Institute of Actuaries of India

Solutions for ACET June 2017

Mathematics

1. B. $y = f(x) = 2x - 3$ or, $x = \frac{y+3}{2} = f^{-1}(y)$. Thus $f^{-1}(x) = \frac{x+3}{2}$.

2. C. $p + q = \frac{(\sqrt{5}-\sqrt{3})^2+(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{16}{2} = 8$. Thus $(p+q)^2 = 64$.

3. D. $|\vec{a}| = |\vec{b}| = |\vec{a}+\vec{b}| = 1$. $|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$. Thus, $\vec{a}\cdot\vec{b} = -\frac{1}{2}$. It follows that $|\vec{a}-\vec{b}|^2 = 3$ and $|\vec{a}-\vec{b}| = \sqrt{3}$.

4. A. $\int \log_e x \, dx = (x\log_e x - x) + c$. But $\log_2 x = \log_e x \,.\log_2 e$. Hence the result.

5. B. Let $\vec{a} = \hat{\imath} - \hat{\jmath}$ and $\vec{b} = \hat{\jmath} - \hat{k}$. So $\vec{a}\cdot\vec{b} = 0 - 1 + 0 = -1$. Further, $|\vec{a}| = |\vec{b}| = \sqrt{2}$. If α is the angle between $\vec{a}$ and $\vec{b}$, we have $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = -\frac{1}{2}$. Thus, $\alpha = 2\pi/3$.

6. D. $f(x) = \log_e(\log_e x)$. Therefore, $f'(x) = \frac{1}{\log_e x} \times \frac{1}{x}$.

7. C. $\int_0^a f(x)dx + \int_0^a f(-x)dx = \int_0^a f(x)dx + \int_{-a}^0 f(x)dx = \int_{-a}^a f(x)dx$.

8. B. The given matrix contains 3 columns of I_3 and $|I_3| = 1 \neq 0$. Hence rank = 3.

9. C. The expression is an identity if $(a^2 - 3a + 2) = 0$, $(a^2 - 5a + 6) = 0$, $a^2 - 4 = 0$ hold at the same time. Thus $a = 2$.

10. C. $I = \int_0^\pi \frac{\sin x}{1+\sin x+\cos x} dx = \int_0^\pi \frac{\sin(\pi-x)}{1+\sin(\pi-x)+\cos(\pi-x)} dx = \int_0^\pi \frac{\sin x}{1+\sin x-\cos x} dx = J$. Thus $I = J$.

11. D. Note that $0 \le |K\vec{a}| = |K|\cdot|\vec{a}| \le 4 \times 5 = 20$. Thus maximum value = 20.

12. A. Clearly $0 \le \frac{x^2}{1+x^2} < 1$, for any real x. or, $\cos^{-1} 1 < \cos^{-1}\left(\frac{x^2}{1+x^2}\right) \le \cos^{-1} 0$ or, $0 < f(x) \le \frac{\pi}{2}$.

13. B. $\frac{dy}{dx} = m = \frac{2x}{1+x^2}$ or, $|m| = \frac{2|x|}{1+|x|^2} \le 1$, as $(1-|x|)^2 \ge 0$. Thus $-1 \le m \le 1$.

14. A. Put $z = \log_e x$, so given integral $= \int_0^{73} \pi \sin(\pi z)\, dz = 2$, as $\cos(73\pi) = -1$.

15. D. Let the requisite sum be p. Hence, $x = \frac{p \times x \times x}{100}$, or, $p = \frac{100}{x}$, assuming $x \neq 0$.

16. B. Let $f(x) = ax^2 + bx + c$, so $f'(x) = 2ax + b$ and $f''(x) = 2a$. As per given condition $2 = f(0) = c, 3 = f'(0) = b, 4 = f''(0) = 2a$, so that $a = 2, b = 3, c = 2$. Thus $\int_{-1}^{1} f(x)dx = \int_{-1}^{1}(2x^2 + 3x + 2)dx = \frac{16}{3}$.

17. D. $C^2 = \begin{pmatrix} a^2 & 0 \\ a+1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 7 & 1 \end{pmatrix} = D$. Thus a^2=1 and $a + 1 = 7$, which contradict each other. Hence a is non-existent.

18. C. Limit of $f(x)$ as x tends to zero is $\lim_{x \to 0} \frac{\tan 3x}{\sin 2x} = \lim_{x \to 0} \frac{\sin 3x}{3x} \times \frac{1}{\cos 3x} \times \frac{2x}{\sin 2x} \times \frac{3}{2} = \frac{3}{2}$.

Statistics

19. A. It is generally known that A.M. ≥ G.M. ≥ H.M. The three means are equal only when all the numbers are equal, but they are said to be distinct. Thus, B is not correct.

20. B. As 77 corresponds to the highest frequency 21.

21. C. $p = \frac{1}{5}, q = \frac{4}{5}$. Variance = $npq = \frac{4n}{25}$. Now $\frac{4n}{25} = \frac{6}{25}$ cannot hold for any integer n. Options A, B and D are possible for different values of n.

22. A. Note that $\beta = \rho \frac{s_y}{s_x}$. The transformation does not change ρ or s_y but changes s_x by the multiplying factor $\frac{9}{5}$. Hence the answer.

23. C. n, total number of matches = 30, even. Median = $\frac{\frac{n}{2}\text{th observation}+\left(\frac{n}{2}+1\right)\text{th observation}}{2} = \frac{\text{15th observation}+\text{16th observation}}{2} = \frac{3+3}{2} = 3$.

24. B. Required probability = $P(A \cap B^c) + P(A^c \cap B) = [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = P(A) + P(B) - 2P(A \cap B)$, as $A \cap B^c, A \cap B$ and $A^c \cap B$ are mutually exclusive events.

25. C. Nine distinct and ordered digits can be chosen from 10 (0 to 9) numbers in ${}^{10}P_9 = 10!$ ways, out of which 0 will appear in the first place in $9_{P_8} = 9!$ ways. Thus the number of numbers with distinct digits = 10! – 9! = 9 × 9!. The total number of 9 digit numbers is 9×10^8. Therefore, the probability that the selected number has distinct digits is $\frac{9 \times 9!}{9 \times 10^8} = \frac{9!}{10^8}$.

26. B. X: event that the sum is 7 and Y: event that 2 appears at least once. So $X \cap Y = \{(2,5),(5,2)\}$. $P(X) = \frac{6}{36}$, $P(Y) = \frac{11}{36}$ and $P(X \cap Y) = \frac{2}{36}$. Required probability $P(Y|X) = \frac{P(X \cap Y)}{P(X)} = \frac{2}{6} = \frac{1}{3}$.

27. D. Sample space = $\{1, 1, 1, 2, 2, 5\}$. Clearly, $P(1) = \frac{3}{6}$, $P(2) = \frac{2}{6}$, $P(3) = \frac{1}{6}$. Mean = $\left(1 \times \frac{3}{6}\right) + \left(2 \times \frac{2}{6}\right) + \left(5 \times \frac{1}{6}\right)$ = 2.

28. C. The probability of correct answer $p = \frac{1}{4}$; so the probability of wrong answer $q = 1 - \frac{1}{4} = \frac{3}{4}$. Clearly $P(r) = {}^{n}C_{r} p^{r} q^{n-r}$, where r is the number of correct answers and $n = 5$. Required probability = $P(4) + P(5) = {}^{5}C_{4}\left(\frac{1}{4}\right)^{4}\left(\frac{3}{4}\right) + {}^{5}C_{5}\left(\frac{1}{4}\right)^{5} = \frac{1}{4^{3}}$.

29. D

30. B

31. D. $Var(X^{*}) = Var\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^{2}} Var(X - \mu) = \frac{1}{\sigma^{2}} Var(X) = 1$.

Data Interpretation

32. A. City T contributes the largest summand to the sum in Problem 31. The number is 10500.

33. B. City P has the same second lowest number of movie watchers which is 4000.

    4000= 2000/(1-50%)

34. **D**. The total number is

$$50 \times \frac{2000}{100-50} + 20 \times \frac{3000}{100-20} + 75 \times \frac{1400}{100-75} + 40 \times \frac{3000}{100-40} + 60 \times \frac{7000}{100-60} = 2000 + 750 + 4200 + 2000 + 10500 = 19450.$$

35. **A**. The average sales cannot be as small as Rs. 642 lakh, as there are many large values to offset the few marginally smaller values. For the same reason, Rs. 383 lakh is too small a value for average cost.

36. **C**. 4%. 4% = 20% of 20% .

37. **D**. Cannot be determined. Only percentage or occurrence of proteins in bones is provided but not the overall percentage of bones in the body.

38. **B**. 10 lakhs. Let the total production in 2001 be x.

$$(1.2 \times 1.2 \times 20\% - 20\%)x = 2200, \text{ so } x = 25000.$$

$$x \times 40\% \times 100 = 25000 \times 40\% \times 100 = 1000000.$$

English

39. D

40. D

41. C

42. B

43. B

44. A

45. B

46. B

47. C

48. A

49. A

50. B

51. B

52. D

53. C

54. A

55. C

56. D

57. C

58. A

59. B

60. D

61. B

62. A

Logical Reasoning

63. A

64. A. Diagram A has a black dot inside a triangle, which is inside a circle, which in turn is inside a rectangle. In diagram B, the black dot is intact, the order of containment between the second and third items (triangle and circle) is reversed and the outermost rectangle is missing.

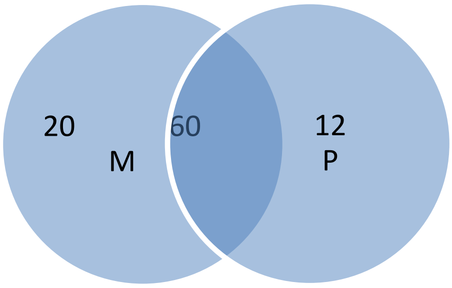
    In diagram C, the second and third items, in order of containment, are a square and a triangle. By reversing their order and dropping the outermost rectangle, we have the shape given in option (1).

65. D. Drizzle is weaker/slower than rain; walking is slower than running.

66. B. In a normal year the first day and the last day of the year is the same. In a leap year the last day is one day more than the first day. In this question it is a leap year. So 16(Jan)+ 29(Feb) + 15(March) = 60 days.

67. D. The lady is the wife of grandson of Amit's mother i.e., the lady is the wife of son of Amit. Hence, Amit is the father-in-law of the lady.

68. B. $100 - 20 - 60 - 12 = 8$.



69. B. A is sitting next to B, while A and C are sitting together. Therefore, A is sitting in between B and C.

    (The given facts imply that the seated persons from left to right are E B A C D, but this is not needed to answer the question.)

70. A. The number of hours from 8:00 p.m. on Thursday to 8:00 a.m. on Monday is 84. In 84 hours, the watch lost 12 minutes. But to show the correct time, the watch has to lose 5 minutes. This would happen in $\frac{5}{12} \times 84 = 35$ hours. 35 hours from 8:00 p.m. on Thursday is 7:00 a.m. on Saturday.