Institute of Actuaries of India

ACET June 2022 Solutions

Mathematics

1. D. Given that $f(x) = 5x^3 + 9$. Let $x_1, x_2 \in R$. $f(x_1) = f(x_2) \Longrightarrow x_1^3 = x_2^3 \Longrightarrow (x_1 - x_2)(x_1^2 + x_2^2 + x_1x_2) = 0$. Therefore, either $(x_1 - x_2) = 0$ (that is, $x_1 = x_2$), or $x_1^2 + x_2^2 + x_1x_2 = 0$ (that is, $x_1 = x_2 = 0$, since x_1 and x_2 are both real). Thus, $x_1 = x_2$ and f is one-one.

   Let $5x^3 + 9 = y \in R$. Then, $x^3 = \frac{y-9}{5}$ and $x = \sqrt[3]{\frac{y-9}{5}}$. Hence f is onto.

2. A. $\lim_{x\to 0} \frac{(4+x)^{\frac{1}{2}}-2}{x} = \lim_{x\to 0} \frac{(4+x)^{\frac{1}{2}}-2}{x} \times \frac{(4+x)^{\frac{1}{2}}+2}{(4+x)^{\frac{1}{2}}+2} = \lim_{x\to 0} \frac{4+x-4}{x\left[(4+x)^{\frac{1}{2}}+2\right]}$. This is in the 0/0 form.

   Applying the l'Hopital rule, we have the limit simplifying to $\lim_{x\to 0} \frac{1}{(4+x)^{\frac{1}{2}}+2} = \frac{1}{4}$.

3. C. The given polynomial is $2x^3 + x^2 - 5x + 2$. One of the zeroes is 1, then $(x - 1)$ is a factor of the given polynomial and when divided by it we get the quotient as $2x^2 + 3x - 2$. The factors of $2x^2 + 3x - 2$ are $(x + 2)$and $(2x - 1)$.

   Thus, the other two zeroes of the polynomials are -2 and $\frac{1}{2}$.

4. A. $4\log_x 3 + 2\log_x 2 = 2 \Longrightarrow \log_x 3^4 + \log_x 2^2 = 2 \Longrightarrow \log_x(81 \times 4) = 2 \Longrightarrow 2 = \log_x(18)^2 = 2\log_x(18)$. Hence $x = 18$.

5. B. The $(r+1)^{\text{th}}$ term in the expansion of $(x+y)^n$ is $T_{r+1} = \binom{n}{r} x^{n-r} y^r$; $r = 0,1, \dots, n$. Hence, $T_{20} = \binom{39}{19} 3^{39-19} a^{19}$ and $T_{21} = \binom{39}{20} 3^{39-20} a^{20}$. $T_{20} = T_{21} \Rightarrow \binom{39}{19} 3^{20} a^{19} = \binom{39}{20} 3^{19} a^{20}$. Hence $a = 3$.

6. B. Given that in an *AP*, $S_n = \frac{n}{2}[2a + (n-1)d]$. Here $n = 15$, $a = 12$ and $S_{15} = 705$. Thus, $705 = \frac{15}{2}[(2 \times 12) + 14d]$, giving $d = 5$. Hence, the 20^{th} term is $a + (20-1)d = 12 + (19 \times 5) = 107$.

7. A. Given that $x + iy = \frac{(4-3i)(3+2i)}{(1+3i)(3-i)} = \frac{12+8i-9i-6i^2}{3-i+9i-3i^2} = \frac{12-i+6}{3+8i+3} = \frac{18-i}{6+8i} = \frac{(18-i)(6-8i)}{(6+8i)(6-8i)} = \frac{108-144i-6i\mp 8i^2}{36-64i^2} = \frac{108-144i-6i-8}{36+64} = \frac{100-150i}{100} = 1 - 1.5i$; $i^2 = -1$. Hence, the complex conjugate of $x + iy$ is $1 + 1.5i$.

8. C. Let $\cos^{-1}\frac{4}{5} = \theta \Rightarrow \cos\theta = \frac{4}{5}$.

   Hence, $\sin\left(\cos^{-1}\frac{4}{5}\right) = \sin\theta = \pm\sqrt{1-\cos^2\theta} = \pm\sqrt{1-\left(\frac{4}{5}\right)^2} = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$.

9. B. Let the correct value of y_2 be $13 + e$, where e is the amount of error. Since the polynomial is order 3, $\Delta^3 y$ is constant. The difference table is:

y	Δy	$\Delta^2 y$	$\Delta^3 y$
0			
	2		
2		9+e	
	11+e		11-3e
13+e		20-2e	
	31-e		-10+3e
44		10+e	
	41		
85			

From the table, we have, $11 - 3e = -10 + 3e$ giving $e = 3.5$, i.e., $y_2 = 16.5$.

10. D. Let O be the origin. The position vector $\overrightarrow{OP}$ is $2\vec{i} + 3\vec{j} - 4\vec{k}$ and the position vector $\overrightarrow{OQ}$ is $3\vec{i} - 5\vec{j} + 2\vec{k}$. Hence, $\overrightarrow{PQ}$ is $\left(3\vec{i} - 5\vec{j} + 2\vec{k}\right) - \left(2\vec{i} + 3\vec{j} - 4\vec{k}\right) = \vec{i} - 8\vec{j} + 6\vec{k}$ and $\left|\overrightarrow{PQ}\right| = \sqrt{1^2 + (-8)^2 + 6^2} = \sqrt{1 + 64 + 36} = \sqrt{101}$.

11. B. The condition that $\vec{a} = 3\vec{i} + c\vec{j} + 2\vec{k}$ and $\vec{b} = 2\vec{i} + \vec{j} + 4\vec{k}$ are perpendicular to each other is $\vec{a} \circ \vec{b} = 0$.
$\vec{a} \circ \vec{b} = \left(3\vec{i} + c\vec{j} + 2\vec{k}\right) \circ \left(2\vec{i} + \vec{j} + 4\vec{k}\right) = 0 \Rightarrow (3 \times 2) + (c \times 1) + (2 \times 4) = 0$.
This implies $c = -14$.

12. A. $y = \log_e \sqrt{\sin \sqrt{e^x}}$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{\sin \sqrt{e^x}}} \cdot \frac{1}{2\sqrt{\sin \sqrt{e^x}}} \cdot \cos \sqrt{e^x} \frac{1}{2\sqrt{e^x}} e^x = \sqrt{e^x} \frac{\cot \sqrt{e^x}}{4}.$$

13. D. $y = 2x^x$; $\log_e y = \log_e 2 + x \log_e x$. $\frac{d \log_e y}{dx} = \frac{1}{y} \cdot \frac{dy}{dx} = x\frac{1}{x} + \log_e x = 1 + \log_e x$.
Hence, $\frac{dy}{dx}$ at $x = e$ is $2e^e(1 + \log_e e) = 2e^e(1 + 1) = 4e^e$.

14. A. $f(x) = \frac{\log_e x}{2x}$; $x > 0$.
Then $f'(x) = \frac{1}{2x^2}(1 - \log_e x)$ and $f''(x) = -\frac{1}{2x^3}(1 + 2(1 - \log_e x))$.

$$f'(x) = \frac{1}{2x^2}(1 - \log_e x) = 0 \Rightarrow \log_e x = 1 = \log_e e \Rightarrow x = e.$$

$$f''(x) \text{ at } x = e \text{ is } -\frac{1}{2e^3}(1 + 2(1 - \log_e e)) = -\frac{1}{2e^3} < 0.$$

Hence, the maximum of $f(x)$ is attained at $x = e$.

15. A. Let $e^{2x} - e^{-2x} = t$. Then, $(2e^{2x} + 2e^{-2x})dx = dt$.

$$\int \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}\,dx = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log t + C = \frac{1}{2}\log(e^{2x}-e^{-2x}) + C.$$

16. D. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\,dx$.

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin x+\cos x}{\sin x+\cos x}\,dx = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}.$

Hence, $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}\,dx = I = \frac{\pi}{4}.$

17. B. $A = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$= (1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + (\sin^2\theta + 1) = 2(1+\sin^2\theta).$

Thus, the value of A at $\theta = \frac{\pi}{4}$ is 3.

18. D. $M = \begin{bmatrix} \sin x & \tan x \\ -\tan x & \sin x \end{bmatrix}$ and $M^T = \begin{bmatrix} \sin x & -\tan x \\ \tan x & \sin x \end{bmatrix}$. $M + M^T = \begin{bmatrix} 2\sin x & 0 \\ 0 & 2\sin x \end{bmatrix}$.

$M + M^T = I \Rightarrow 2\sin x = 1 \Rightarrow x = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}.$

19. D. Since A and B are symmetric matrices, $A = A^T$ and $B = B^T$. Now,
$(AB - BA)^T = (AB)^T - (BA)^T = B^TA^T - A^TB^T = BA - AB = -(AB - BA).$
Thus, $AB - BA$ is skew symmetric.

20. C. $M = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$; $Adj\ M = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$.

$Det\ M = 2(-1) - 1(4) + 3(8-7) = -2 - 4 + 3 = -3 \neq 0.$

Hence, $M^{-1} = \frac{Adj\ M}{Det\ M} = -\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}.$

Statistics

21. B. The ordered observations are 5, 7, 10, 12, 15, x, $x+2$, 23, 25, 26, 28, y.
Median = (6th observation +7th observation)/2 = $\frac{x+x+2}{2} = x+1 = 18 \Rightarrow x = 17$.
Observations are 5, 7, 10, 12, 15, 17, 19, 23, 25, 26, 28, y. Mean = 18.5.
So $\frac{5+7+10+12+15+17+19+23+25+26+28+y}{12} = 18.5 \Rightarrow y + 187 = 18.5 \times 12 \Rightarrow y = 35$.
$(x, y) = (17, 35)$.

22. D. Mean of $ax_1, \dots, ax_n$ is $\frac{ax_1 + \cdots + ax_n}{n} = a\bar{x}$.
Standard deviation of $ax_1, \dots, ax_n$ is
$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(ax_i - a\bar{x})^2} = \sqrt{a^2\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{a^2}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} = |a|s$.

23. B. Observations are 6, 5, 6, 5, 1, 2, 5, 6, 4, 1, 2, 5, 6, 5, 7, 6, 5, 9, 10, 8.
Arranging in increasing order gives
1, 1, 2, 2, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 8, 9, 10.
Mean of the original observations = 104/20 = 5.2.
Median of original observations = 5.
If highest 10% and lowest 10% of the observations are deleted, then the new observations are 2, 2, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 8.
The mean of the new observations = 83/16= 5.1875.
The median of the new observations = (8th obs +9th obs)/2 = $\frac{5+5}{2} = 5$.
The mode of the new observations is 5.
The range of the new observations = 8-2 = 6.

24. A. Let the average marks of Section A and B are $\bar{x}_1$ and $\bar{x}_2$ respectively.
Given that $\bar{x}_1 = 78$. The average marks of class X = $\frac{48\bar{x}_1 + 52\,\bar{x}_2}{48+52} = 65$.
This implies $48 \times 78 + 52\,\bar{x}_2 = 6500 \Rightarrow \bar{x}_2 = 53$.

25. B. A. If each observation is multiplied by a constant k, then new quartiles will be k times old quartiles. Hence the interquartile range will be k times the old interquartile range.
B. For a positively skewed distribution mean > median > mode.
C. For a positively skewed distribution, the frequency curve has longer tail towards the right.
D. For a negatively skewed distribution, the frequency curve has longer tail towards the left.

26. C. $\bar{x} = 10$, $\bar{y} = 15$, $s_y = 2.5$, $s_x = 1.5$. The regression line of x on y is

$$x - \bar{x} = r\frac{s_x}{s_y}(y - \bar{y}) \Rightarrow x - 10 = 0.75 \times \frac{1.5}{2.5}(y - 15)$$
$$\Rightarrow x = 0.45y + 3.25$$

The regression line of y on x is

$y - \bar{y} = r\frac{s_y}{s_x}(x - \bar{x}) \Rightarrow y - 15 = 0.75 \times \frac{2.5}{1.5}(x - 10) \Rightarrow y = 1.25x + 2.5.$

27. B. Probability of getting one white and one black ball is $= \frac{10\times 6}{16\times 16} + \frac{6\times 10}{16\times 16} = \frac{15}{32}$.

28. C. If E and F are independent events, then
E and $\bar{F}$ are independent
$\bar{E}$ and F are independent
$\bar{E}$ and $\bar{F}$ are independent
$P(E|\bar{F}) = P(E)$, since E and $\bar{F}$ are independent. So $P(E|\bar{F}) > P(E)$ is not true.

29. D. Let E be the event that a light bulb is defective.
$P(M_1) = 0.50,\ P(M_2) = 0.20,\ P(M_3) = 0.30.$
$P(E|M_1) = 0.04,\ P(E|M_2) = 0.01,\ P(E|M_3) = 0.02.$
$P(M_3|E) = \frac{P(E|M_3)P(M_3)}{P(E)}.$
$P(E) = P(E|M_1)P(M_1) + P(E|M_2)P(M_2) + P(E|M_3)P(M_3)$
$= 0.04 \times 0.50 + 0.01 \times 0.20 + 0.02 \times 0.30 = 0.028.$
$P(M_3|E) = \frac{0.02\times 0.3}{0.028} = \frac{3}{14}.$
Probability that the defective item is not manufactured by M_3 is
$P(\bar{M}_3|E) = 1 - P(M_3|E) = 1 - \frac{3}{14} = \frac{11}{14}.$

30. C. Area A of the equilateral triangle A is $\frac{\sqrt{3}}{4}X^2$. Expected area is $E(A) = \frac{\sqrt{3}}{4}E(X^2)$.
$X \sim Uniform(0, 2)$. $E(X^2) = \int_0^2 \frac{x^2}{2}dx = \frac{4}{3}$. $E(A) = \frac{\sqrt{3}}{4} \times \frac{4}{3} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$..

31. A. X denotes the number of heads. $X \sim Binomial\left(5, \frac{1}{2}\right)$.
$P(X = 3) = \binom{5}{3}\left(\frac{1}{2}\right)^5 = 5/16.$

32. C. $E(X) = 0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2) + 3 \times P(X = 3) =$
$P(X = 1) + 2 \times 0.2 + 6P(X = 1) = 7P(X = 1) + 0.4.$
$E(X) = 1.8 \Rightarrow 7P(X = 1) + 0.4 = 1.8 \Rightarrow P(X = 1) = 0.2.$
Also, $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1.$
$\Rightarrow P(X = 0) + 3P(X = 1) + P(X = 2) = 1.$
$\Rightarrow P(X = 0) + 3 \times 0.2 + 0.2 = 1.$
$\Rightarrow P(X = 0) = 0.2.$

33. D. $\text{Var}(X) = E\left(X - E(X)\right)^2 = E(X^2) - \left(E(X)\right)^2.$
$E(X) = \sum_{i=1}^{m} p_i x_i,\ E(X^2) = \sum_{i=1}^{m} p_i x_i^2.$
So $\text{Var}(X) = \sum_{i=1}^{m} p_i x_i^2 - (\sum_{i=1}^{m} p_i x_i)^2.$

34. B. Given that $P(X \leq -1) = 0.25$ and $P(X \leq 1) = 0.75$

$P(X \le -1) = \Phi\left(\frac{-1-\mu}{\sigma}\right)$ and $P(X \le 1) = \Phi\left(\frac{1-\mu}{\sigma}\right)$, where Φ is the cdf of $N(0,1)$.
Now $\Phi\left(\frac{-1-\mu}{\sigma}\right) = 0.25 \Rightarrow \frac{-1-\mu}{\sigma} = \Phi^{-1}(0.25) = Q_1$
$\Phi\left(\frac{1-\mu}{\sigma}\right) = 0.75 \Rightarrow \frac{1-\mu}{\sigma} = \Phi^{-1}(0.75) = Q_3$.
We have $\mu + \sigma Q_1 = -1, \mu + \sigma Q_3 = 1$.
Solving, we get $\mu = \frac{(Q_1+Q_3)}{Q_1-Q_3}$ and $\sigma = \frac{2}{Q_3-Q_1}$.

35. C. $P(X > a+b | X > b) = \frac{P(X>a+b, X>b)}{P(X>b)} = \frac{P(X>a+b)}{P(X>b)}$.
$P(X > b) = \sum_{x=b+1}^{\infty} p(1-p)^{x-1} = (1-p)^b$.
$P(X > a+b) = (1-p)^{a+b}$.
$P(X > a+b | X > b) = \frac{(1-p)^{a+b}}{(1-p)^b} = (1-p)^a = P(X > a)$.

36. A. $\text{Var}(XY) = E(X^2Y^2) - \left(E(XY)\right)^2 = E(X^2)E(Y^2) - \left(E(X)\right)^2\left(E(Y)\right)^2$.
$X \sim N(2,3) \Rightarrow E(X) = 2$ and $E(X^2) = Var(X) + \left(E(X)\right)^2 = 3 + 4 = 7$.
$Y \sim Uniform(2,4) \Rightarrow E(Y) = \frac{2+4}{2} = 3$ and $E(Y^2) = \int_2^4 \frac{y^2}{2} dy = 28/3$.
So $\text{Var}(XY) = 7 \times 28/3 - 2^2 \times 3^2 = 88/3$.

37. A. $P(X = 5) = 2P(X = 4) \Rightarrow \frac{e^{-\lambda}\lambda^5}{5!} = 2 \times \frac{e^{-\lambda}\lambda^4}{4!} \Rightarrow \lambda = 10$.
$P(X < 2) = P(X = 0) + P(X = 1) = e^{-10}[1 + 10] = 11e^{-10}$.

38. C. $f(x) = \lambda e^{-\lambda(x-\theta)}, x > \theta, \lambda > 0$.
$E(X) = \int_\theta^\infty \lambda x e^{-\lambda(x-\theta)} dx = \lambda \int_0^\infty (y+\theta) e^{-\lambda y} dy = \int_0^\infty \lambda y e^{-\lambda y} dy +$
$\theta \int_0^\infty \lambda e^{-\lambda y} dy = \frac{1}{\lambda} + \theta$.
$E(X^2) = \int_\theta^\infty \lambda x^2 e^{-\lambda(x-\theta)} dx = \int_0^\infty \lambda (y+\theta)^2 e^{-\lambda y} dy$
$= \int_0^\infty \lambda y^2 e^{-\lambda y} dy + 2\theta \int_0^\infty \lambda y e^{-\lambda y} dy + \theta^2 \int_0^\infty \lambda e^{-\lambda y} dy = \frac{2}{\lambda^2} + \frac{2\theta}{\lambda} + \theta^2$.
$\text{Var}(X) = E(X^2) - \left(E(X)\right)^2 = \frac{2}{\lambda^2} + \frac{2\theta}{\lambda} + \theta^2 - \left(\theta + \frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$.

39. A. $\int_0^1 \int_0^1 f(x,y) dx dy = 1$.
$\int_0^1 \int_0^1 f(x,y) dx dy = \int_0^1 \int_0^y kxy(1-y) dx dy = k \int_0^1 y(1-y) \int_0^y x \, dx \, dy =$
$\frac{k}{2} \int_0^1 (y^3 - y^4) dy = \frac{k}{40}$.
So $k = 40$.

$$E(XY) = \int_0^1 \int_0^1 xyf(x,y) dx dy = \int_0^1 \int_0^y 40x^2(y^2 - y^3) dx dy$$
$$= \int_0^1 \frac{40}{3}(y^5 - y^6) dy = \frac{40}{3}\left[\frac{1}{6} - \frac{1}{7}\right] = \frac{20}{63}.$$

$$E(X) = \int_0^1 \int_0^1 xf(x,y)dxdy = \int_0^1 \int_0^y 40x^2(y-y^2)dxdy = \int_0^1 \frac{40}{3}(y^4-y^5)dy$$
$$= \frac{40}{3}\left[\frac{1}{5}-\frac{1}{6}\right] = \frac{4}{9}.$$
$$E(Y) = \int_0^1 \int_0^1 yf(x,y)dxdy = \int_0^1 \int_0^y 40x(y^2-y^3)dxdy = \int_0^1 20(y^4-y^5)dy$$
$$= 20\left[\frac{1}{5}-\frac{1}{6}\right] = \frac{2}{3}.$$
$\text{Cov}(X,Y) = E(XY) - E(X)E(Y) = \frac{20}{63} - \frac{8}{27} = \frac{60-56}{189} = \frac{4}{189} > 0.$
Therefore correlation is positive.

40. D. The correlation coefficient between X and X^2 is
$\text{Corr}(X,X^2) = \text{Cov}(X,X^2)/\sqrt{Var(X)Var(X^2)}$
$\text{Cov}(X,X^2) = E(X^3) - E(X)E(X^2)$
$E(X) = -1 \times p + 0 \times (1-2p) + 1 \times p = 0$
$E(X^3) = (-1)^3 \times p + 0 \times (1-2p) + 1^3 \times p = 0$
$\text{Cov}(X,X^2) = 0 - 0 \times E(X^2) = 0$. Hence $\text{Corr}(X,X^2) = 0$.

Data Interpretation

41. C. The number of days with sales at least 50000 in Outlet I $= 50 - 25 = 25$.
The proportion of days $= 25/50 = 0.50$.

42. A. The percentage of days with sales less than 40000 in Outlet I $= (18/50) \times 100 = 36$.

43. B. The proportion of days with sales Rs.20000 or more but less than Rs.50000 in Outlet II $= \frac{28-8}{50} = 20/50 = 0.40$.

44. D. A. The percentage of days having sales at least Rs. 70000 in Outlet I
$= ((50 - 40)/50) \times 100 = 20$.
The percentage of days having sales at least Rs. 70000 in Outlet II
$= ((50 - 38)/50) \times 100 = 24$. This is NOT less than 20.

B. The percentage of days having sales at least Rs. 30000 in Outlet II
$= ((50 - 15)/50) \times 100 = 70$.
The percentage of days having sales at least Rs. 30000 in Outlet I
$= ((50 - 12)/50) \times 100 = 76$.
So the the percentage of days having sales at least Rs. 30000 in Outlet II is less than that in outlet I.

C. The percentage of days having sales less than Rs.60000 in Outlet I
$= (35/50) \times 100 = 70$.
The percentage of days having sales less than Rs.60000 in Outlet II
$= (32/50) \times 100 = 64$.
So the percentage of days having sales less than Rs.60000 in Outlet I is more than that of Outlet II only by 6%.

D. The percentage of days having sales at least Rs.50000 but less than Rs.80000 in outlet I $= (42 - 25) \times 100/50 = 34$.
The percentage of days having sales at least Rs.50000 but less than Rs.80000 in outlet II $= (45 - 28) \times 100/50 = 34$.

45. D. The number of students indicated for MS $= 65 + 14 + 12 + 3 = 94$.

46. A. The number of students who do not like CS $= 65 + 14 + 52 + 3 = 134$.

47. D. The number of students who like only one type song $= 65 + 13 + 52 = 130$.

48. A. The number of students who like both MS and CS $= 12 + 3 = 15$.

49. C. The number of applications is maximum in Sep for all age groups.

50. D. The number of applications in the age group [45, 60) is less than that of the other age groups for all the months.

51. B. A. The number of applications is in the age group [18, 25) is more than 250 in all months except Mar and May. In Mar, it is slightly less than 250 and in May it is slightly less than that of Mar.
But the number of applications is more than 300 in the months of Aug, Sep and Oct.
So total number of applications exceeds 3000.

B. It is clear that maximum number of applications received in the age group [25, 35) in all months. The number of applications is at least 400 in all the months. Also the number of applications exceeds 450 in Feb, Jun, Sep and Dec.
So total number of applications must be greater than
$8 \times 400 + 4 \times 450 = 3200 + 1800 = 5000$.

C. The total number of applications considering all age groups:
It exceeds 3000 in the age group [18, 25).
It exceeds 5000 in the age group [25, 35).
In the age group [35, 45), the minimum number of applications is about 280 in Oct.
It exceeds 400 in 4 months, 300 in 6 months and 350 in one month. So the total number of applications must be greater than
$280 + 4 \times 400 + 6 \times 300 + 350 = 4030$.

The number of applications in the age group [45, 60) is greater than $12 \times 170 = 2040$.
So total number of applications is more than 3000+5000+4030+2040 = 14070.

D. The number of applications received in the age group is maximum in Sep.

English

52. B.
53. B.
54. D.
55. C.
56. C.
57. C.
58. A.
59. D.
60. C.
61. D.
62. A.

Logical reasoning

63. A. The only son of B's grandfather must be B's father. A must be a son of his brother's father, who is B 's father too. Therefore, B and A must be siblings. Gender of B is not mentioned. A is the only appropriate option among the available ones.

64. A. Since 1996 was a leap year, it had 366 days, ie, two days in excess of 52 weeks. Those two days included a Monday and a Tuesday.

65. B. Each hour is allocated $\frac{360}{12} = 30$ degrees. The hour hand moves by 30 degrees every hour.

66. B. $1000 - 100 = 900$.

67. C. A is not correct, since Yatika can skip a class even if Sujatha or Arsal skips it. For the same reason B is not correct. C is a valid inference from negation of the given statement. D is not correct, since there can be other circumstances of Zafar's absence – apart from Sujatha's attendance.

68. A. The number of persons who play at least one of the two games is $85 - 8 = 77$. Out of these, $77 - 45 = 32$ persons do not play football, and therefore they play basketball only. Therefore, the number of basketball players who also play football is $37 - 32 = 5$.

69. D. The immediate neighbours of Hemant (wearing blue) wear yellow and arrange. The second position to his left is occupied by the person wearing white. Therefore, the only slot for the person wearing purple is the second place to the right of Hemant. This is precisely where Elly seats. Therefore, the person wearing purple must be Elly.

70. C. Since only a mango can be an apple, there is no possibility of some apple being guava.

    Even though some guava is grape and some guava is mango, but the guava that is grape need not be mango. Thus, one cannot conclude that some grape is mango.

    The possibility of some non-grape mango being orange is not ruled out being any of the statements.

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