

Institute of Actuaries of India

ACET May 2024 Indicative Solutions

Mathematics

1. C.

Using De Morgan's property of sets, the rest can be proved. For C, it should be $\bar{A} \cap \overline{(B \cap C)} = \bar{A} \cap (\bar{B} \cup \bar{C})$

2. D.

Let $x = \sqrt{42 + x}$, then $x^2 - x - 42 = 0$,
which has roots -6 and 7 . Since x is positive, $x = 7$.

3. A.

The determinant is -4 , Adjoint matrix is $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 3 \end{bmatrix}$, hence the inverse

matrix is $\begin{bmatrix} -0.25 & 0.75 & -0.25 \\ 0.75 & -0.25 & -0.25 \\ -0.25 & -0.25 & 0.75 \end{bmatrix}$

4. D.

Direct formula, can also be obtained as follows

$$\int (x + \sin x) dx = \int x dx + \int \sin x dx = x^2/2 + (-\cos x) + C = x^2/2 - \cos x + C$$

5. A.

$K^T = \begin{bmatrix} 4 & 5 & 6 \\ 16 & 20 & 24 \\ 20 & 25 & 30 \\ 24 & 30 & 36 \end{bmatrix}$ Therefore, $KK^T = \begin{bmatrix} 20 & 25 & 30 \\ 24 & 30 & 36 \end{bmatrix}$. The determinant is 0. All

submatrices also have determinant 0. Therefore, the rank is 1

6. C

The magnitude of vector is $\sqrt{1 * 1 + 2 * 2 + 2 * 2} = \sqrt{9} = 3$

7. B

$$(x + iy)(2 + 3i) = (2x - 3y) + (3x + 2y)i.$$

Since this is conjugate of $-5+6i$, i.e., $-5-6i$,

we have $2x-3y = -5$ and $3x+2y = -6$.

Solving for x and y , we have $x = -28/13$, $y = 3/13$.

8. D

The area is given by $\int_1^3 2x \, dx$ is x^2 from 1 to 3, hence $9-1 = 8$.

9. B

Given, $n(C) = 48$, $n(F) = 36$, $n(B) = 29$. $n(C \cup F \cup B) = 64$, $n(C \cap F \cap B) = 4$.

$n(C \cap F) + n(F \cap B) + n(B \cap C) = 4 + 48 + 36 + 29 - 64 = 53$

$n(C \cap F) + n(F \cap B) + n(B \cap C) - 3 * n(C \cap F \cap B) = 53 - 12 = 41$.

10. B

$|x-1|$ is not differentiable at 1 and $|2x+1|$ is not differentiable at $-1/2$

11. A.

$n(X-Y) = 75$ implies that $n(X \text{ intersection } Y) = 25$. Hence $n(Y-X) = 100$.

$n(X \cup Y) = 75 + 25 + 100 = 200$.

12. D.

13. D. Since $\log_x y = \log_a y / \log_a x$.

14. C. By using gamma function definition, the integral part of this equals 6!

15. C. Using linear interpolation on $\log_a 1.8 = 0.255$ and $\log_a 1 = 0$, we have

$$(y-y_1)/(x-x_1) = (y_2-y_1)/(x_2-x_1)$$

$$(y-0)/(1.5-1) = (0.255-0)/(1.8-1) \Rightarrow y = 0.5 * 0.255 / 0.8 = 0.159$$

16. D. Cosine function is not one-to-one and hence not invertible

17. B. Infinite. It is discontinuous at all Integers.

18. A. Let $y = (\sin^{-1} x) \log(x)$

Differentiate on both sides w.r.t x

$$dy/dx = d/dx(\sin^{-1}(x)) * \log x + \sin^{-1} x * (d/dx(\log x))$$

$$dy/dx = \frac{\log x}{\sqrt{1-x^2}} + \frac{\sin^{-1} x}{x}$$

19. D. $f(x) = y = (3x-5)/2 \Rightarrow 2y = 3x-5 \Rightarrow 2y+5 = 3x$

$$\Rightarrow x = (2y+5)/3 \Rightarrow f^{-1}(y) = (2y+5)/3.$$

20. A.

$$\log_4 x + \log_8 x + \log_{16} x = 13/3$$

$$\Leftrightarrow 1/\log_x 4 + 1/\log_x 8 + 1/\log_x 16 = 13/3$$

$$\Leftrightarrow 1/(2\log_x 2) + 1/(3\log_x 2) + 1/(4\log_x 2) = 13/3$$

$$\Leftrightarrow 13/(12\log_x 2) = 13/3$$

$$\Leftrightarrow 1/(4\log_x 2) = 1$$

$$\Leftrightarrow \log_x 2 = 1/4$$

$$\Leftrightarrow x = 2^4 = 16$$

Statistics

21. B $5/18$. Probability for sum 6 and 8 is $5/36$. Getting either of them is $5/18$.

22. C

Case 1: Selecting 0 blue marble: $\binom{20}{7}$

Case 2: Selecting 1 blue marble: $\binom{10}{1} \times \binom{20}{6}$

Case 3: Selecting 2 blue marbles: $\binom{10}{2} \times \binom{20}{5}$

Case 4: Selecting 3 blue marbles: $\binom{10}{3} \times \binom{20}{4}$

Total ways: $\binom{20}{7} + \binom{10}{1} \times \binom{20}{6} + \binom{10}{2} \times \binom{20}{5} + \binom{10}{3} \times \binom{20}{4}$

Solving we get $77,520 + (10 \times 38,760) + (45 \times 15,504) + (120 \times 4,845)$

$= 77,520 + 3,87,600 + 6,97,680 + 5,81,400$

$= 17,44,200$

23. C. Using the formulae $n_1(m_1) + n_2(m_2) = (n_1 + n_2)m$ and

$n_1(m_1^2 + s_1^2) + n_2(m_2^2 + s_2^2) = (n_1 + n_2)(m^2 + s^2)$, we get $s_2 = 4$.

where n , m and s denote number of observations, mean and standard deviation respectively.

24. D. Calculating the mean, we have $5p + 200 = (40 + p) \cdot 5 \Rightarrow p = 10$

25. C. 25th and 26th observations are in value 5 and frequency for value 5 is 10

26. D. Mean doesn't change, median is now 4 and mode is 3.

27. D. $E[X^6] = 1^6 \cdot (0.4) + 0^6 \cdot (0.6) = 0.4$

28. B.

$P(X=4) = \frac{e^{-\lambda} \lambda^4}{4!}$ and $P(X=5) = \frac{e^{-\lambda} \lambda^5}{5!}$. Dividing and equating to $5/4$, gives $\lambda = 4$

29. C. Exponential is continuous not discrete

30. A. From the given probabilities, the mean is 12. Since in exponential distribution, $\text{Var}(X) = E[X]^2 = 144$.

31. C. The number of permutations of the word TENNESSEE is $9!/(4!*2!*2!) = 3780$. The number of permutations in which all Es are together = $6!/(2!*2!) = 180$. Hence the number of permutations in which all the 4 Es are not together is $3780-180 = 3600$.

32. D. Case 1: when King of spades is selected, we have $1C1 * 36C3 = 7,140$
Case 2: when King of spades is not selected, we have $3C1 * 12C1 * 36C2 = 22,680$. Total = $7,140+22,680 = 29,820$.

33. A. The median for first half set is 2, i.e., $Q1 = 2$. The median for 2nd half set is 21, i.e., $Q3=21$. $IQR = Q3-Q1 = 21-2 = 19$.

34. B. $Corr(3X,-Y+2) = Cov(3X,-Y+2)/(SD(3X)*SD(-Y+2))$
 $= (3*(-1)*Cov(X,Y))/(3*1*SD(X)*SD(Y))$
 $= -3*Corr(X,Y)/3 = -Corr(X,Y) = -0.5$

35. C. The number of combinations where all three cards are different is $52C1*39C1*26C1 = 52,728$
The number of combinations where all three are aces is $4C3 = 4$
Probability = $4/52,728 = 1/13,182$.

36. D. $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 0.5+0.8-0.4 = 0.9$

37. C.

38. B. Number of 3 digit numbers = $9*10*10 = 900$. Number of 3 digit number without the digit 3 = $8*9*9 = 648$. Hence 3 digit numbers with at least one digit as 3 is $900-648 = 252$.

39. C.

The number of permutations is $9!/(2!) = 181,440$

40. B.

The number of combinations are $12C5*8C3 = 44,352$.

Data Interpretation

41. C. 54

Total foreign capped players sold = $6+6+5+6+4+6+5+4 = 42$

Total foreign uncapped players sold = $3+2+3+2+5+2+3+3 = 23$

Unsold foreign players = $119 - 42 - 23 = 54$

42. D. Bangalore

Kolkata has 23 players,

Delhi, Chennai, Hyderabad, Jaipur and Ahmedabad has 25 players each,

Mumbai has 26 players and Bangalore has 27 players.

43. B. 89

Number of uncapped players auctioned = $333 - 176 = 157$

Number of uncapped players sold = $3+2+3+2+5+2+3+3 +$

$6+7+5+5+4+7+6+5 = 68$

Number of uncapped players unsold = $157 - 68 = 89$

44. B. 34

Since 91 Indian capped players were sold, to minimize, consider there were none unsold. Then the foreign capped players auctioned is $176 - 91 = 85$. Since the total foreign player auctioned is 119, the uncapped foreign players auctioned is $119 - 85 = 34$.

45. D. 2019

There is a slight decrease in number of associates from 2018 to 2019.

46. A. 515

The graph clearly shows the point just above 500.

47. C. 2019

The steep increase in the line clearly shows that the maximum increase is in 2018 to 2019.

48. C. 483

The associates are around 150 and fellows are little more than 300 as seen in the graph.

49. B. John

$$\text{Leonard: } 18.6 * 12.3 + 19.2 * 11.6 + 20.4 * 14.2 + 16.0 * 12.5 + 15.5 * 13.2 = 11.46$$

$$\text{John: } 18.5 * 12.3 + 17.9 * 11.6 + 17.7 * 14.2 + 18.4 * 12.5 + 17.5 * 13.2 = 11.48$$

$$\text{Michael: } 18.5 * 12.3 + 16.3 * 11.6 + 16.6 * 14.2 + 15.3 * 12.5 + 16.8 * 13.2 = 10.65$$

$$\text{Robin: } 17.1 * 12.3 + 16.6 * 11.6 + 15.6 * 14.2 + 18.4 * 12.5 + 17.4 * 13.2 = 10.84$$

50. A. Monica

$$\text{Monica: } 13.7 * 12.3 + 14.2 * 11.6 + 15.2 * 14.2 + 15.0 * 12.5 + 15.4 * 13.2 = 9.42$$

$$\text{Claire: } 13.6 * 12.3 + 15.8 * 11.6 + 14.5 * 14.2 + 16.7 * 12.5 + 17.4 * 13.2 = 9.95$$

$$\text{Robin: } 17.1 * 12.3 + 16.6 * 11.6 + 15.6 * 14.2 + 18.4 * 12.5 + 17.4 * 13.2 = 10.84$$

$$\text{Michael: } 18.5 * 12.3 + 16.3 * 11.6 + 16.6 * 14.2 + 15.3 * 12.5 + 16.8 * 13.2 = 10.65$$

51. B. Monica, Season 2

$$\text{Monica, Season 1: } 13.7 * 12.3 = 1.69$$

$$\text{Monica, Season 2: } 14.2 * 11.6 = 1.65$$

$$\text{Claire, Season 1: } 13.6 * 12.3 = 1.67$$

$$\text{Claire, Season 2: } 15.8 * 11.6 = 1.83$$

English

52. B. will be relaxing

53. A. if.

54. B. Abundant

55. C. Brief or short lived

56. A. Dull

57. D. Et cetera

58. C. Specifically

59. B. Find out

60. A. The cat sits on the window sill, enjoying the warmth of the sun, and occasionally flicking its tail

61. A. The dog chased the cat around the yard until they both got tired and fell asleep under the tree.

62. D. i, iii, i, respectively.

Logical Reasoning

63. A. group of Lion is a Pride; a group of Dog is a Pack

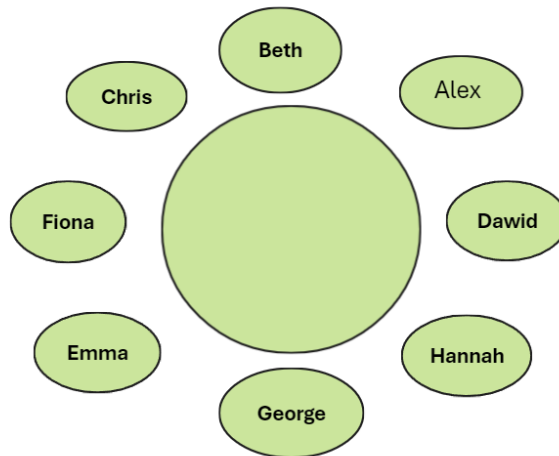
64. D. Reverse the word then take the 2nd alphabet from it to get the code.

WORLD -> DLROW -> FNTQY. SATURN -> NRUTAS -> PTWVCU

65. B. Every hour it moves 30° . So in 5 hours $6 \times 30^\circ = 150^\circ$

66. A. David

67. C. Alex and Beth



68. A. Uncle's father Grandfather, Son of grandfather is uncle, Daughter of Uncle is Sister/Cousin.

69. A. Sunday. After 400 it will be same day as present day i.e. Saturday. A day after Saturday is Sunday.

70. B. 242. Its alternative subtraction series. First 2 is subtracted then 4 is subtracted.