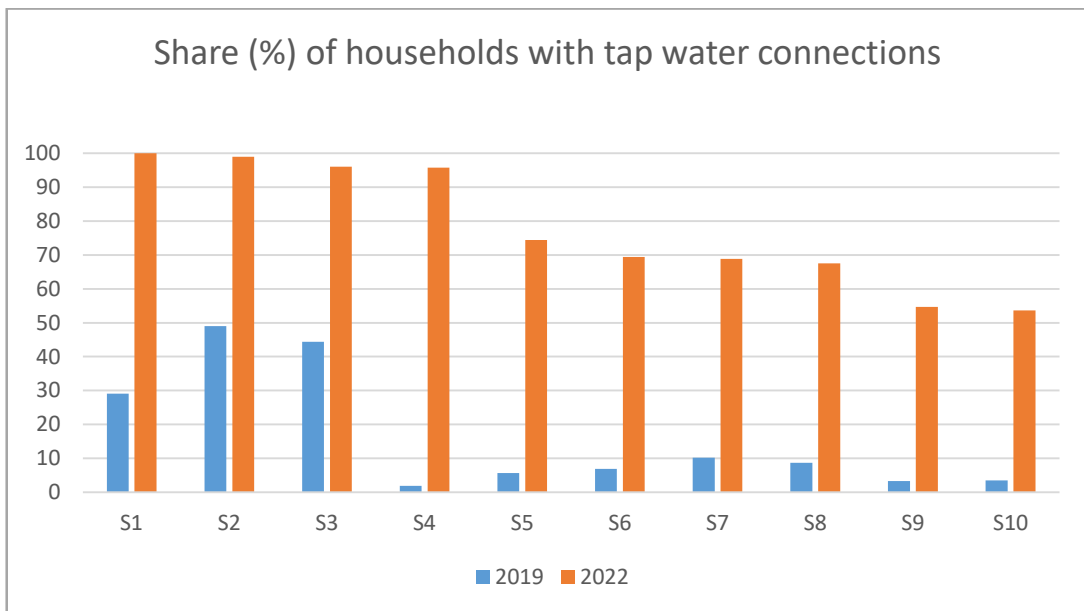
44. Which state has maximum increase in percentage of households with tap water connections from 2019 to 2022?

    A. S4.
    B. S5.
    C. S9.
    D. S10.

<div align="right">1 mark</div>

45. Which state had 100% tap water connections in 2022?

    A. S1.
    B. S2.
    C. S3.
    D. S4.

<div align="right">1 mark</div>

46. The minimum and maximum share (%) of households with tap water connections in 2019 are in states

    A. S9 and S2.
    B. S10 and S3.
    C. S4 and S2.
    D. S4 and S3.

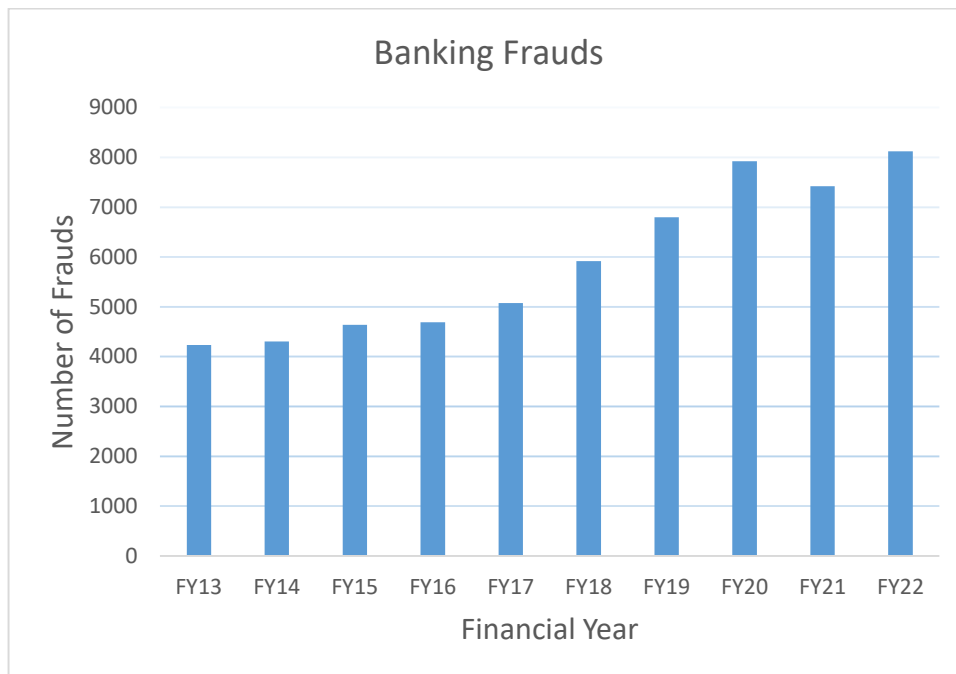<div align="right">1 mark</div>

47. The number of states having more than 65% households with tap water connections in 2022 is

    A. All states.
    B. 8.
    C. 7.
    D. 6.

<div align="right">1 mark</div>

The following bar diagram displays the number of banking frauds for 10 financial years from FY13 to FY22. Answer question 48 based on this diagram.

Banking Frauds

48. Which of the following statements is true?

   A. The minimum number of frauds occurred in FY14.
   B. The maximum number of frauds occurred in FY20.
   C. The number of frauds increased in every year over the previous year.
   D. The number of frauds decreased over the previous year is in FY21.

2 marks

The following pie chart shows budget allocation in different sectors in a financial year. The total budget allocation in that year is 40 lakh crore. Answer questions 49-51 based on this information.
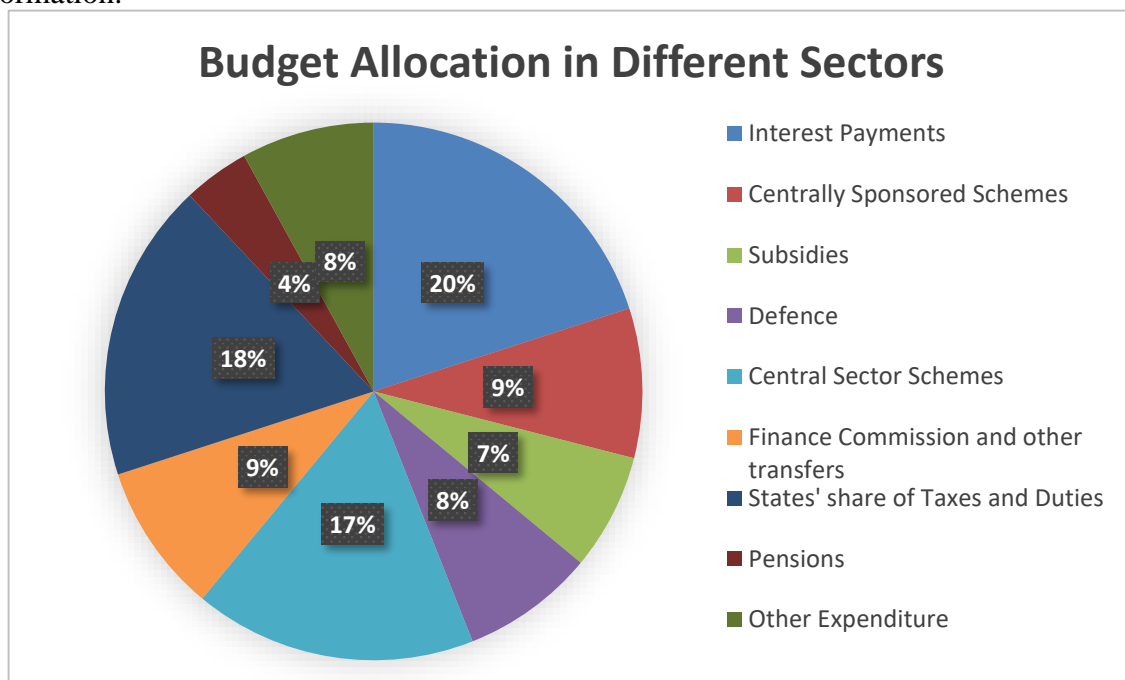
49. Total budget allocation on Centrally sponsored Schemes and Central Sector Schemes is

   A. 3.6 lakh crore.
   B. 6.8 lakh crore.
   C. 10.4 lakh crore.
   D. 12 lakh crore.

50. Total budget allocation for the sectors with less than 10% allocation is

    A. 14 lakh crore.
    B. 18 lakh crore.
    C. 22 lakh crore.
    D. 25 lakh crore.

51. The difference between maximum and minimum sectorwise allocations is

    A. 5.2 lakh crore.
    B. 6.4 lakh crore.
    C. 16 lakh crore.
    D. 20 lakh crore.

English

52. Meteorology is

   A. a science that deals with phenomena of weather.
   B. a science that deals with metals and other minerals.
   C. a science that deals with the composition and origin of meteors and other cosmic objects.
   D. a science that studies the physical system of measurements.

   1 mark

53. An avalanche is

   A. a series of waves in the sea caused by the displacement of a large volume of water.
   B. a large air mass that rotates around a strong center of low atmospheric pressure.
   C. a slide of large masses of snow and mud down a mountain.
   D. a sudden, very heavy rainfall, usually local in nature and of brief duration.

   1 mark

54. Contrite is

   A. a binding agreement between two or more persons that is enforceable by law.
   B. a feeling or expressing pain or sorrow for sins or offenses.
   C. to make or work out a plan.
   D. to injure the underlying soft tissue or bone.

   1 mark

55. Expurgate is

   A. to edit by omitting or modifying parts considered indelicate.
   B. to kill on a large scale.
   C. to remove, destroy or wipe out completely.
   D. to extinguish by crushing.

   1 mark

56. One word for the phrase "to make certain that (something) will occur" is

   A. assure.
   B. reassure.
   C. insure.
   D. ensure.

   1 mark

57. One word for the phrase "capable of being interpreted in two or more ways" is

   A. definite.
   B. confusing.
   C. doubtful.
   D. ambiguous.

   1 mark

58. One word for the phrase "to become familiar with someone/ something" is

   A. effeminate.

B. antagonize.
C. acquaint.
D. appreciate.

<div align="right">1 mark</div>

59. A person who has no money is called

A. affluent.
B. pauper.
C. disabled.
D. hapless.

<div align="right">1 mark</div>

60. Choose the correct sentence:

A. Ramesh claims that cats could be the best pets.
B. Ramesh claims that cats as the best pets.
C. Ramesh claims that cats make of the best pets.
D. Ramesh claims that cats make the best pets.

<div align="right">2 marks</div>

61. Choose the correct sentence:

A. The helium-filled balloon rising in the air.
B. The helium-filled balloon was rising up the air.
C. The helium-filled balloon rose up the air.
D. The helium-filled balloon rose into the air.

<div align="right">2 marks</div>

Read the passage below and answer Question No. 62.

Mount Everest, along with the rest of the Himalayas, inches further skyward every year. It towers 8,848.86m (29,032ft) above sea level, according to China and Nepal's most recent official joint survey, whose borders run across its summit. But it isn't the only giant in these lands – 10 of the world's 14 peaks higher than 8,000m (26,247ft) above sea level can be found in the Himalayan range. The history of measuring the tallest mountain in the world stretches back to 1852. In Asia, the height of Mount Everest was a mystery. It was known only as "Peak XV". Radhanath Sikdar, an Indian mathematician, had been employed by the British to work on their Great Trigonometrical Survey. They wanted to gather a more accurate geographical picture of the territory they were occupying so they could control it more effectively for trade or military purposes. Sikdar used trigonometry. He measured Everest's summit's horizontal and vertical angles from other mountaintops whose positions and heights were already known. In doing so he made a momentous discovery: the tallest mountain ever recorded. According to his calculations, the mountain stood at 8,839.8m (29,002ft) tall. Though the technology behind measuring mountains has advanced since the 1850s, his figure was astonishingly accurate, just nine meters off the latest official height. Despite Sikdar's findings, the mountain eventually was named after his previous boss, British surveyor Sir George Everest, who had retired several years before Sikdar's discovery.

I. The British wanted

   i. to develop agricultural land near the base of Mount Everest.
   ii. to control the territory that included Mount Everest.
   iii. to habitat the area that included Mount Everest.

II. Height of the Mount Everest was first measured by

   i. Nepalese and Chinese.

      ii.   Sir George Everest.

      iii.   Radhanath Sikdar.

III.   Which of the following statements is correct?

      i.   It is observed that there are fourteen peaks in the world higher than 8,000 meters.

      ii.   It is observed that there are ten peaks in the Himalayas higher than 8,000 meters.

      iii.   Both i and ii.

62. The correct answers to I, II and III are

    A. ii, iii, iii, respectively.
    B. i, ii, iii, respectively.
    C. iii, ii, iii, respectively.
    D. iii, ii, i, respectively.

                                        3 marks

Logical Reasoning

63. In a college with 100 students, 60 students play hockey, 84 play kabaddi and 72 play basketball. If all the students play at least one game, what is the minimum number of students who play all the three sports?

    A. 16.
    B. 22.
    C. 36.
    D. 58.

    2 marks

64. In the question below are given some statements followed by some conclusions. You have to take the given statements to be true even if they seem to be in variance with commonly known facts. Read all the conclusions and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts.

    Statements:
    I.   Samosas are Jalebi.
    II.   All Jalebis are Tikki.
    III.   All Tikkis are Barfi.

    Conclusions:

    I.   All Jalebis are Barfi.
    II.   All Tikkis are Samosas.
    III.   All Samosas are Barfi.
    IV.   All Barfi are Jalebi.

    A. Only conclusions I and II follow.
    B. Only conclusions I and III follow.
    C. Only conclusions II and III follow.
    D. All conclusions follow.

    2 marks

65. Alexa leaves her house and moves 50 meters southwards. She then turns left and goes another 20 meters. Then turning to the North, she moves another 30 meters and then starts walking towards her house. In which direction is she walking now?

    A. North.
    B. North -East.
    C. South -East.
    D. North -West.

    1 mark

66. A clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?

    A. 144.
    B. 150.
    C. 168.
    D. 180.

    1 mark

67. In 2016 Shailendra celebrated his birthday on Friday. What will be the first year after 2016 when Shailendra will celebrate his birthday on a Wednesday given that he was not born in January or February?

    A. 2021.
    B. 2023.
    C. 2020.
    D. 2025.

    1 mark

68. On a circular dart board, 15 balloons are numbered from 1 to 15 and arranged in the clockwise direction in the same order. A boy shot 14 balloons on the board by shooting every alternate balloon in the clockwise direction beginning from the balloon numbered 2. Which numbered balloon did he shoot at the tenth shot?

    A. 13.
    B. 10.
    C. 5.
    D. 9.

    1 mark

69. A large cube of dimensions $7 \times 7 \times 7$ is painted with black colour on all the faces and is cut into 343 small cubes of dimensions $1 \times 1 \times 1$. How many of the smaller cubes will be painted on 4 sides?

    A. 0.
    B. 8.
    C. 16.
    D. 21.

    1 mark

70. Ankit is a son of Shiv's father's sister. Sahil is the son of Nimrit who is the mother of Gaurav and grandmother of Shiv. Kishore is the father of Tina and grandfather of Ankit. Nimrit is the wife of Kishore.
    How is Ankit related to Nimrit?

    A. Nephew.
    B. Son.
    C. Grandson.
    D. Data inadequate.

    1 mark

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