Institute of Actuaries of India

ACET February 2020 Solutions

Mathematics

1. C. f is both one-to-one and onto.

2. A. $f \circ g(x) = f\big(g(x)\big) = f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right) + 1 = x.$
   $g \circ f(x) = g\big(f(x)\big) = g(2x+1) = \frac{(2x+1)-1}{2} = x.$

3. C. Let $f(x) = x^3 - 4x - 7$. Then $f(0) = -7\ (-ve)$, $f(1) = -10(-ve)$, $f(2) = -7\ (-ve)$ and $f(3) = \ 8\ (+ve)$. Since $f(x)$ is continuous, there exists a positive root between 2 and 3.

4. D. $= \tan^{-1}x; f(0) = 0.$

   $f^{(i)}(x) = \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots \quad ; f^{(i)}(0) = 1 = 1!$
   $f^{(ii)}(x) = -2x + 4x^3 - 6x^5 + \cdots \quad ; f^{(ii)}(0) = 0$
   $f^{(iii)}(x) = -2 + 12x^2 - 30x^4 + \cdots \quad ; f^{(iii)}(0) = -2 = -(2)!$
   $f^{(iv)}(x) = 24x - 120x^3 + \cdots \quad ; f^{(iv)}(0) = 0$
   $f^{(v)}(x) = 24 - 360x^2 + \cdots \quad ; f^{(v)}(0) = 24 = 4!$

   $$f(x) = \tan^{-1}x = f(0) + \frac{f^{(i)}(0)}{1!}x + \frac{f^{(ii)}(0)}{2!}x^2 + \frac{f^{(iii)}(0)}{3!}x^3 + \frac{f^{(iv)v}(0)}{4!}x^4 + \frac{f^{(v)}(0)}{5!} + \cdots$$
   $$= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots.$$

5. B. $\frac{\binom{n}{k}}{\binom{n}{k-1}} = \frac{n-k+1}{k} = \frac{10}{5} = 2$ implies $n - 3k + 1 = 0$

   $\frac{\binom{n}{k}}{\binom{n}{k+1}} = \frac{k+1}{n-k} = \frac{10}{10} = 1$ implies $n - 2k - 1 = 0$

   These imply $n = 5$ and $k = 2$. Thus $\binom{n}{k+2} = \binom{5}{4} = 5.$

6. A. $\frac{3x+5}{x^2(x-1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$ implies $3x + 5 = A(x-1) + Bx(x-1) + Cx^2.$
   Letting $x = 0$ gives $A = -5$ and $x = 1$ gives $C = 8$. Thus, $C^A = 8^{-5} = \frac{1}{8^5}.$

7. D. Let $S_n = 1.6 + 2.7 + 3.8 + \cdots + n(n+5) = \sum_{i=1}^{n} i(i+5)$

   $$= \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} 5i = \frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}$$
   $$= \frac{n(n+1)(2n+1) + 15n(n+1)}{6}$$
   $$= \frac{n(n+1)(2n+1+15)}{6} = \frac{n(n+1)(n+8)}{3}.$$

8. A. Let α, β be the roots of the quadratic equation. Arithmetic mean: $\frac{\alpha+\beta}{2} = a$; Geometric mean: $\sqrt{\alpha\beta} = b \rightarrow \alpha\beta = b^2$. The required equation is

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

$$\text{i.e.,} \quad x^2 - 2ax + b^2 = 0.$$

9. B. $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \cdots + \frac{1}{\log_{25} n} = \log_n 2 + \log_n 3 + \ldots + \log_n 25$.

$$= \log_n(2 \times 3 \times \ldots \times 25) = \log_n 25! = \frac{1}{\log_{25!} n}.$$

10. A. $\lim_{x\to 0} \frac{x-\sin x}{x \sin x}$ has the $\frac{0}{0}$ form.

By L' Hopital's rule, we have $\lim_{x\to 0} \frac{1-\cos x}{x\cos x+\sin x}$, which also has $\frac{0}{0}$ form.

Again, by L' Hopital's rule, we have

$$\lim_{x\to 0} \frac{1-\cos x}{x\cos x+\sin x} = \lim_{x\to 0} \frac{\sin x}{-x\sin x+2\cos x} = 0.$$

11. C. Let O be the origin.

Then, $\overrightarrow{OA} = 3\vec{\imath} - 7\vec{\jmath} - 7\vec{k}$ and $\overrightarrow{OB} = 5\vec{\imath} + 4\vec{\jmath} + 3\vec{k}$.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\vec{\imath} + 11\vec{\jmath} + 10\vec{k}.$$

$$|\overrightarrow{AB}| = \sqrt{2^2 + 11^2 + 10^2} = \sqrt{225} = 15.$$

Thus, the direction cosines of $\overrightarrow{AB}$ are $\left(\frac{2}{15}, \frac{11}{15}, \frac{10}{15}\right)$.

12. D. Let $\vec{a} = 3\vec{\imath} + \vec{\jmath} + 4\vec{k}$ and $\vec{b} = \vec{\imath} - \vec{\jmath} + \vec{k}$. Then $\vec{a}\cdot\vec{b} = (3\times 1) - (1\times 1) + (4\times 1) = 6$. Also $|\vec{a}| = \sqrt{9+1+16} = \sqrt{26}$; $|\vec{b}| = \sqrt{1+1+1} = \sqrt{3}$. Hence $\cos\theta = \frac{|\vec{a}\cdot\vec{b}|}{|\vec{a}||\vec{b}|} = \sqrt{\frac{6}{13}}$.

13. B. $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$AB = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 10 & 3 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 8 & 1 \end{bmatrix}.$$

$$AB + BA + I = \begin{bmatrix} 0 & -1 \\ 10 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 8 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 18 & 5 \end{bmatrix}.$$

14. A. If the matrix $\begin{bmatrix} x-1 & 2 & 0 \\ 2 & x-4 & 0 \\ 0 & 0 & x-3 \end{bmatrix}$ is singular, then its determinant is 0.

i.e., $(x-1)(x-4)(x-3) - 2\times 2(x-3) = 0$,

i.e., $(x-3)\big((x-1)(x-4) - 4\big) = 0$, i.e., $(x-3)(x^2-5x) = 0$,

i.e., $(x-3)x(x-5) = 0$ implying $x = 0$ or $x = 3$ or $x = 5$.

15. C. The rank of the matrix is 2, as it has two linearly independent columns.

16. B. $y = (kx - 5)e^{4x}$ then, $\frac{dy}{dx} = (kx - 5)4e^{4x} + ke^{4x}$.

$\frac{dy}{dx} = -11$ at $x = 0$ implies $-5(4) + k = -11$. Hence, $k = 9$.

17. D. $x = a\sin\theta$ and $y = b\cos\theta$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-b\sin\theta}{a\cos\theta} = -\frac{b}{a}\tan\theta.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{b}{a}\tan\theta.\right) = -\frac{b}{a}\left(\frac{d}{d\theta}\tan\theta\right)\frac{d\theta}{dx}$$

$$= -\frac{b}{a}(\sec^2\theta)\ \frac{1}{a\cos\theta} = -\frac{b}{a^2}sec^{\,3}\theta$$

18. A. Let $f(x) = \log_e\left(\frac{2-x}{2+x}\right)$. Hence,

$$f(-x) = \log_e\left(\frac{2+x}{2-x}\right) = \log(2+x) - \log(2-x)$$

$$= -(\log(2-x) - \log(2+x)) = -\log\left(\frac{2-x}{2+x}\right) = -f(x).$$

This implies f is an odd function. Hence, $\int_{-1}^{1}\log_e\left(\frac{2-x}{2+x}\right)dx = 0$.

19. B. Let $x^2 = t$ implies $2xdx = dt$

$$\int x^3\, e^{x^2}dx = \frac{1}{2}\int t\, e^t dt = \frac{1}{2}\left[t\, e^t - \int e^t\, dt\right] = \frac{1}{2}\, e^t(t-1) + c$$

$$= \frac{1}{2}\, e^{x^2}(x^2 - 1) + c.$$

20. B. By Trapezoidal rule

$$\int_0^1 f(x)dx = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\big(f(x_1) + \cdots + f(x_{n-1})\big)\right]$$

$$= \frac{0.2}{2}[(0.5 + 1) + 2(0.96154 + 0.86207 + 0.73529 + 0.60976)]$$

$$= 0.1[1.5 + 6.33732] = 0.1[7.83732] = 0.783732.$$

Statistics

21. B. All possible cases: $\binom{6}{3} = 20$.

Cases favourable to the event: {(1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (1, 3, 5), (2, 4, 6)}.

Number of cases favourable to the event (selected numbers are in AP) = 6.

22. A. Total number of arrangements = $\frac{9!}{3!2!}$.

Number of arrangement when all the T's occur together = $\frac{7!}{2!}$.

Required probability = $\frac{7!}{2!} / \frac{9!}{3!2!} = \frac{6}{9\times 8}$ = 1/12.

23. C. Suppose $E_1 =$ the event that C_1 fails; $E_2 =$ the event that C_2 fails.

$P(C_1 \text{ fails alone}) = P(E_1\bar{E}_2) = 0.25$. $P(C_1 \text{ and } C_2 \text{ fail}) = P(E_1E_2) = 0.25$.

$P(E_1) = P(E_1E_2) + P(E_1\bar{E}_2) = 0.25 + 0.25 = 0.50$

$P(C_2 \text{ fails } | C_1 \text{has failed}) = P(E_1E_2)/P(E_1) = \frac{0.25}{0.50}$ = ½.

24. D. $P(A) = 0.6$, $P(B) = 0.2$ and $P(A|B) = 0.5$.

$P(AB) = P(B) \times 0.5 = 0.2 \times 0.5 = 0.1.$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A\,B)]$

$= 1 - [0.6 + 0.2 - 0.1] = 0.3.$

$P(\bar{A}\,|\bar{B}) = \frac{P(\bar{A}\,\bar{B})}{P(\bar{B})} = 0.3/0.8 = 3/8.$

25. A. Let $E_1 =$ the event that a car is manufactured by plant I; $E_2 =$ the event that a car is manufactured by plant II; S = the event that the car is of standard quality.

$P(E_1) = 0.6, \qquad P(E_2) = 0.4, \qquad P(S|E_1) = 0.9, \qquad P(S|E_2) = 0.8.$

Therefore, $P(E_1|S) = \frac{P(E_1)P(S|E_1)}{P(E_1)P(S|E_1)+P(E_2)P(S|E_2)} = \frac{0.6\times 0.9}{0.6\times 0.9+0.4\times 0.8}$ = 54/86 = 27/43.

26. B. $\sum_{i=1}^{20}(x_i - 30) = -20$. This implies $\sum_{i=1}^{20} x_i = 600 - 20 = 580$; $\bar{x} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{580}{20} = 29.$

27. D. Let $n_i =$ number of students in group i, i =1, 2; $\bar{x}_i =$ average marks of group i students, i =1, 2; $\bar{x}$ = average marks of all the students = $(n_1\,\bar{x}_1 + n_2\,\bar{x}_2)/(n_1 + n_2)$. Then

$$(n_1 + n_2) \times 70 = n_1 \times 68 + n_2 \times 73.$$

This implies $2\,n_1 = 3\,n_2$, i.e., percentage of group 1 students = $\frac{3}{5} \times 100 = 60$.

28. C. Median = $\frac{x+x+2}{2} = 6$. This implies $x = 5$.

Mean = $(-1 + 0 + 3 + 5 + 7 + 9 + 12 + 13)/8 = 6.$

29. A. Standard deviation s = 5.2.

Coefficient of variation cv = $\frac{s}{\bar{x}} \times 100$ = 10.4. So $\bar{x}$ = 50.

If each observation is increased by 2, then new $\bar{x}$ = 52; s remains unchanged.

New CV = $\frac{5.2}{52} \times 100 = 10\%$.

30. B. $Q_1 = 10 + 0.25 \times 20 = 15$, $Q_2 = 10 + 0.5 \times 20 = 20$,
$Q_3 = 10 + 0.75 \times 20 = 25$. Interquartile range = $Q_3 - Q_1 = 25 - 15 = 10$.

31. A. $E(X) = 10$, i.e.,
$0 \times P(X = 0) + 5 \times P(X = 5) + 10 \times P(X = 10) + 15 \times P(X = 15) = 10$.
This implies 5 $P(X = 5) + 10 \times 0.3 + 15 \times 2\, P(X = 5) = 10$, i.e., $P(X = 5) = 0.2$.
Therefore, $P(X = 0) + P(X = 5) + P(X = 10) + P(X = 15) = 1$,
i.e., $P(X = 0) + 0.2 + 0.3 + 0.4 = 1$. So $P(X = 0) = 0.1$.

32. C. $E(X) = 5$ and $E(X^2) = 25$.

$$Var(X) = E(X^2) - \left(E(X)\right)^2 = 25 - 25 = 0, \text{ i.e., } X = E(X) = 5.$$

$$E\left(X + E(X)\right)^3 = E(5+5)^3 = 1000.$$

33. D. X is a continuous random variable with $P(X > x) = e^{-0.5\,x}$.
Distribution function of X is $F(x) = 1 - e^{-0.5\,x}$.
PDF of X is $f(x) = 0.5\, e^{-0.5\,x}$.
$E(X) = \int_0^\infty x\, f(x) dx = \int_0^\infty x \times 0.5 e^{-0.5\,x}\, dx = \frac{1}{0.5} = 2$.

34. A. $X \sim Poisson(\lambda)$; $P(X = 0) = 0.1$. This implies $e^{-\lambda} = 0.1$, that is $\lambda = -\ln 0.1$.
$P(X = 0) = 0.1$ and $P(X = 1) = \lambda\, e^{-\lambda} = -0.1 \times \ln 0.1$.
So $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.1 + 0.1 \times \ln 0.1 = 0.9 + 0.1 \times \ln 0.1$.

35. D. $X \sim$ Binom (n, p). $E(X) = np$, Var$(X) = npq$.
$np = 4$ and $npq = 2$. $q = 1/2$, That is $p = 1/2$. $n = 8$.
$P(X \geq 1) = 1 - P(X = 0) = 1 - \left(\frac{1}{2}\right)^8 = 255/256$.

36. C. $1 = \frac{1}{4} \int_{-c}^{c} |x| dx = \frac{1}{4}\left(\frac{2c^2}{2}\right) = \frac{c^2}{4}$. This implies $c = 2$.

37. D. $X \sim N(\mu, \sigma^2)$, where = $\mu = 5$, $\sigma^2 = 1.5$.
$E(Y) = E(2\, X^2 + 3) = 2\, E(X^2) + 3$.
$E(X^2) = \mu^2 + \sigma^2 = 25 + 1.5 = 26.5$.
$E(Y^2) = 2 \times 26.5 + 3 = 56$.

38. C. $P(Y < X) = P(X = 0, Y = -1) + P(X = 1, Y = 0) + P(X = 1, Y = -1) = 0.1 + 0.15 + 0.05 = 0.3$.

39. B. Let ρ be the correlation coefficient between X and Y.

$$\text{Cov}\left(X + kY, X + \frac{2}{3}Y\right) = \text{Var}(X) + k\,\text{Cov}(X,Y) + \frac{2}{3}\text{Cov}(X,Y) + \frac{2k}{3}\text{Var}(Y)$$

$$= 4 + \left(k + \frac{2}{3}\right)\text{Cov}(X,Y) + \frac{2k}{3} \times 9 = 4 + \left(k + \frac{2}{3}\right) \times 6\rho + 6k$$

$$= 4(1+\rho) + 6k(1+\rho) = (4+6k)(1+\rho) = 0 \text{ when } k = -2/3.$$

40. B. Regression of y on x: $x + 5y + 3 = 0$.

This is equivalent to $(z-2)+5\left(\frac{w-3}{2}\right)+3=0$, i.e., $2z-4+5w-15+6=0$.

Data Interpretation and Data Visualization

Answer for 41-43: Summary table

Salary in Rs. (x)	Cumulative percentage of employees	Percentage of employees	Number of employees
$x < 15000$	30%	30%	60
$15000 \le x < 18000$	50%	20%	40
$18000 \le x < 25000$	70%	20%	40
$25000 \le x < 30000$	85%	15%	30
$30000 \le x < 40000$	95%	10%	20
$40000 \le x$	100%	5%	10

41. A. The number of employees whose salary is Rs. 30000 or above = 20+10 = 30.

42. D. The percentage of employees, whose salary is Rs. 15000 or above but less than Rs. 30000, is $20\% + 20\% + 15\% = 55\%$.

43. B. The ratio of the numbers of employees having salary less than Rs. 15000 to those having salary Rs. 30000 or above is $= 60 : (20+10) = 2:1$.

Answer for 44 and 45: Frequency Distribution

Number of visits	Frequency
1	1
2	3
3	7
4	9
5	7
6	7
7	3
8	3
Total	40

44. B. 4 has highest frequency. So mode is 4.

45. D. The number of customers of visited 5 or more days is $7+7+3+3=20$. The proportion is $20/40=0.50$.

Answer for 46 and 47: Summary table

Age in Years	Type of Work					
	Manual			Non-manual		
	Male	Female	Total	Male	Female	Total
10 - 15	15	30	45	0	0	0
16 - 20	22	25	47	30	5	35
21 - 30	35	50	85	70	40	110
31 - 45	30	45	75	45	20	65
46 - 60	20	15	35	20	10	30
above 60	8	5	13	5	0	5
Total	130	170	300	170	75	245

46. C. Total work-force = 300+245 = 545. Number of male workers = 130+170 = 300.
The percentage of males in the overall work-force of the community is $(300/545) \times 100 = 55.04\%$.

47. C. The average age of female non-manual workers in the age group 31-60 years:
Mid value of 31-45 = 38, frequency = 20
Mid value of 46 – 60 = 53, frequency = 10
The average age of female non-manual workers in the age group 31-60 years = $(20 \times 38 + 10 \times 53)/(20 + 10) = 43$ years.

48. A. Percentage of female manual workers:
10-15: $(30/45) \times 100 = 66.67$ (Highest)
16-20: $(25/47) \times 100 = 53.19$.
21-30: $(50/85) \times 100 = 58.52$.
31-45: $(45/75) \times 100 = 60$.
46-60: $(15/35) \times 100 = 42.86$.
60 and above: $(5/13) \times 100 = 38.46$.

49. C. The maximum number of homicides occurs in 2003.

50. B. Years in which number of homicides is less than that in the year 1996: 2010, 2014, 2017, 2018.

51. B. The expenses on the components other than prizes is approximately = 201.32 × 0.43 = 86.57.

English

52. D

53. B

54. B

55. D

56. A

57. B

58. A

59. C

60. D

61. C

62. D

Logical Reasoning

63. D. No. of people who read *Hindustan Times* = 185.
  No. of people who read *Amar Ujala* = 127.
  No. of people who read *Dainik Jagaran* = 212.
  Now,
  No. of people who read both *Hindustan Times* and *Amar Ujala* = 20.
  No. of people who read both *Amar Ujala* and *Dainik Jagaran* = 35.
  No. of people who read both *Hindustan Times* and *Dainik Jagaran* = 29.
  Let No. of people who read *Hindustan Times*, *Amar Ujala* and *Dainik Jagaran* $= x$.
  So, no. of people reading only *Hindustan Times* is $185 - 20 - 29 + x = 136 + x$;
  only *Amar Ujala* is = $127 - 20 - 35 + x = 72 + x$;
  only *Dainik Jagaran* is $= 212 - 35 - 29 + x = 148 + x$;
  Now, $(136 + x) + (72 + x) + (148 + x) + (20 - x) + (29 - x) + (35 - x) + x + 50 = 500$, i.e., $490 + x = 500$
  So, $x = 10$. This is the number who read all the 3 dailies.
  So, No. of people who read only one paper is $136 + 10 + 72 + 10 + 148 + 10 = 386$.

64. A. At 3 O'clock, Minute hand is at 12 while the Hour hand is at 3. Again the minute hand has to sweep through (30×5) ie 150° for reaching the figure 5 to show 25 mins.
  Simultaneously the Hour hand will also rotate for 25 mins. Thus, starting from the mark 3, the hour hand will cover an angle $= (25 \times 30) / 60 = 12.5°$.
  Since the minute hand has covered 60 degrees from mark 3 to mark 5,
  the angle between Hour and Minute hands $= (60 - 12.5) = 47.5°$.

65. C. The sitting order, from left to right, is UVWX.

66. D. The two months together have 61 days. The gap between the second day of the first month and the last day of the second month is 59 days, which amounts to 8 weeks and 3 days. Therefore, the last day of the second month is a Wednesday.

67. A. Sum is 9, minimum is even. The minimum cannot be 2, as the other one has to be 7. The minimum cannot be 6, as the other one has to be 3 (smaller than 6). So the minimum is 4, maximum is 5, product is 20.

68. C. If the given statement is true, then both Ramesh and Suresh are of type A. If the given statement is false, then Suresh is of type B and Ramesh is of type A.

69. C. $1 + 4 + 9 + 16 + 25 = 55$.

70. B. Converse of Statement 2 is not necessarily true. Therefore conclusion I does not follow.
  Statement 1 says that some boys are students all of whom, by statement 2, are engineers. Therefore, Conclusion II follows.